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Stochastic Cooling Enhanced Steady-State Micro-Bunching

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Overview

Introduction

SSMB Scenarios

- Global Microbunching

- Local/Reversible Microbunching

Combine Optical Stochastic Cooling with SSMB

Summary

Introduction

Radiation from a Point Charge and a Charged Particle Beam

- ▶ Single particle radiation

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2\omega^2}{16\pi^3\epsilon_0c^3} \left| \int_{-\infty}^{\infty} [\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \vec{\mathbf{v}})] e^{i\omega(t' - \hat{\mathbf{n}} \cdot \vec{\mathbf{r}}(t')/c)} dt' \right|^2.$$

- ▶ Once the **prescribed motion of an electron** is given, the radiation property is determined.
- ▶ For the radiation of N electrons, the superimposed radiation is

$$\frac{d^2W}{d\omega d\Omega} = \frac{e^2\omega^2}{16\pi^3\epsilon_0c^3} \left| \int_{-\infty}^{\infty} e^{i\omega t'} \hat{\mathbf{n}} \times \left[\hat{\mathbf{n}} \times \left(\sum_{j=1}^N \vec{\mathbf{v}}_j(t') e^{-i\frac{\omega}{c} \hat{\mathbf{n}} \cdot \vec{\mathbf{r}}_j(t')} \right) \right] dt' \right|^2.$$

- ▶ The radiation from an electron beam is determined by **both the electron prescribed motion and the beam phase space distribution**.

Microbunching Enables Coherent Radiation Generation

- ▶ Approximation: we consider only the different phase terms for different particles, and assume the relative change of beam size in the radiator is small. Then

$$\left. \frac{d^2 W}{d\omega d\Omega} \right|_{\text{beam}} = \left. \frac{d^2 W}{d\omega d\Omega} \right|_{\text{point}} N_e^2 |b(\omega)|^2,$$

$$\text{Bunching factor: } b(\omega) = \frac{1}{N_e} \sum_{j=1}^{N_e} e^{-i\frac{\omega}{c} \hat{\mathbf{n}} \cdot \vec{r}_j} = \frac{1}{N_e} \sum_{j=1}^{N_e} e^{-i\frac{\omega}{c} (x_j \sin \theta \cos \varphi + y_j \sin \theta \sin \varphi + z_j)}.$$

- ▶ For relativistic particles, most radiation is in the forward direction, the bunching factor can be further simplified as the Fourier transform of current distribution

$$b(\omega) = \frac{1}{N_e} \sum_{j=1}^{N_e} e^{-i\frac{\omega}{c} z_j} = \int_{-\infty}^{\infty} \rho(z) e^{-i\frac{\omega}{c} z} ds.$$

- ▶ Incoherent radiation: phase randomly distributed, $\langle b(\omega) \rangle = \frac{1}{N_e}$
- ▶ Coherent radiation: radiation from different electrons add in phase, $\langle b(\omega) \rangle \sim 1$

Normal bunch



Incoherent radiation $P \propto N_e$

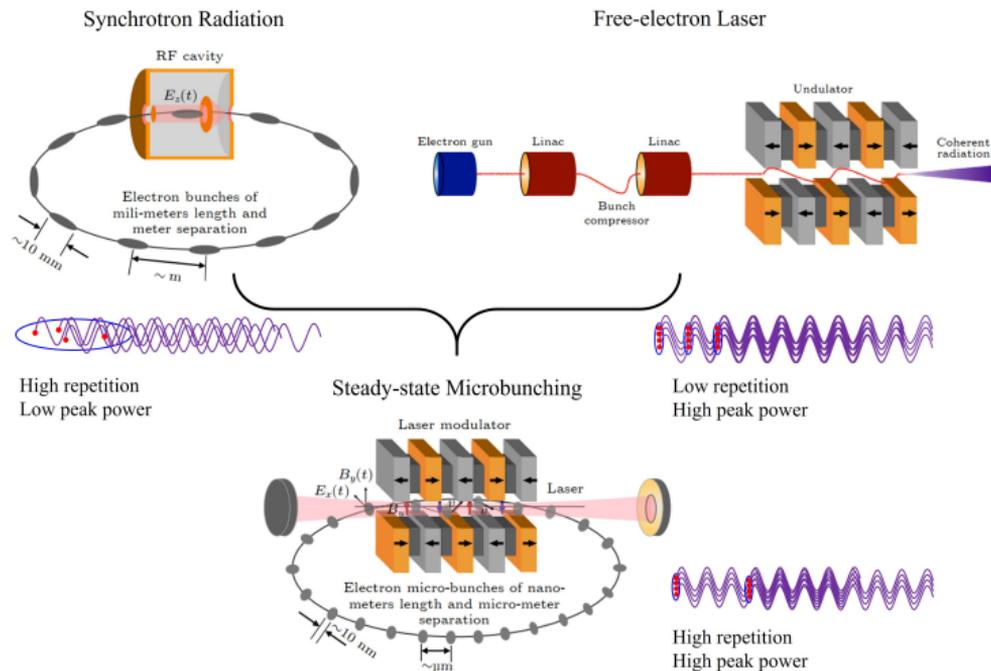
Microbunching



Coherent radiation $P \propto N_e^2$

Steady-State Microbunching (SSMB)²

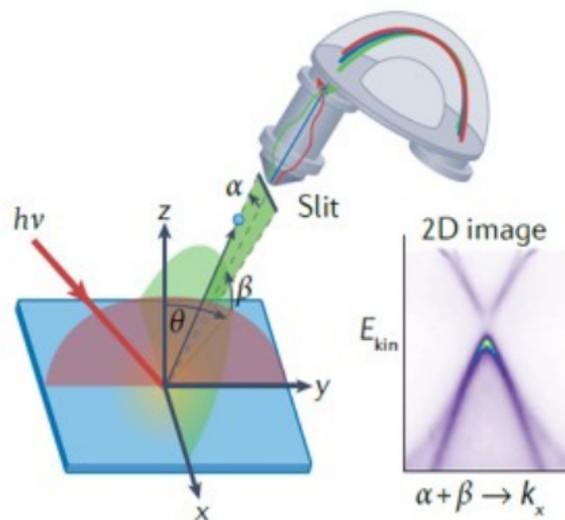
- High-average-power narrow-band radiation, with wavelength ranging from THz to soft X-ray.



