

DISPERSION SUPPRESSION FOR WEDGE-BASED FINAL COOLING AT A 10 TeV MUON COLLIDER*

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Abstract

Achieving a luminosity of $\gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ in a 10 TeV Muon Collider, given the short lifetime of a muon, requires reducing the 6D emittance of the muon beam through a process known as ionization cooling. In the final stage of this cooling process, the transverse emittance must be reduced to $\sim 22 \mu\text{m}$, typically by allowing longitudinal emittance growth up to downstream acceptance limits. While the current International Muon Collider Collaboration designs involve 40 T solenoids to reach the transverse emittance target, such high-field solenoids come with several challenges, including mechanical stress management, quench protection, and potential limitations in relying on High Temperature Superconductor technology. Designed as an alternative to using such solenoids while simultaneously reaching target transverse emittance, the previously proposed wedge-based, reverse emittance-exchange cooling scheme requires excellent dispersion suppression. In this study, we design and simulate a dispersion suppressor channel for the wedge-based final cooling design that reduces dispersion in the target direction to a target value of $D_x \sim 0.001 \text{ m}$.

INTRODUCTION

In the realm of planned next-generation facilities following the completion of the High Luminosity Large Hadron Collider (HL-LHC) at CERN, a circular muon collider stands out from others due to its potential to reach a multi-TeV center of mass energy with greater energy and power efficiency. For a feasible realization of the International Muon Collider Collaboration (IMCC) 10 TeV muon collider design, obtaining luminosities of $\gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ at the interaction point through emittance reduction at the cooling stage of the collider is crucial [1, 2]. Fig. 1 shows the evolution of the muon beam's transverse and longitudinal emittance throughout all cooling stages. The IMCC baseline Final Cooling design features liquid hydrogen (LH_2) absorbers and RF cavities within high-field matching solenoids. In more recent designs, the field strength of these solenoids gets up to $\gtrsim 40 \text{ T}$ [3] at the absorbers.

Creating and maintaining this ultra-high field regime is one of the major challenges of the design, as such magnets introduce large electromagnetic stresses, the need for quench protection and thermal management (a summary of further progress and challenges can be found in [4]). In this paper, we outline a previously proposed alternative final cooling concept that doesn't rely on high-field solenoids, discuss the requirements for the realization of the full design with

a focus on dispersion suppression, and finally demonstrate dispersion suppression with our design.

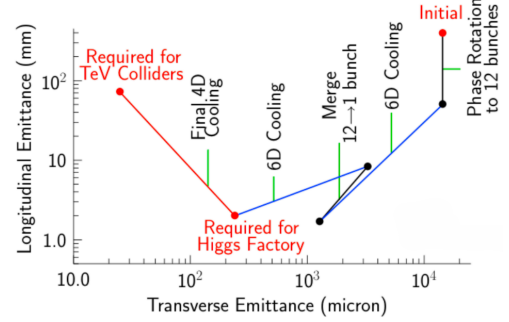


Figure 1: Fernow-Neuffer plot of the muon cooling process [5, 6]. The Final 4D Cooling stage investigated in this study is shown in a red line.

WEDGE-BASED FINAL COOLING DESIGN OVERVIEW

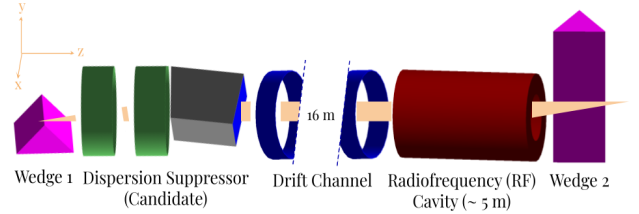


Figure 2: Wedge-based final cooling lattice modeled in G4Beamline with the candidate dispersion suppressor design (two quadrupoles and a sector dipole). The orange band indicates the beam trajectory along z . The first wedge is oriented in the x direction, whilst the second wedge at the end is oriented towards the y direction, facilitating ionization cooling in x and y directions respectively.

The optimized wedge-based final cooling design is based on the concept outlined in [7, 8]. A diagram of the updated design can be found in Fig. 2. We consider a starting distribution of $145 \mu\text{m}$ transverse emittance with a momentum spread of $\sigma_p = 1.0 \text{ MeV}/c$, corresponding to stage B10 of rectilinear 6D ionization cooling design (originally [9] involved 8 stages, but more recent improvements on the original design involve stage B10 [10]). The muon beam is initialized with a mean momentum of $p_\mu = 100 \text{ MeV}/c$ where an emittance exchange can be realized through a single wedge-shaped solid "dense carbon" diamond material [8] (referred to as just "wedge" hereafter). Dispersion is introduced in the wedge through the wedge's position-dependent non-uniform thickness, exploiting existing momentum-position correlation to preferentially reduce the longitudinal emittance while

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increasing transverse emittance, achieving reverse-emittance exchange.

The magnets in the dispersion suppression section succeeding the first wedge are positioned to primarily mitigate the dispersion in the x direction. This dispersion correction section design is composed of elements whose fields are linear in transverse coordinates, i.e., dipoles and quadrupoles, which also focus off-momentum particles to the centerline. Prior to this study, the effects of this dispersion section for the final cooling channel have been shown only through manual subtraction of the dispersion from the post-wedge particle distribution.

