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Collective Effects in Muon Ionization Cooling

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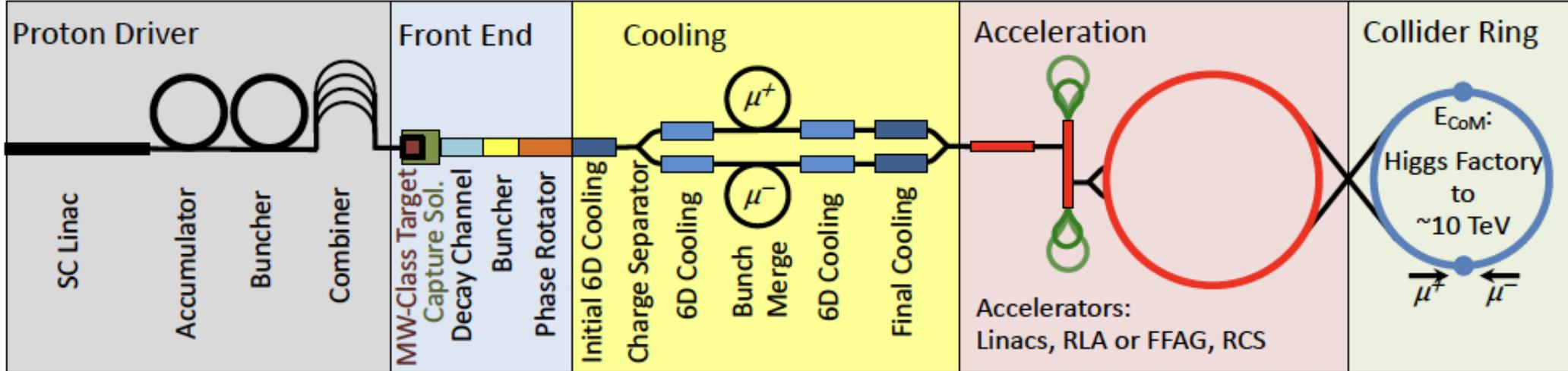
Contents

- Overview of the muon cooling channel for a muon collider
  - Determine beam parameters to achieve the desired luminosity
- Mechanism of beam-induced plasma formation within the absorber material
  - Evaluate focusing strength with collective fields
- Estimation of collective electromagnetic fields in ionization cooling
  - Evaluate focusing strength with collective fields
- Effect of plasma on stopping power
  - Increasing stopping power using plasma
- Summary

Overview of the Muon Collider Concept

Proton beam-based muon source and cooling chain

- Our story starts from luminosity...

Number of particles cross at an Interaction Point (IP) per second

$$\mathcal{L} = \frac{N_{\mu^+} \cdot N_{\mu^-} \cdot f \cdot n_b}{4\pi \cdot \sigma_x \cdot \sigma_y} > 10^{34} \quad \text{cm}^{-2} \text{ s}^{-1}$$