²D. F. Ratner and A. W. Chao, Phys. Rev. Lett. 105, 154801 (2010). Deng. X. et al. Nature 2021.

Potential Applications of SSMB

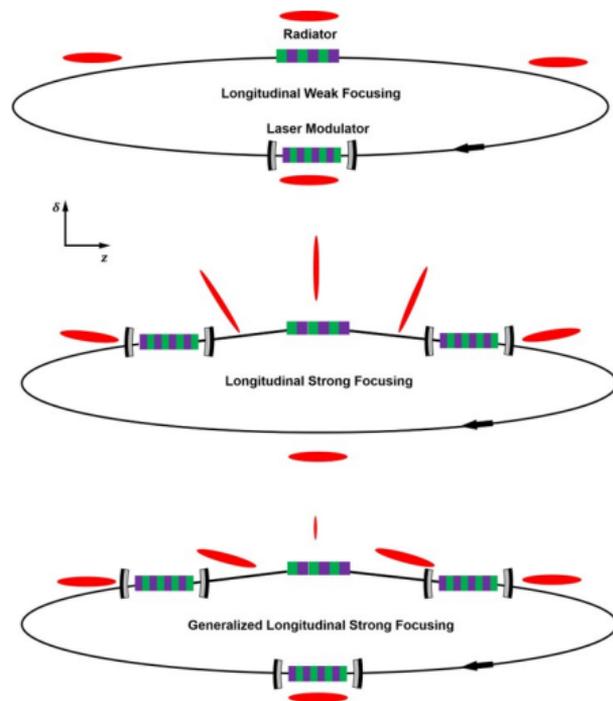
- ▶ High-resolution ARPES
 - ▶ High photon flux: orders of magnitude higher than a conventional synchrotron source
 - ▶ Tunable wavelength
 - ▶ Quasi-continuous waveform output



SSMB Scenarios

Global Microbunching Scenarios of SSMB³

- ▶ There is a laser modulator bunching system in the ring, playing a similar role to RF bunching
 - ▶ Longitudinal weak focusing (LWF):
 $\nu_s \ll 1$, β_z nearly unchanged. Target radiation wavelength $\lambda_R \gtrsim 100$ nm.
 - ▶ Longitudinal strong focusing (LSF):
 $\nu_s \gtrsim 1$, β_z changes significantly. Target radiation wavelength can be as short as $\lambda_R \gtrsim 10$ nm, require a high modulation laser power.
 - ▶ Generalized longitudinal weak focusing (GLSF): transverse-longitudinal coupling, bunch length changes significantly. Target radiation wavelength can be as short as $\lambda_R \gtrsim 10$ nm, with a lower modulation laser power required compared to LSF.



³X. Deng, et al., PRAB 23, 044002 (2020). X. Deng, et al., PRAB 24, 094001 (2021). Y. Zhang, et al., PRAB 24, 090701 (2021). Z. Li, et al., PRAB 26, 110701 (2023). X. Deng, et al., arXiv.2404.06920. PhD Theses of Z. L. Pan, X. J. Deng, Y. Zhang, Z.Z. Li.

Key of Global Microbunching Scenarios of SSMB

- ▶ Longitudinal weak focusing (LWF) and longitudinal strong focusing (LSF): small longitudinal emittance, small longitudinal β -function

$$\epsilon_z \equiv \langle J_{III} \rangle = \frac{C_L \gamma^5}{2c\alpha_{III}} \oint \frac{\beta_z}{|\rho|^3} ds,$$
$$J_{III} \equiv \frac{(z + D'_x x - D_x x')^2 + [\alpha_z (z + D'_x x - D_x x') + \beta_z \delta]^2}{2\beta_z},$$
$$\sigma_z = \sqrt{\epsilon_z \beta_z}.$$

- ▶ Generalized longitudinal weak focusing (GLSF): small vertical emittance, small chromatic \mathcal{H} -function

$$\epsilon_y \equiv \langle J_{II} \rangle = \frac{C_L \gamma^5}{2c\alpha_{II}} \oint \frac{\mathcal{H}_y}{|\rho|^3} ds,$$
$$J_{II} \equiv \frac{(y - D_y \delta)^2 + [\alpha_y (y - D_y \delta) + \beta_y (y' - D'_y \delta)]^2}{2\beta_y},$$
$$\sigma_z = \sqrt{\epsilon_y \mathcal{H}_y}.$$

Theoretical Minimum Longitudinal Emittance

- ▶ Evolution of β_z in a bending magnet:

$$\begin{aligned}\beta_z(\alpha) &\equiv \beta_{55}'''(\alpha) = 2|\mathbf{E}_{III5}(\alpha)|^2 = 2|(\mathbf{B}(\alpha)\mathbf{E}_{III}(0))_5|^2 \\ &= \left(\sin\alpha \frac{\alpha_{z0}}{\sqrt{\beta_{z0}}} D_{x0} + \rho(1 - \cos\alpha) \frac{\alpha_{z0}}{\sqrt{\beta_{z0}}} D'_{x0} + \sqrt{\beta_{z0}} - \rho(-\alpha + \sin\alpha) \frac{\alpha_{z0}}{\sqrt{\beta_{z0}}} \right)^2 \\ &\quad + \left(-\sin\alpha \frac{1}{\sqrt{\beta_{z0}}} D_{x0} - \rho(1 - \cos\alpha) \frac{1}{\sqrt{\beta_{z0}}} D'_{x0} + \rho(-\alpha + \sin\alpha) \frac{1}{\sqrt{\beta_{z0}}} \right)^2.\end{aligned}$$

- ▶ The theoretical minimum emittance is realized when in each bending magnet center $\alpha_{z0} = 0$, $\beta_{z0} \approx \frac{\rho\theta^3}{120\sqrt{7}}$, $D_{x0} \approx -\frac{\rho\theta^2}{40}$, $D'_{x0} = 0$, and $\epsilon_{z,\min} = C_q \frac{\gamma^2}{J_z} \frac{\theta^3}{60\sqrt{7}}$, with $C_q = 3.8319 \times 10^{-13}$ m for electrons.
- ▶ It is difficult to make the optimal conditions given above satisfied in all bending magnets. A more practical way to realize small longitudinal emittance is to letting each half of the bending magnet is isochronous ⁴

$$\alpha_{z0} = 0, \beta_{z0} \approx \frac{\rho\theta^3}{12\sqrt{210}}, D_{x0} \approx -\frac{\rho\theta^2}{24}, D'_{x0} = 0, \epsilon_{z,\min,ISO} = C_q \frac{\gamma^2}{J_z} \frac{\theta^3}{6\sqrt{210}}.$$

⁴Deng, et al., arXiv.2404.06920. accepted by NST.

Local/Reversible Microbunching Scenarios of SSMB ⁵

- ▶ Microbunching occurs only locally at the radiator. Laser modulation and reverse modulation. Conventional RF bunched, or coasting beam outside the insertion.

- ▶ HGHG SSMB: HGHG + reverse HGHG, key parameter is the energy spread

$$\delta = \delta + A \sin(k_L z), \quad z = z + R_{56} \delta,$$

$$b_n = J_n(-nk_L R_{56} A) \exp \left[-(nk_L R_{56} \sigma_\delta)^2 / 2 \right].$$

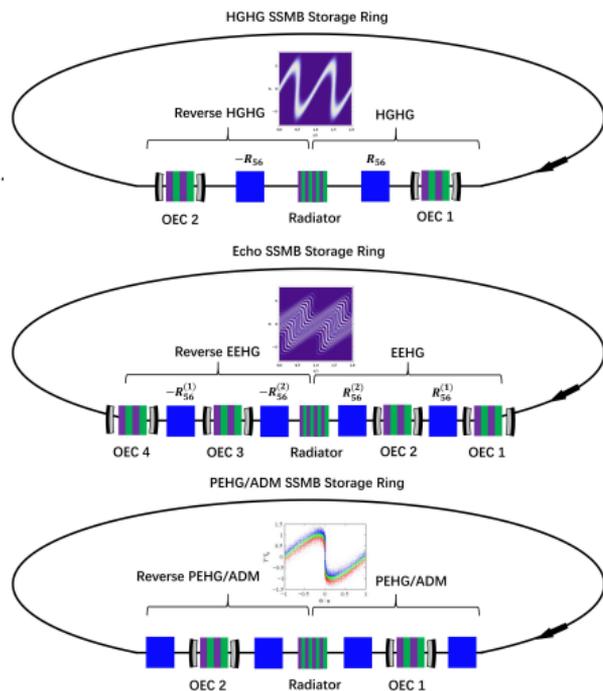
- ▶ Echo SSMB: EEHG + reverse EEHG, key parameter is the energy spread

$$b_n \approx \frac{0.67}{(n+1)^{1/3}} J_1(\zeta y) \exp \left[-\zeta^2 / 2 \right],$$

$$\zeta \equiv -k_L \left(R_{56}^{(1)} - n R_{56}^{(2)} \right) \sigma_\delta, \quad y \equiv A_1 / \sigma_\delta.$$

- ▶ PEHG/ADM SSMB: PEHG + reverse PEHG, key parameter is y -emittance

$$b_n = J_n(-nk_L R_{56} A) \exp \left[-(nk_L)^2 \epsilon_y \mathcal{H}_y / 2 \right]$$



⁵Deng, Pan, Zhao, Li, Chao & Tang, Reversible Microbunching in an Electron Storage Ring, submitted and to be published, 2025.

Key of Local/Reversible Microbunching: Perfect Modulation Cancellation

- ▶ Assume there is a longitudinal coordinate deviation Δz from its ideal location at the reverse modulation, then there will be a leakage of modulation

$$\Delta\delta = -A \{ \sin[k_L(z + \Delta z)] - \sin(k_L z) \}.$$

- ▶ Single-pass growth of energy spread or vertical emittance should be note larger than the natural diffusion per turn to keep the equilibrium parameters

$$\Delta\sigma_\delta^2 = \frac{(Ak_L)^2}{2} \sigma_{\Delta z}^2 \lesssim \frac{\sigma_\delta^2}{N_{z,damping}/4}, \quad \Delta\epsilon_y = \frac{(Ak_L)^2}{2} \sigma_{\Delta z}^2 \mathcal{H}_{yM} \lesssim \frac{\epsilon_y}{N_{y,damping}/2}.$$

- ▶ A **universal criterion** of tolerance of Δz , applies for **all** reversible microbunching schemes ($N_{y,damping} \approx 2N_{z,damping}$)⁶

$$\sigma_{\Delta z} \lesssim \frac{\lambda_R}{\pi} \sqrt{\frac{2}{N_{z,damping}}}.$$

For instance, if $\lambda_R = 10$ nm, and $N_{z,damping} = 2000$, we need $\sigma_{\Delta z} \lesssim 0.1$ nm, which is close to the lower limit in our present lattice optimization.