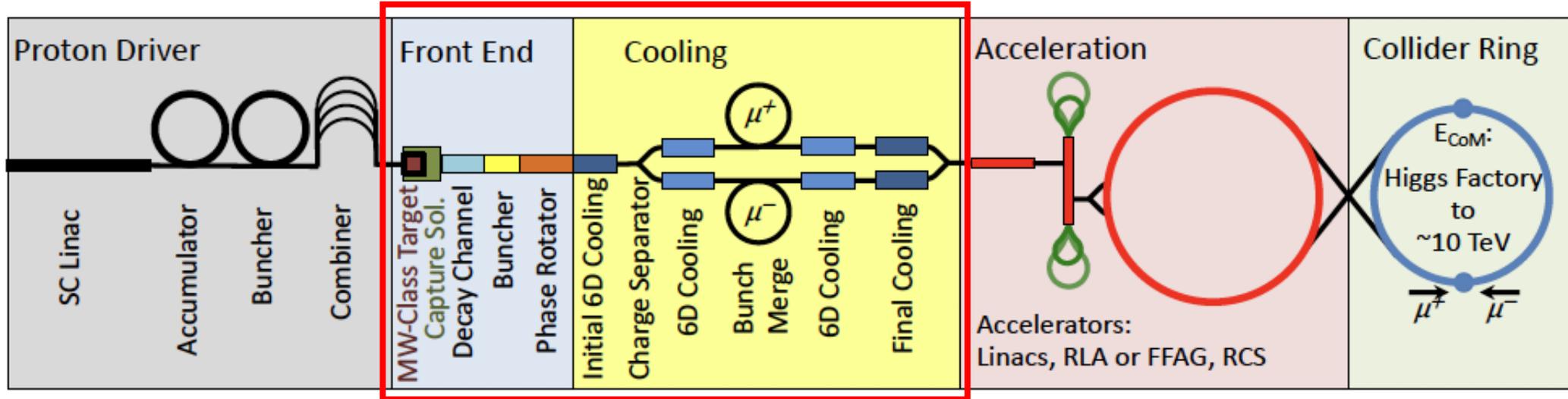Beam crossing area at IP

N: Num of μ per bunch ($\sim 10^{12}$)
f: Bunch revolution (~ 1000)
n_b: Repetition rate (~ 5 Hz)
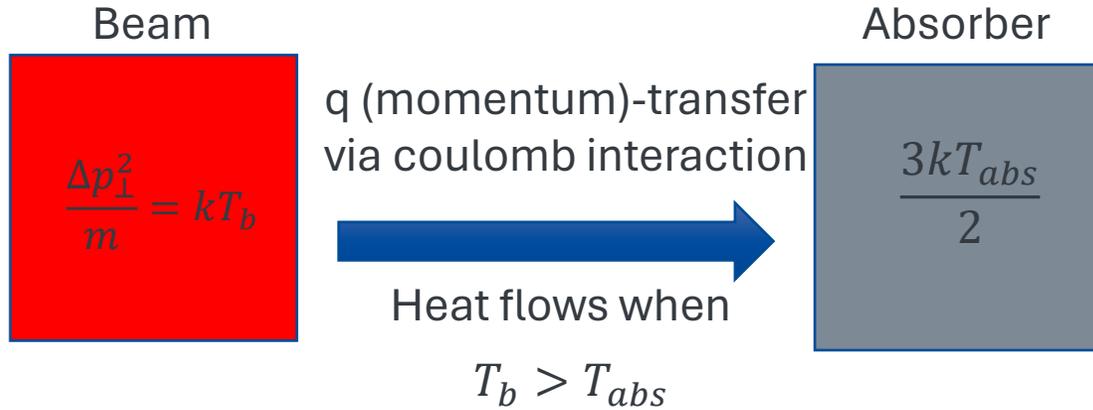σ: RMS Beam spot size at collision ($\sim 10^{-4}$ cm)

Muon Beam Cooling Requirements
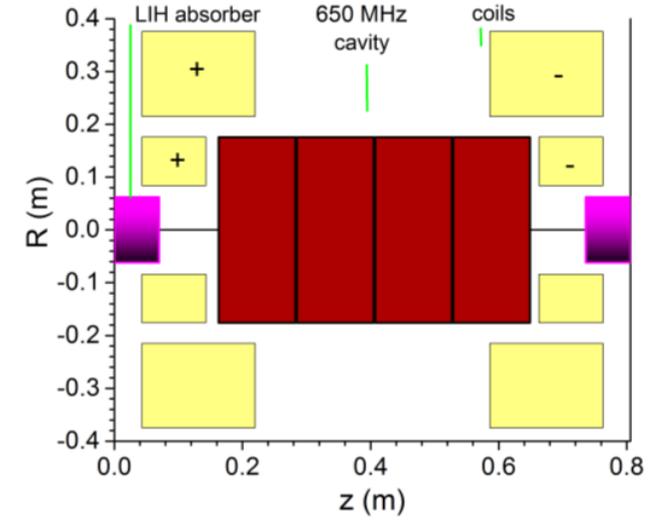
Proton beam-based muon source and cooling chain

- Required cooled muon beam for each sign: $n_\mu \sim 10^{12-13}$, $\varepsilon_{t,n} \sim 20$ mrad
  - Starting proton beam intensity: $\sim \mathbf{10^{15}}$ **protons/spill**
  - Required $\mu/proton$ yield: $> \mathbf{0.1}\ \boldsymbol{\mu/p}$
    - *> 10 % capture and transmission efficiencies in the Front End*
  - Acceptable μ transmission efficiency in cooling: $\mathbf{10^{12-13}}$ **muons/bunch**
    - *Transmission efficiency through whole cooling channels > 1 %*
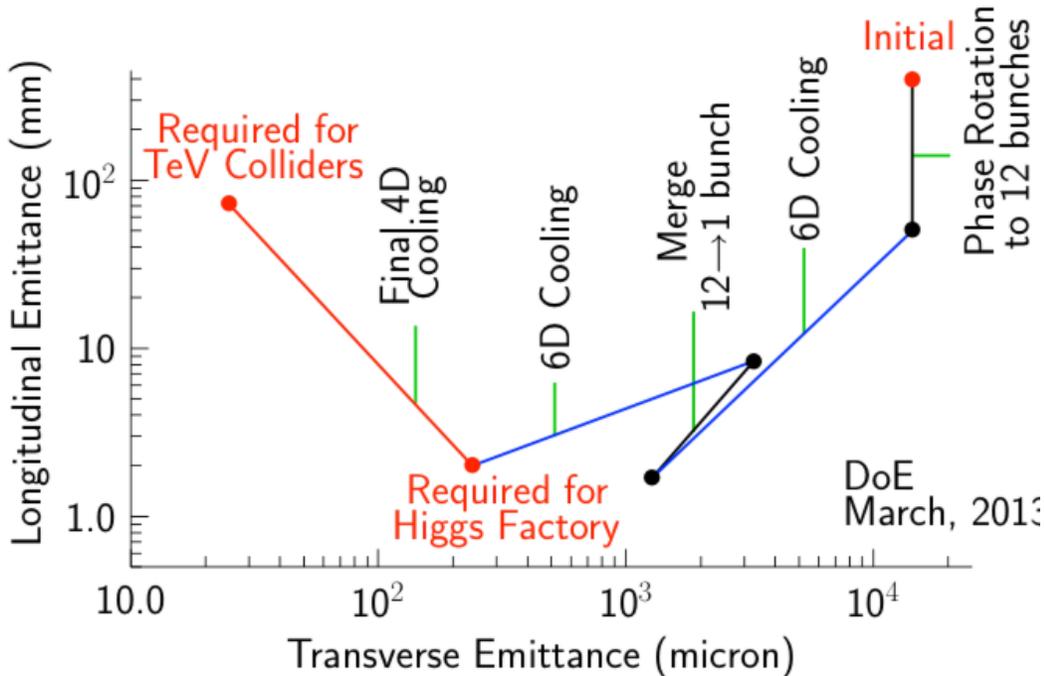 - Beam spot size reduces from > 100 mm at capture to < 1 mm at the end of cooling