⁶Deng, Pan, Zhao, Li, Chao, & Tang, Reversible Microbunching in an Electron Storage Ring, submitted and to be published, 2025.

Non-perfect Modulation Cancellation May Arise from Various Issues ⁷

- ▶ Quantum excitation
- ▶ Intrabeam scattering
- ▶ Linear optics mismatch
- ▶ Lattice nonlinearity
- ▶ Coherent undulator radiation
- ▶ Laser phase noises
- ▶ etc.

⁷Deng, Pan, Zhao, Li, Chao, & Tang, Reversible Microbunching in an Electron Storage Ring, submitted and to be published, 2025.

Motivations of an Enhanced Damping Rate in SSMB

- ▶ In all SSMB schemes, microbunching globally or locally, a faster damping can
 - ▶ Lower the equilibrium beam emittances or energy spread
 - ▶ Push the potential of SSMB to an even shorter wavelength range
 - ▶ Mitigate the lattice design and other technical challenges for example the required laser power level or phase noise level
 - ▶ Fight again intrabeam scattering and stabilize other collective beam instabilities
- ▶ SSMB is a low-energy storage ring, typically several hundred MeVs
 - ▶ For effective laser modulation: $A \propto \frac{\sqrt{P_L L_u}}{\gamma^2}$, maximal optical enhancement cavity power at present is about 700 kW (refer to the talk of A. Martens in this workshop)
 - ▶ For small equilibrium emittances and energy spread: $\epsilon_z \propto \gamma^2 \theta^3$, $\sigma_\delta \propto \gamma$
- ▶ The laser modulator induced energy modulation strength

$$A = \frac{eV_L}{E_0} = \sqrt{\frac{4P_L Z_0}{\lambda_L}} \frac{eK}{\gamma^2 m_e c^2} [JJ] \sqrt{Z_R} \tan^{-1} \left(\frac{L_u}{2Z_R} \right),$$

with $\chi = K^2 / (4 + 2K^2)$ and $[JJ] \equiv J_0(\chi) - J_1(\chi)$.

- ▶ A low beam energy means a slow natural radiation damping. For example, a 300 MeV electron with 1.25 T bending magnet losses about 896 eV per turn, which means $N_{z,\text{damping}} \approx 3.35 \times 10^5$. $U_0 = C_\gamma E_0^4 / \rho$, $N_{z,\text{damping}} \approx E_0 / U_0$.

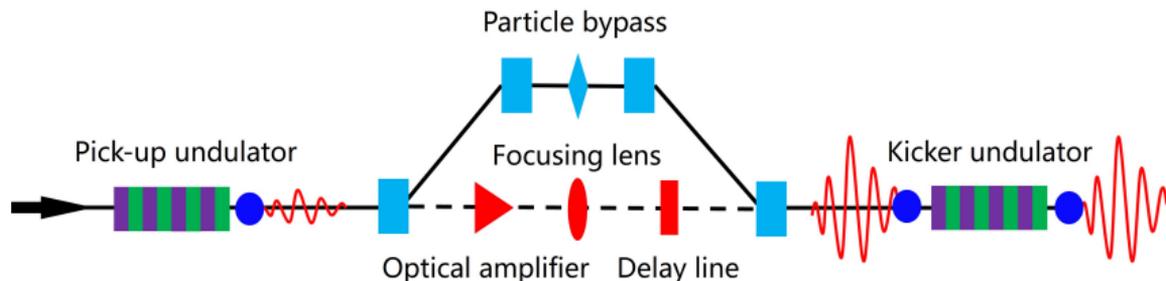
Why Optical Stochastic Cooling (OSC)

- ▶ Application of damping wigglers:
 - ▶ While damping wiggler is a natural solution for a faster damping, it requires a high radiation power loss. For example, if we aim at $N_{z,damping} = 2000$ for $E_0 = 600$ MeV, then we need $U_{0wiggler} = 300$ keV (50 times stronger than natural damping). Then the overall radiation power will be 150 kW for a 500 mA beam.
 - ▶ Such a large loss will lower the overall efficiency of such an SSMB light source. Also it take extra efforts to handle such high-power radiation.
 - ▶ It also increases the overall size of such a light source, even if superconducting wigglers is used (80 m total length for 4 T wigglers in the above case).
 - ▶ Damping wiggler has also quantum excitation contribution. A too strong damping wiggler may increase the beam emittance and energy spread.
- ▶ The potential advantages of applying optical stochastic cooling (OSC):
 - ▶ A more compact storage ring
 - ▶ A higher overall efficiency from wall electricity to light delivered to users
 - ▶ Smaller equilibrium emittances and energy spread, enhancing the potential of SSMB
 - ▶ Fight against collective effects like intrabeam scattering

Basics of Optical Stochastic Cooling

Some Basics of Optical Stochastic Cooling ⁹

- ▶ Cooling mechanism: use each electron's radiation generated in the pick-up undulator to correct its own momentum deviation in the kicker undulator. The kicks of nearby electrons' radiations (in slippage length) is a heating effect.
- ▶ Bandwidth: the shorter the radiation slippage length $N_u \lambda_R$, thus the better ability to identify each electron, the larger the bandwidth. Optical wavelength is much shorter than that of the microwave, thus can realize a much larger bandwidth.
- ▶ Mixing: update of nearby particles in the slippage length, or the overlap of Schottky bands in the feedback system bandwidth.
- ▶ The mechanism of OSC has recently been experimentally demonstrated in IOTA storage ring ⁸



⁸Jarvis. et al. Nature, 2022. Lebedev. et al. J. Instrum. 2021.

⁹Van der Meer 1984, Mikhailichenko and Zolotarev, PRL, 1993. Zolotarev and Zholents, PRE, 1994.

Some Basics of Optical Stochastic Cooling ¹¹

- ▶ Damping speed: assume perfect mixing, the theoretical maximal damping rate (appropriate amplification applied) is $\frac{1}{\tau} = \frac{W}{N}$, with W the bandwidth and N the total number of particles in the ring. A factor of 2 smaller if betatron or synchrotron oscillation is considered.
- ▶ For the case of OSC we have

$N_{z,damping,optimal}$ = (twice) the number of electrons in the radiation slippage length

- ▶ For a fast damping in OSC, a short radiation wavelength is required, for example EUV ¹⁰. Put in some numbers to get a feeling, if $\lambda_R = 200$ nm, $N_u = 5$, $I_P = 1$ A, then the number of electrons in the slippage length is $N_s = \frac{I_P N_u \lambda_R}{ec} = 2.08 \times 10^4$, which is one order of magnitude smaller than the natural damping time in our previous 300 MeV storage ring example $N_{z,damping} \approx 3 \times 10^5$.
- ▶ OSC is a natural longitudinal damping mechanism. Damping in other directions can be accomplished by coupling optics manipulation.

¹⁰Zholents, et al. PRAB, 2021.

¹¹Van der Meer 1984, Mikhailichenko and Zolotorev, PRL, 1993. Zolotorev and Zholents, PRE, 1994. Jarvis. et al. Nature, 2022. Lebedev. et al. J. Instrum. 2021.

Damping Rate in Linear Approximation

- ▶ Symplectic transfer matrix from the pick-up s_1 to kiker s_2 undulator: $\mathbf{X}_2 = \mathbf{R}\mathbf{X}_1$.
- ▶ Self radiation corrective kick:

$$\Delta\delta = -A \sin(k_R \Delta z)$$

with $k_R = 2\pi/\lambda_R$ the wavenumber of OSC radiation, and $\Delta z = R_{51}x_1 + R_{52}x_1' + R_{53}y_1 + R_{54}y_1' + R_{56}\delta_1$, $R_{55} = 1$ assumed.

- ▶ Using Hamiltonian perturbation theory, we have the sum rule for the OSC damping rates of three eigen modes

$$\alpha_{I,0} + \alpha_{II,0} + \alpha_{III,0} = \frac{Ak_R R_{56}}{2}.$$

The subscript 0 is used here for the damping rates to denote that they are calculated by linearizing the energy kick around the zero-crossing phase.

- ▶ The OSC damping rate of each eigen mode in a general coupled lattice is ¹²

$$\alpha_{I,0} = -\frac{Ak_R}{2} \left(R_{51}\hat{\beta}_{51}^I + R_{52}\hat{\beta}_{52}^I + R_{53}\hat{\beta}_{53}^I + R_{54}\hat{\beta}_{54}^I + R_{56}\hat{\beta}_{56}^I \right),$$

$$\alpha_{II,0} = -\frac{Ak_R}{2} \left(R_{51}\hat{\beta}_{51}^{II} + R_{52}\hat{\beta}_{52}^{II} + R_{53}\hat{\beta}_{53}^{II} + R_{54}\hat{\beta}_{54}^{II} + R_{56}\hat{\beta}_{56}^{II} \right),$$

$$\alpha_{III,0} = -\frac{Ak_R}{2} \left(R_{51}\hat{\beta}_{51}^{III} + R_{52}\hat{\beta}_{52}^{III} + R_{53}\hat{\beta}_{53}^{III} + R_{54}\hat{\beta}_{54}^{III} + R_{56}\hat{\beta}_{56}^{III} \right).$$

Generalized β functions are defined based on the eigenvectors of the one-turn map.

¹²Deng, FLS2023-TU4P30.

Amplitude-dependent Damping Rate

- ▶ Considering the sinusoidal nature of the kick waveform, the damping rates will be different for particles with different betatron or synchrotron amplitudes,

$$\alpha_I = 2\alpha_{I,0} \frac{J_1(k_R a_I) J_0(k_R a_{II}) J_0(k_R a_{III})}{k_R a_I}, \quad \alpha_{II} = 2\alpha_{II,0} \frac{J_0(k_R a_I) J_1(k_R a_{II}) J_0(k_R a_{III})}{k_R a_{II}},$$
$$\alpha_{III} = 2\alpha_{III,0} \frac{J_0(k_R a_I) J_0(k_R a_{II}) J_1(k_R a_{III})}{k_R a_{III}},$$

with J_n the n -th order Bessel function of the first kind, and

$$a_I = \sqrt{2J_I [\beta_{11}^I R_{51}^2 + 2\beta_{12}^I R_{51} R_{52} + \beta_{22}^I R_{52}^2]}, \quad a_{II} = \sqrt{2J_{II} [\beta_{33}^{II} R_{53}^2 + 2\beta_{34}^{II} R_{53} R_{54} + \beta_{44}^{II} R_{54}^2]},$$
$$a_{III} = \sqrt{2J_{III} [\beta_{55}^{III} R_{55}^2 + 2\beta_{56}^{III} R_{55} R_{56} + \beta_{66}^{III} R_{56}^2]},$$

where $J_{I,II,III}$ are the generalized Courant-Snyder invariants of the particle.¹³

- ▶ The first roots of $J_0(x)$ and $J_1(x)$ are $\mu_{01} \approx 2.405$ and $\mu_{11} \approx 3.83$. The range of oscillation amplitude which gives a positive damping rate is called cooling range. Usually we need at least 3σ cooling range to damp most particles in the beam.