Ionization Cooling and Emittance Evolution

Beam

Absorber

$$\frac{\Delta p_\perp^2}{m} = kT_b$$

q (momentum)-transfer via coulomb interaction

$$\frac{3kT_{abs}}{2}$$

Heat flows when

$$T_b > T_{abs}$$

Ionization Cooling Channel

D. Stratakis, CERN Workshop 2021

D. Stratakis, CERN Workshop 2021

- **Maximum heat transfer occurs when $\Delta T = T_b - T_{abs}$ is largest**

- **Strongly focusing optics maximize ΔT**

- **Cooling simulation demonstrates 6D phase space compression by 10^6**

- <u>**However, collective plasma effects in absorbers are not included in the past simulation study→ We will cover this**</u>

Phase Space Compression

Emittance evolution formula

Betatron function

Stopping Power

$$\frac{d\varepsilon_n}{ds} = \frac{\beta\gamma}{2} \cdot \widehat{\beta}_x \sigma_\theta^2 - \frac{\varepsilon_n}{\beta^2 E} \cdot \left(\frac{dE}{ds}\right)$$

Heating term

Cooling term

In this presentation, we will discuss modulation of the <u>stopping power and beam focusing mechanism</u> due to collective effects

Review of Stopping Power

Stopping cross section

$$S = \int T(p) d\sigma$$

p: **Impact parameter**

$$m_1, e_1 \quad v$$

$$m_2, e_2 \quad T = \frac{q^2}{2m_2}$$

$$d\sigma = 2\pi p \cdot dp \sim 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \frac{dT}{T^2}$$
:Differential cross section in Coulomb interaction
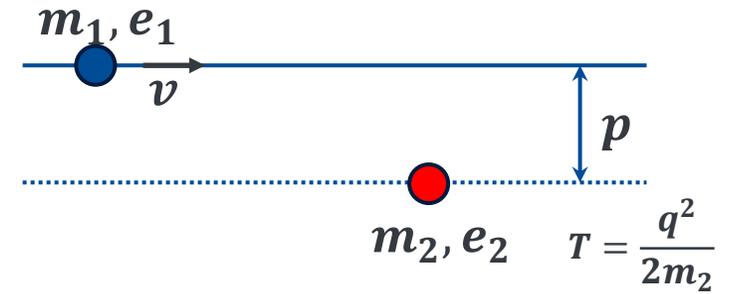
$$S = \int_{T_{min}}^{T_{max}} 2\pi \frac{e_1^2 e_2^2}{m_2 v^2} \frac{dT}{T} = 4\pi \frac{e_1^2 e_2^2}{m_2 v^2} \left[\frac{1}{2} \ln(T^2) \right]_{T_{min}}^{T_{max}}$$
Unit: $eV \cdot cm^2$

Coulomb logarithm = $ln(\Lambda)$

Stopping power

Represents the mean energy loss due to Coulomb interaction with target electrons

$$\left\langle -\frac{dE}{dx} \right\rangle = S \cdot N$$
:The formula is valid for single particle collision process

Bethe Equation

Stopping power formula including relativistic effect for medium energy ($p \sim$ 200 MeV/c)

$$\left\langle -\frac{dE}{dx} \right\rangle_{bound} = K \frac{Z}{A} \frac{z^2}{\beta^2} \left[\frac{1}{2} ln \left(\frac{2m_e c^2 \beta^2 \gamma^2}{I^2} W_{max} \right) - \beta^2 \right]$$

Coulomb logarithm = $ln(\Lambda)$

Lowest energy T_{min} is determined from bound atomic electrons

I is a mean excitation energy of bound atomic electrons

$$I = exp \left(\frac{\sum_i f_i \cdot h\nu_i}{\sum_i f_i} \right) \qquad f_i: \text{ Transition probability}$$
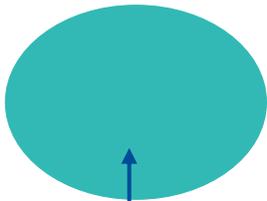
Plasma in Ionization Cooling

Plasma Parameter of Ionized Electrons in Muon Collider System

$n_e \, [cm^{-3}]$
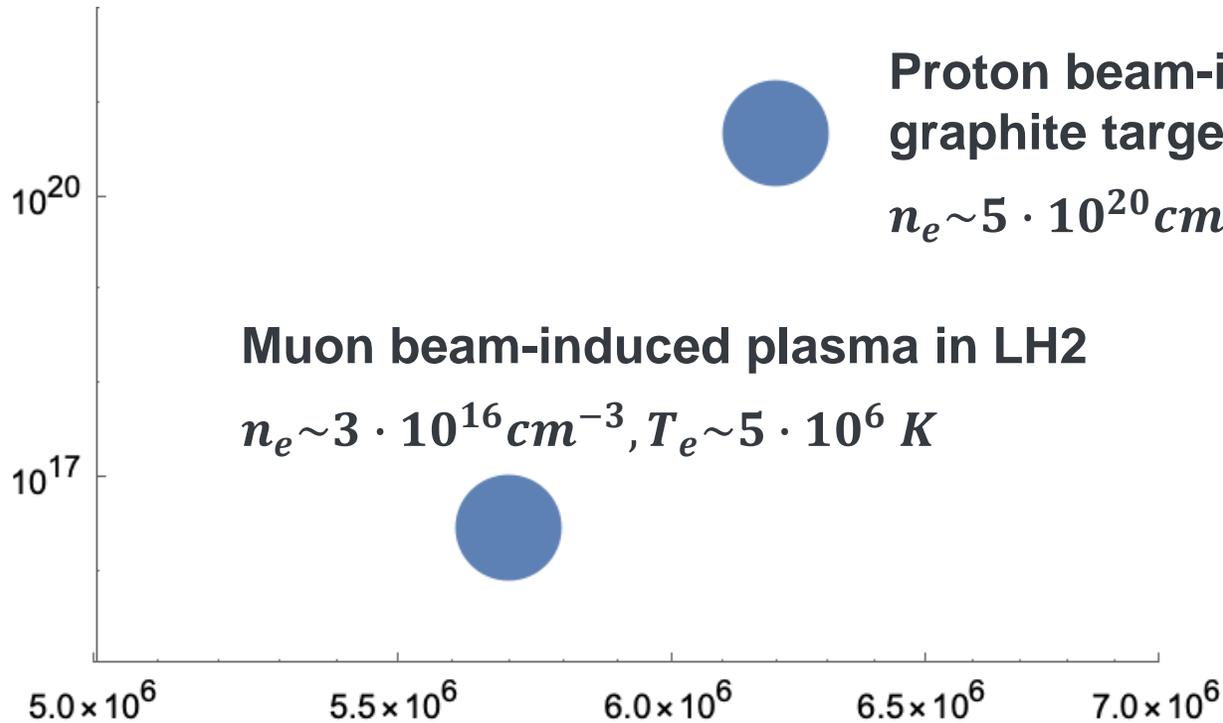
M. Chung et al, PRL111, 184802 (2013)