¹³Deng, FLS2023-TU4P30.

Energy Heating induced by Other Particles' Radiation

- ▶ Energy heating due to radiation kicks of nearby particles within the slippage length ¹⁴

$$\Delta\delta = \begin{cases} -A \frac{2\pi N_u - |\phi|}{2\pi N_u} \sin \phi, & |\phi| \leq 2\pi N_u, \\ 0, & |\phi| > 2\pi N_u. \end{cases}$$

$$\frac{d\overline{\Delta\delta^2}}{dn} = \lambda(s) \frac{N_u \lambda_R}{3} \left(1 - \frac{3}{8\pi^2 N_u^2} \right) A^2.$$

- ▶ For a Gaussian bunch, if only OSC damping and diffusion are considered,

$$\frac{d\sigma_\delta^2}{dn} = -2\alpha_{\text{LOSC}}\sigma_\delta^2 + \frac{I_P}{ec} \frac{N_u \lambda_R}{6} A^2,$$

with $\alpha_{\text{LOSC}} = \frac{Ak_R R_{56}}{2}$, the equilibrium energy spread is $\sigma_{\delta\text{OSC}0} = \sqrt{\frac{\frac{I_P}{ec} \frac{N_u \lambda_R}{6} A^2}{\alpha_{\text{LOSC}}}}$.

¹⁴Lebedev. et al. J. Instrum. 2021.

Equilibrium Energy Spread

- ▶ If we consider the combined affects of radiation damping, quantum excitation and other diffusion,

$$\frac{d\sigma_\delta^2}{dn} = -2\alpha_{LRD}\sigma_\delta^2 + 2\alpha_{LRD}\sigma_{\delta 0}^2 + \Delta\sigma_\delta^2/2,$$

The equilibrium energy spread is given by $\sigma_\delta = \sigma_{\delta 0} \sqrt{1 + \frac{\Delta\sigma_\delta^2}{4\alpha_{LRD}}}$, with $\sigma_{\delta 0} = \sqrt{\frac{C_q}{J_s} \frac{\gamma^2}{\rho}}$ the natural energy spread given by radiation damping and quantum excitation.

- ▶ If we want OSC to be effective, then we need the equilibrium energy spread given by OSC alone be smaller than that determined without the OSC.
- ▶ If we consider the combined affects of OSC, radiation damping, quantum excitation and other diffusion,

$$\frac{d\sigma_\delta^2}{dn} = -2(\alpha_{LOSC} + \alpha_{LRD})\sigma_\delta^2 + \frac{I_P}{ec} \frac{N_u \lambda_R}{6} A^2 + 2\alpha_{LRD}\sigma_{\delta 0}^2 + \Delta\sigma_\delta^2/2,$$

The new equilibrium is $\sigma_{\delta OSC} = \sqrt{\frac{\frac{I_P}{ec} \frac{N_u \lambda_R}{6} A^2 + 2\alpha_{LRD}\sigma_{\delta 0}^2 + \Delta\sigma_\delta^2/2}{2(\alpha_{LOSC} + \alpha_{LRD})}}$.

- ▶ There is an optimal A to minimize the equilibrium energy spread.

Radiation Kick Strength

- ▶ The radiation kick strength is determined by the details of pick-up, kicker, optical system and amplifier if there is.
- ▶ Assume identical pick-up and kicker planar undulator. Assume the optical system is refractive. Assume perfect linear amplifier. The radiation kick strength is ¹⁵

$$A = \frac{1}{4\pi\epsilon_0} \frac{(e\gamma K k_u)^2}{3\gamma m_e c^2} 2L_u [JJ] F_h(K, \gamma\theta_m) \sqrt{G},$$

with θ_m the angular acceptance of the focusing lens, G the radiation power amplification factor and

$$F_h(K, \infty) \approx \frac{1}{1 + 1.13K^2 + 0.04K^3 + 0.37K^4}, \quad 0 \leq K \leq 4.$$

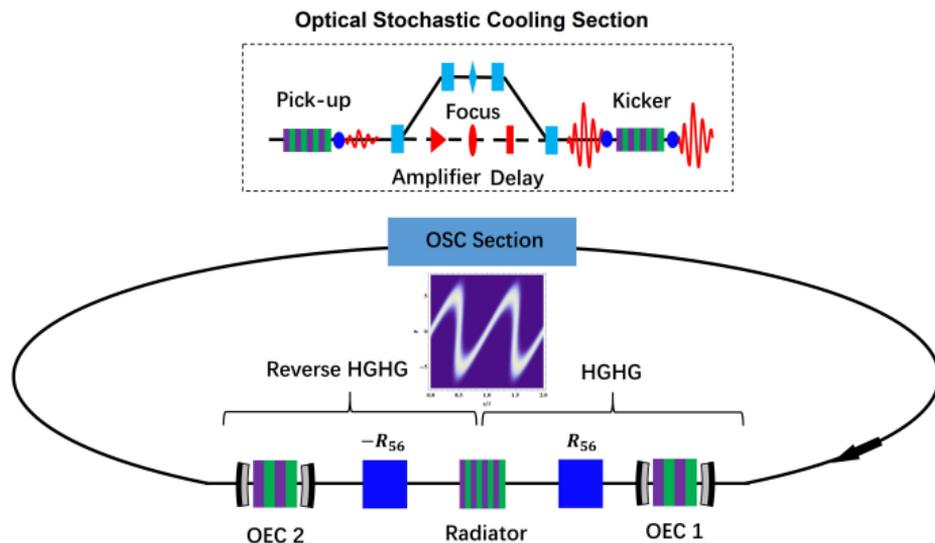
- ▶ Note that for radiation wavelength shorter than 200 nm, there may be no appropriate amplifier available.