$$n_e = \frac{\langle -dE/dx \rangle}{W_{ion}}$$

Proton beam-induced plasma in graphite target

$n_e \sim 5 \cdot 10^{20} cm^{-3}, T_e \sim 6 \cdot 10^6 \, K$

Muon beam-induced plasma in LH2

$n_e \sim 3 \cdot 10^{16} cm^{-3}, T_e \sim 5 \cdot 10^6 \, K$

Fusion plasma

$n_e \sim 10^{13-15} \, cm^{-3}, T_e \sim 10^8 \, K$

10^{20}

10^{17}

$T_e \, [K]$

5.0×10^6 5.5×10^6 6.0×10^6 6.5×10^6 7.0×10^6

Wakefield plasma

$n_e \sim 10^{16-18} \, cm^{-3}, T_e \sim 10^{4-6} \, K$

F. Sauli, CERN 77-09 (1977)

$$T_e = \frac{\bar{\varepsilon}}{k_B} = \frac{\int_{E_0}^{E_{max}} E \cdot P(E) dE}{k_B}, P(E) = K \frac{Z}{A} \frac{1}{\beta^2} \frac{1}{E^2}$$

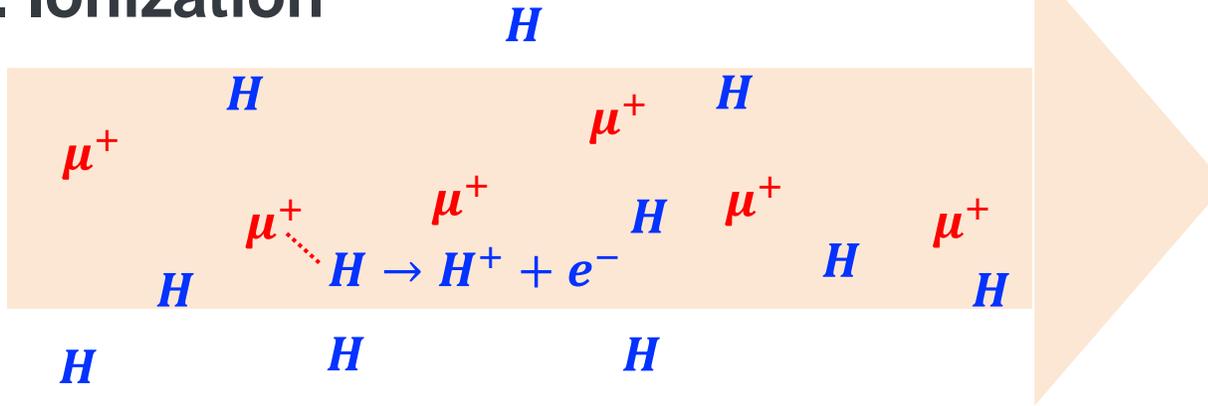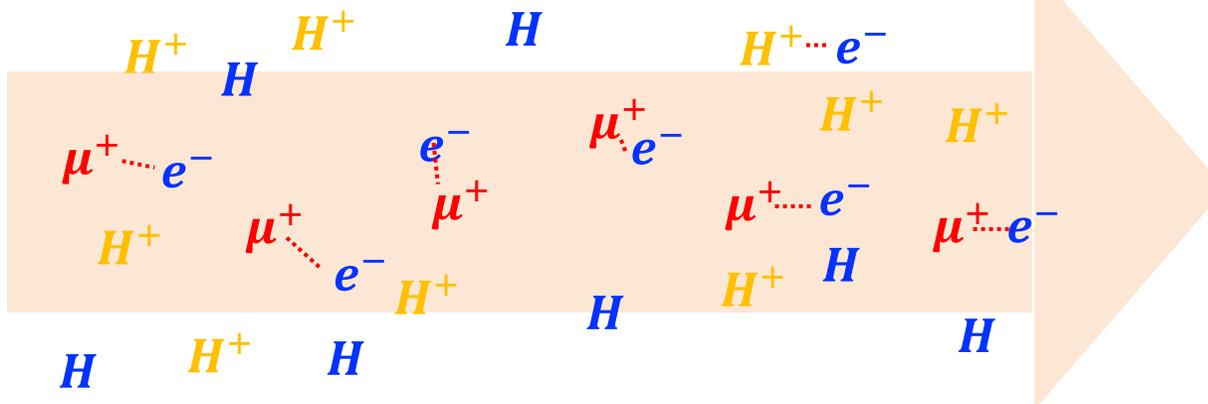# Plasma Sheath Formation