¹⁵Lebedev. et al. J. Instrum. 2021.

OSC + Reversible/Local Microbunching

“OSC + HGHG SSMB” as An Example Combination

- ▶ Coasting beam in the storage ring, microbunching only at the radiator
- ▶ Mixing condition: $|\sigma_\delta \eta C_0| \gg N_u \lambda_R$, easy to satisfy in reversible microbunching schemes since **we do not require a small phase slippage factor η**
- ▶ RF cavity or induction linac can be used for energy compensation. In our example, we use induction linac to accelerate a long coasting beam in the ring, 1 A peak current, 500 mA average current, 50% duty factor. The edges of the acceleration waveform can be used to form a barrier bucket



An Example 1 kW 100 nm OSC-HGHG-SSMB Radiation Source

Para.	Value	Description	Para.	Value	Description
E_0	300 MeV	Beam energy	$\sigma_{\Delta z}$	0.2 nm	rms Δz between 2 laser modulators
C_0	50 m	Circumference	$\Delta\sigma_{\delta,ins}^2$	1.79×10^{-12}	Growth of σ_{δ}^2 per pass of the insertion
η	1×10^{-3}	Phase slippage factor	$\Delta\sigma_{\delta,OSC}^2$	1.85×10^{-12}	Growth of σ_{δ}^2 per pass of OSC
I_P	1 A	Peak current	$\Delta\sigma_{\delta,QE}^2$	1×10^{-12}	Growth of σ_{δ}^2 from quantum excitation
f	50%	Beam filling factor	λ_{RO}	266 nm	OSC radiation wavelength
I_A	0.5 A	Average current	λ_u	3 cm	OSC undulator period
B_{ring}	1.25 T	Bending field of dipoles	B_0	1.14 T	OSC undulator magnetic field
U_0	896 eV	Radiation loss per turn in dipoles	K_u	3.2	OSC undulator parameter
$\sigma_{\delta 0}$	2.87×10^{-4}	Natural energy spread	$N_u = 5$	5	Number of undulator period
$\tau_{\delta RD}$	55.8 ms	Natural longitudinal R.D. time	L_u	15 cm	OSC undulator length
$\sigma_{\delta NoOSC}$	$> 4.8 \times 10^{-4}$	Energy spread with other diffusion	N_s	2.77×10^4	No. of electrons in a slippage length
$\frac{\alpha_{LOSC}}{\alpha_{LRD}}$	10	Ratio of OSC and R.D. damping	G	178	Power amplification factor
$\sigma_{\delta OSC}$	1.9×10^{-4}	Energy spread with OSC	R_{56}	179 μm	R_{56} between two undulators
$\tau_{\delta OSC}$	5.1 ms	Damping time with OSC	$\frac{\mu 01}{k_{OSC} R_{56} \sigma_{\delta 0}}$	3	Cooling range
$N_{z,damping}$	3×10^4	Damping time in revolution number	λ_R	100 nm	Radiation wavelength
$\tau_{\delta,IBS}$	≈ 100 ms	Longitudinal IBS diffusion time	b_8	0.1	Bunching factor
λ_L	800 nm	Modulation laser wavelength	ϵ_{\perp}	6 nm	Transverse emittance
V_L	360 kV	Laser induced modulation voltage	σ_{\perp}	110 μm	Transverse beam size at radiator
A	1.2×10^{-3}	Laser induced modulation strength	λ_u	2 cm	Radiator undulator period
λ_u	4.5 cm	Modulator undulator period	B_0	1.18 T	Radiator peak magnetic field
B_0	1.13 T	Modulator peak magnetic field	K_u	2.2	Undulator parameter
K_u	4.74	Undulator parameter	N_u	300	Number of undulator period
L_u	3.15 m	Modulator undulator length	L_u	6 m	Modulator undulator length
R_y	$L_u/3$	Rayleigh length	P_P	1 kW	Peak radiation power
$P_{L,P}$	2 MW	Modulation laser peak power	P_A	0.5 kW	Average radiation power
$P_{L,A}$	1 MW	Modulation laser average power			

A simplified schematic: 10 m arc + 15 m reversible HGHG insertion + 10 m arc + 15 m OSC cooling section

Coherent Undulator Radiation

- ▶ The coherent planar undulator radiation power at the odd- H -th harmonic from a transversely-round electron beam is ¹⁶

$$P_{H,\text{peak}}[\text{kW}] = 1.183 N_u H \chi [JJ]_H^2 FF_{\perp}(S) |b_{z,H}|^2 I_P^2 [\text{A}],$$

where N_u is the number of undulator periods,

$[JJ]_H^2 = \left[J_{\frac{H-1}{2}}(H\chi) - J_{\frac{H+1}{2}}(H\chi) \right]^2$, with $\chi = \frac{K^2}{4+2K^2}$, and the transverse form factor is

$$FF_{\perp}(S) = \frac{2}{\pi} \left[\tan^{-1} \left(\frac{1}{2S} \right) + S \ln \left(\frac{(2S)^2}{(2S)^2 + 1} \right) \right],$$

with $S = \frac{\sigma_{\perp}^2 \omega}{L_u c}$ is the diffraction parameter and σ_{\perp} the rms transverse electron beam size, $b_{z,H}$ is the bunching factor at the H -th harmonic, and I_P is the peak current.

- ▶ The radiation power of an equivalent helical undulator can be a factor of 2 larger. Helical undulator is assumed in the above table.

¹⁶Deng. Springer Thesis, 2023.