1. Ionization

μ^+ H

μ^+ H μ^+ H

μ^+ μ^+ H μ^+ μ^+

H $H \rightarrow H^+ + e^-$ H H

H H H

- Send high energy μ^+ in neutral hydrogen atom H

- Ionization happens (dashed line)

2. Charge neutralization (within ~10 ps)

H^+ H^+ H $H^+ \cdots e^-$

H

$\mu^+ \cdots e^-$ e^- $\mu^+ e^-$ H^+ H^+

μ^+ μ^+ $\mu^+ \cdots e^-$ $\mu^+ \cdots e^-$

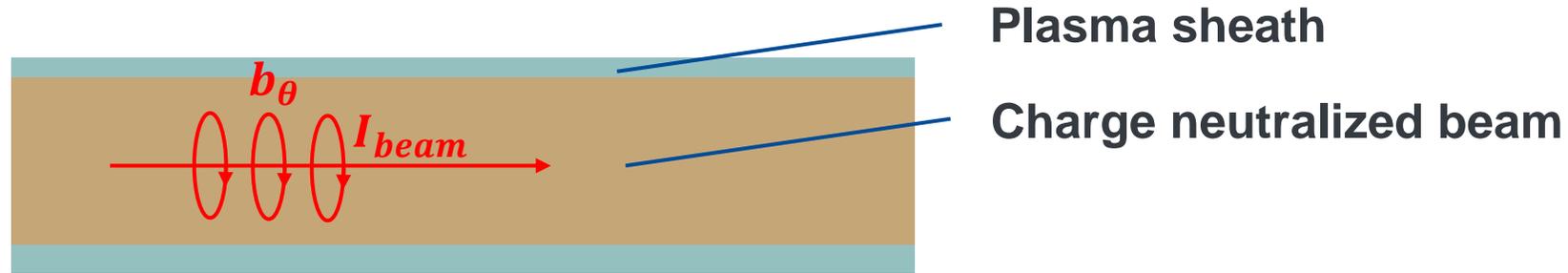H^+ e^- H^+ H

H^+ H H

- Beam attracts ionized electrons near the beam

- Within the beam volume $n_{\mu^+} \sim n_{e^-}, n_{e^-} \sim n_{H^+}$ (gas and beam are <u>neutral</u>)

- Outside beam volume $n_{e^-} < n_{H^+}$ (gas is <u>positively charged</u>)

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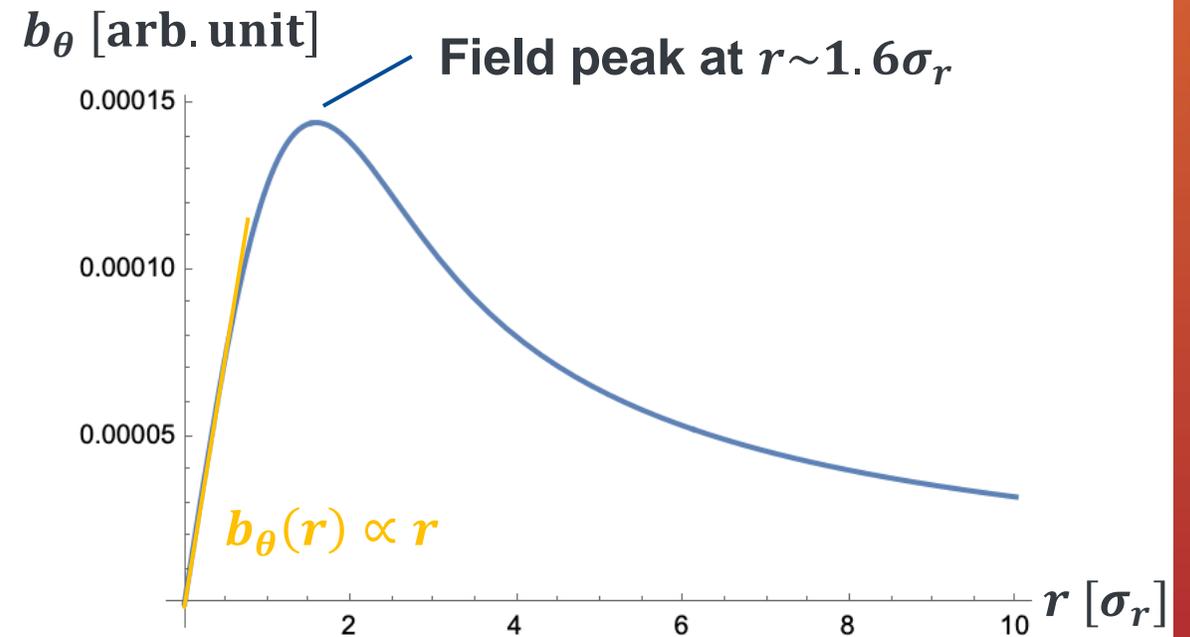# Formulate Azimuthal Magnetic Field from Collective effect

Beam charge is neutralized, but beam current I_{beam} still exists

Plasma sheath

Charge neutralized beam

Beam induced magnetic field

$$b_\theta(r) = \frac{\mu_0 I_{beam}}{2\pi r}\left(1 - e^{-r^2/2\sigma_r^2}\right)$$

b_θ [arb. unit]

Field peak at $r \sim 1.6\sigma_r$

$b_\theta(r) \propto r$

r [σ_r]

Additional Focusing Strength

Focusing strength

$$K = \frac{q}{p} \cdot \frac{db_\theta(r)}{dr} = \frac{q\mu_0 I_{beam}}{2\pi p}\left[-\frac{1-e^{-r^2/2\sigma_r^2}}{r^2} + \frac{e^{-r^2/\sigma_r^2}}{\sigma_r^2}\right]$$

$$1 - e^{-r^2/2\sigma_r^2} = 1 - \left(1 - \frac{r^2}{2\sigma_r^2} + \mathcal{O}(r^4)\right) = \frac{r^2}{2\sigma_r^2} - \mathcal{O}(r^4)$$

For near beam axis (up to around $r \sim \sigma_r$),

$$b_\theta(r) \sim \frac{\mu_0 I_{beam}}{4\pi\sigma_r^2} r$$

$$K \sim \frac{q\mu_0 I_{beam}}{4\pi p \sigma_r^2}$$

- The induced azimuthal field introduces an additional, self-generated focusing force proportional to beam current density – effectively behaving light an embedded plasma quadrupole
- We will validate the focusing model in numerical simulation as the future work

Numbers evaluation (I)

In final cooling stage

$n_\mu = 10^{12}$ /pulse in 0.1 ns

$p = 0.2$ **GeV/c**

$\sigma_r = 0.001$ **m**

$$I = 1.6 \cdot 10^{-19} \times \frac{10^{12}}{10^{-10}} = 1.6 \cdot 10^3$$

$$K = \frac{0.3 \cdot 4\pi \cdot 10^{-7} \cdot 1.6 \cdot 10^3}{4\pi \cdot 0.001^2} = 240 \ m^{-2} \rightarrow \widehat{\beta}_\perp = \sqrt{\frac{1}{K}} \sim 0.065 \ \text{m}$$

- **Net betatron function ~ 10-20 % shorter → stronger focusing → increasing cooling rate within $r \leq \sigma_r$**

- **However, non-linear Lorentz force for $r > \sigma_r$ can drive envelope oscillations and halo growth**

- **Need numerical simulation study**

Modify Stopping Power Formula by including Plasma Polarization

Dielectric term (Free/Collective electrons)

For free electrons (i.e. a plasma), use the dielectric form:

$$\left\langle -\frac{dE}{dx} \right\rangle = \frac{z^2 e^2}{2\pi v^2} \int_{-\infty}^{\infty} \frac{dk}{k} \int_0^{kv} \omega \cdot \mathrm{Im} \left[-\frac{1}{\varepsilon(\omega, k)} \right] d\omega$$

M. D. Barriga-Carrasco

Eur. Phys. J. Plus (2022) 137:375

In the case of collisional plasma, a dielectric term (Drude model):

$$\varepsilon(\omega) \sim 1 - \frac{\omega_p^2}{\omega(\omega + iv)}$$

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Assume $\omega \gg v$ (cold plasma), the lowest energy transfer: $T_{min} \sim \hbar \omega_p$

Modified Stopping Formula

We assume that the total electron density, n_e^{total} is the sum of the bound electrons, n_e^{bound} and the plasma electrons, n_e^{plasma}
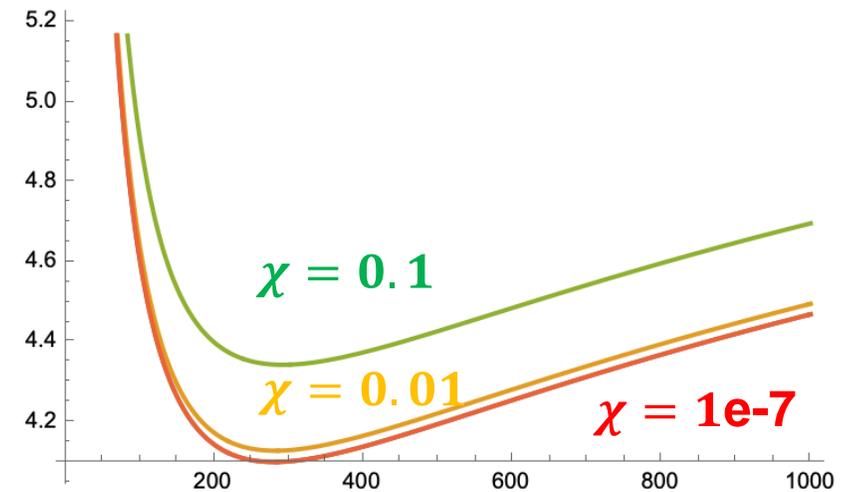
$$n_e^{bound} = (1 - \chi) \cdot n_e^{total}$$
$$n_e^{plasma} = \chi \cdot n_e^{total}$$

$\left\langle -\dfrac{dE}{dx} \right\rangle$ [MeV/(g/cm^2)]

Modified stopping power formula

$$\left\langle -\frac{dE}{dx} \right\rangle = (1 - \chi) \cdot \left\langle -\frac{dE}{dx} \right\rangle_{bound} + \chi \cdot \left\langle -\frac{dE}{dx} \right\rangle_{plasma}$$

$$\left\langle -\frac{dE}{dx} \right\rangle_{plasma} = -K \frac{Z}{A} \frac{z^2}{\beta^2} \left[ln \left(\frac{2 m_e c^2 \beta^2 \gamma^2}{\hbar \omega_p} \right) - \beta^2 \right]$$