Intrabeam Scattering

- ▶ To minimize the IBS diffusion rate, we have used a transversely round electron beam, with a large transverse emittance of $\epsilon_{\perp} = 6$ nm.
- ▶ The IBS diffusion of energy spread for a transversely round beam ($\epsilon_x = \epsilon_y$) is ¹⁷

$$\tau_{\delta, \text{IBS}}^{-1} \approx \frac{\Psi_0 I_P r_e^2 L_C}{8e\gamma^3 \sigma_{\delta}^2 \langle \sigma_x \rangle \epsilon_{\perp}},$$

where Ψ_0 is a constant depending on the lattice optics around the ring.

- ▶ Here for an order of magnitude estimation, we put in some numbers, $\Psi_0 = 1$, $L_C = 10$ m, average transverse beam size around the ring $\langle \sigma_x \rangle = \sqrt{6 \text{ nm} \times 10 \text{ m}} \approx 250 \mu\text{m}$, $\epsilon_{\perp} = 6$ nm, $\sigma_{\delta} = 1.9 \times 10^{-4}$, $I_P = 1$ A,

$$\tau_{\delta, \text{IBS}} = 169 \text{ ms},$$

which is more than one order of magnitude longer than the OSC damping time.

- ▶ For the transverse dimension (full coupling assumed)

$$\tau_{\perp, \text{IBS}} = \frac{\epsilon_{\perp}}{\sigma_{\delta}^2 \langle \mathcal{H}_{\perp} \rangle} \tau_{\delta, \text{IBS}}.$$

As long as $\langle \mathcal{H}_{\perp} \rangle \lesssim 0.2$ m, the IBS is acceptable for the transverse dimensions.

¹⁷V. Lebedev, Accelerator Handbook 2013, pp.155-159.

Microwave Instability

- ▶ The CSR-induced microwave instability threshold bunch current with shielding for a coasting beam is ¹⁸

$$I_b = \frac{3\sqrt{2}\gamma\eta\sigma_\delta^2 I_{Alf} L_b}{\pi^{\frac{3}{2}} h},$$

where $I_{Alf} = 17$ kA is the Alfvén current.

- ▶ Put in the numbers in our table: $E_0 = 300$ MeV, $\eta = 1 \times 10^{-3}$, $L_b = C_0/2 = 25$ m, $\sigma_{\delta 0} = 2 \times 10^{-4}$, assume gap of two parallel metal plates $h = 1$ cm, then

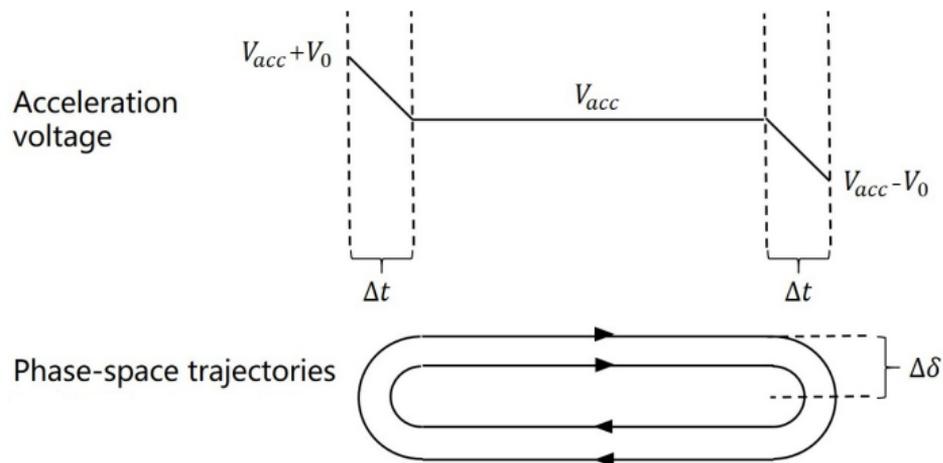
$$I_b = 0.76 \text{ A},$$

which is larger than the 0.5 A average beam current we have applied.

¹⁸Cai, IPAC2011-FRXAA01.

Induction Linac

- ▶ Repetition rate: 6 MHz ($C_0 = 50$ m, $T_0 = 166.7$ ns)
- ▶ Acceleration voltage: $V_{acc} \approx 2$ kV
- ▶ Duty factor: 50%
- ▶ Bucket half-height: $\Delta\delta = \sqrt{\frac{eV_0/E_0}{\eta C_0/c\Delta t}}$. If $E_0 = 300$ MeV, $\eta = 1 \times 10^{-3}$, $C_0 = 50$ m, $\Delta t = 20$ ns, then to realize a half-bucket height $\Delta\delta = 0.02$ ($100\sigma_\delta$ if $\sigma_\delta = 2 \times 10^{-4}$), we need $V_0 = 1$ kV, which should be feasible in practice.



More Issues to be Studied

- ▶ Amplifier available? Optimize the parameters set to lower the required amplification factor
- ▶ Phase locking of two optical enhancement cavities
- ▶ Synchronization between electron and its radiation in the OSC section
- ▶ Other physical processes that may result in growth of equilibrium energy spread

Summary

- ▶ Both OSC and SSMB have great potential, here we propose to combine them for an even brighter and longer future.
- ▶ The application of OSC in reversible/local microbunching SSMB scenarios is feasible and we have presented an example of 1 kW 100 nm radiation source based on this idea, using parameters all reachable using present technology. Such a compact source (circumference ~ 50 m) can be built in universities and institutes with a reasonable cost, and be useful for basic science research.
- ▶ A more ambitious application of OSC in Echo SSMB could push the coherent radiation wavelength to soft X-ray, for example 10 nm.
- ▶ The work on application of OSC in Global Microbunching SSMB scenarios is ongoing and will be reported in the future.

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