$\chi = 0.1$

$\chi = 0.01$

$\chi = $ 1e-7

μ, Kin [MeV]

- Nominal plasma density in Liquid H is $\chi = 1e - 7$
- χ can be 0.01-0.1 in Gaseous H
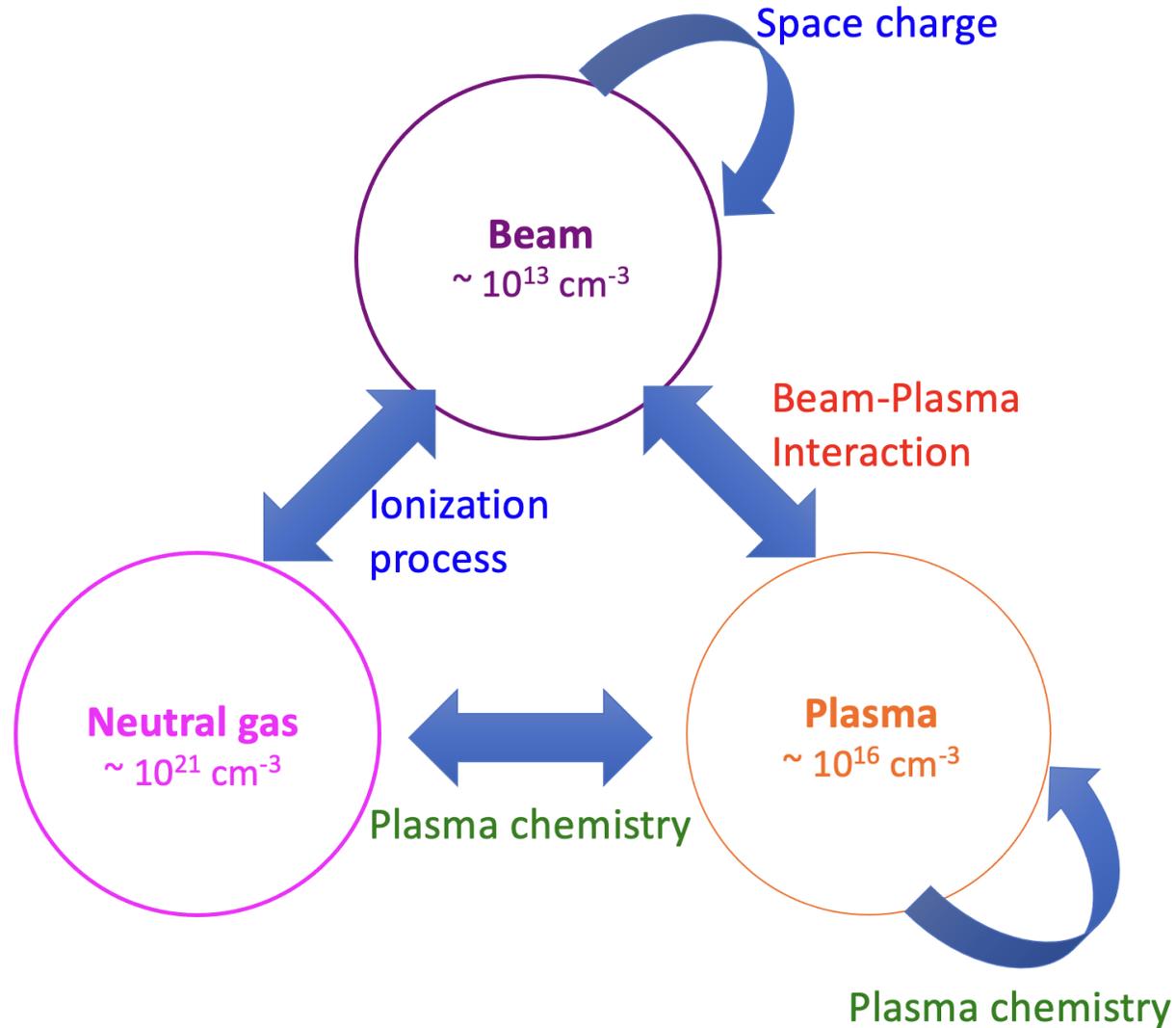- Or, use laser or beam to increase plasma density

Summary

- We considered the collective effect in ionization cooling
- The induced collective fields have the potential to change the cooling performance
  - Muons near beam axis gain additional focusing force, so that the phase space shrinks more than without the collective effect
  - However, muons far from the beam axis gain non-linear focusing force, which could be destructive to the phase stability conditions
  - We propose further simulation study to evaluate the collective field effects
- We also propose a new stopping power formula to include plasma polarization in absorbers
  - This effect is negligible in a nominal cooling beam parameters
  - On the other hand, this mechanism is useful for enhancing ionization cooling without introducing additional stochastic effects
  - It is worth considering an increasing in plasma density using laser and/or beam

Extra Slides
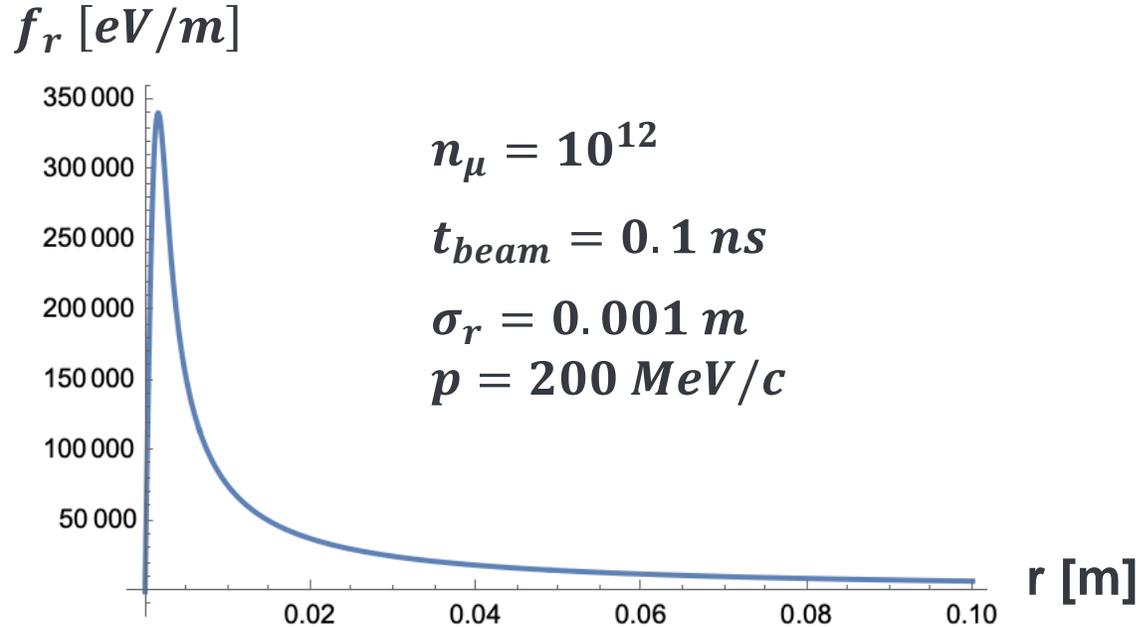
Beam-Plasma-Neutral Atom Interaction

Beam
~ 10^{13} cm^{-3}

Space charge

Beam-Plasma Interaction

Ionization process

Neutral gas
~ 10^{21} cm^{-3}

Plasma chemistry

Plasma
~ 10^{16} cm^{-3}

Plasma chemistry

Electric force in muon beam in final 6D cooling channel section

Transverse direction

$f_r \, [eV/m]$

$n_\mu = 10^{12}$

$t_{beam} = 0.1 \, ns$

$\sigma_r = 0.001 \, m$

$p = 200 \, MeV/c$

r [m]

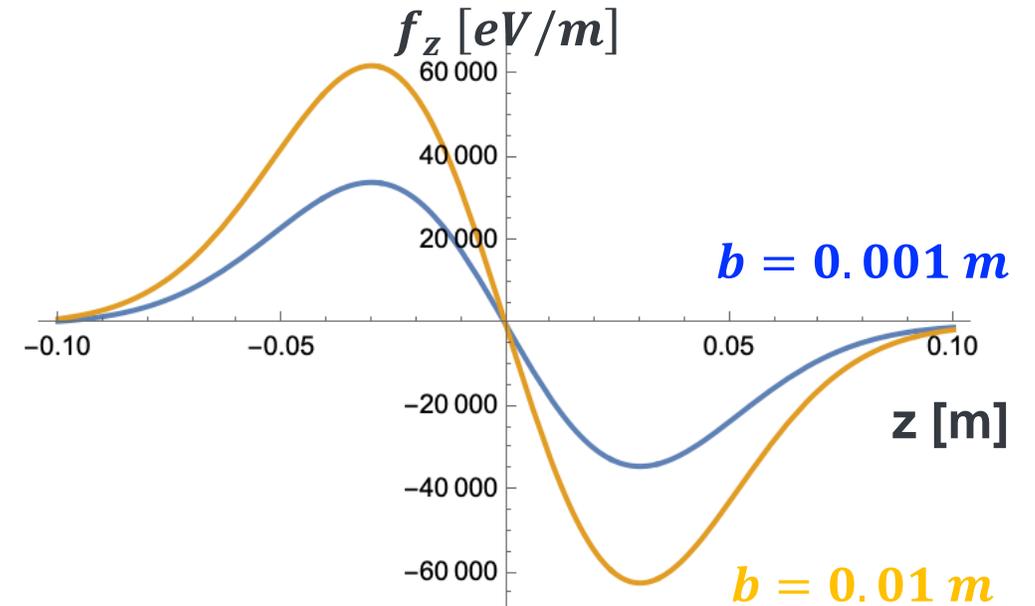**Longitudinal direction (on beam axis)**

$f_z \, [eV/m]$

$b = 0.001 \, m$

z [m]

$b = 0.01 \, m$

$$f_r = eE_r$$

$$\sim \frac{eQ}{2\pi\varepsilon_0\gamma^2 r}\left[1 - exp\left(-\frac{r^2}{2\sigma_r^2}\right)\right]$$

$$f_z = eE_z$$

$$\sim \frac{eQ}{2\pi\varepsilon_0\gamma^2}\left[1 + 2\,ln\left(\frac{b}{\sigma_r}\right)\right]\frac{z}{\sigma_z^2}exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

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Additional Considerations and Future Directions

- Multiple scattering in plasma material
  - Plasma screen charge of nuclei
  - However, a toy model shows that this effect changes only a few percent change (thus not shown in this presentation)

- Complicate plasma chemistry is not included in toy model
  - However, the recombination rate is known to be small because it is a binary process
  - Electron capture by neutral atoms should be investigated further, because it proceeds via a three-body process
  - Electron temperature could be quickly thermalized and reach $T_e \sim T_i$
  - We are preparing plasma simulation study to include those processes

- Production of intense plasma
  - We are considering either beam and laser to generate intense plasma