After the dispersion correction channel, the beam goes through a drift channel to increase the bunch length σ_t so that the subsequent radiofrequency (RF) cavity can do an energy-phase rotation to reduce σ_p , the momentum spread of the particle distribution. Any residual dispersion uncorrected by the dispersion suppressor, which corresponds to a nonzero correlation between transverse position and relative momentum, will cause coupling between transverse and longitudinal planes in the beam line, reducing the effectiveness of the energy-time phase rotation. The first part with the wedge and the dispersion suppression section could be repeated for the y-axis, thus ensuring a complete emittance reduction of all transverse directions.

DISPERSION SUPPRESSION DESIGN

Table 1: Relevant Parameter Values for the Initial Beam After the First Wedge

Name	x Axis	y Axis
ϵ [mm]	0.042	0.146
β [m]	0.035	0.025
α	-2.437	-0.681
D [m]	0.019 89	-0.0002
D'	-0.0402	-0.0128

In order to discuss the optimized design candidate for dispersion suppression, we first motivate the target values for such a design. Table 1 shows the relevant Twiss parameters for the muon beam after traversing through the wedge with wedge length of 7.51 mm and wedge angle of 47.3° and a mean momentum of $p_\mu = 87.33$ MeV/c and a spread of $\sigma_p = 7.15$ MeV/c. In dispersive systems, we can write the RMS beam size as:

$$\sigma_x^2 = \epsilon_x \beta_x + D_x^2 \sigma_\delta^2 \quad (1)$$

where $\sigma_\delta = \Delta p/p_\mu$ [11]. Simply inputting the values from the table, we can observe that the second dispersive term in Eq. 1 is higher than the first geometric term, indicating that the post-wedge beam size that will be propagated downstream the cooling channel is affected by dispersion. For the beam size to be less affected, we render the dispersive contribution subdominant by targeting $D_x^2 \sigma_\delta^2 \lesssim 10\% \epsilon_x \beta_x$. This means that the dispersion value targeted after the dis-

persion suppressor must be at least one order of magnitude lower than its post-wedge value.

Optimization Method

Table 2: Relevant Parameter Values for the Beam After QQB

Name	x Axis	y Axis
ϵ [mm]	0.337	0.148
β [m]	1.25	5.31
α	-2.01	-24.69
D [m]	0.0009	-0.000 76
D'	0.0103	-0.0028
Transmission After QQ [%]	93.5	
Transmission After B [%]	84.6	

Table 3: Optimized QQB Parameters

Parameter	Value
QF_gradient [T/m]	4.47
QD_gradient [T/m]	6.68
Q_radius [mm]	71.41
Drift_length [mm]	10.00
Q_z [mm]	107.9
B_field [T]	1.74
B_angle [deg]	1.63
B_bending_radius [mm]	643.1
B_z [mm]	5.99

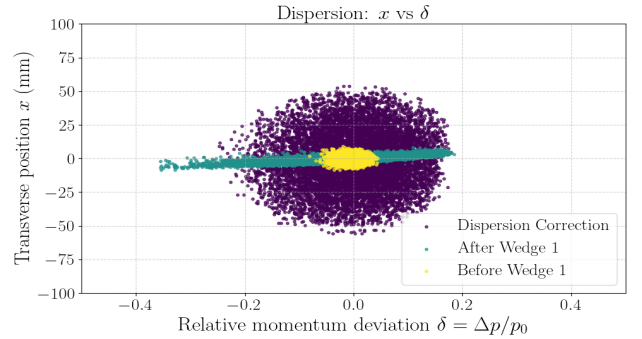


Figure 3: The dispersion vs. δ plot for various stages: before the wedge, after the wedge, and after the dispersion suppressor (with a 2σ cut applied).

To guide the design of the dispersion suppressor, we first consider established approaches such as the Double Bend Achromat (DBA) lattice as in [12] and the FODO-based dispersion suppression scheme as in [11]. The DBA lattice consists of a focusing quadrupole magnet located halfway between a pair of identical dipole magnets, and is typically constructed to enforce $D = D' = 0$ at its entrance and exit. These conditions are not satisfied in our case, where the beam exiting the wedge carries nonzero D, D' . Similarly, FODO-based dispersion suppressors assume the beam comes from a repeating sequence of focusing and bending elements that make up the arc of a storage ring or recirculating accelerator, where dispersion is generated and subsequently canceled by

modifying the bending distribution. This makes the design incompatible with our single-pass cooling channel with an arbitrary D , D' from the wedge, as the periodicity required to analytically cancel dispersion is not present.

We instead adopt a more flexible approach, comprising a quadrupole doublet and a sector dipole (QQB), where the quadrupoles provide transverse focusing and matching and the dipole corrects the residual beam dispersion. To design and refine this QQB lattice, we created two G4Beamline templates: one for the focusing (QF) and defocusing (QD) quadrupoles separated by a drift channel, and another for the sector dipole. A companion Python script performs optimization over the parameters `QF_gradient`, `QD_gradient`, `Drift_length` for the quadrupole doublet (representing the focusing and defocusing quadrupole gradients, the length of the drift channel between the quadrupoles, respectively) and `B_strength`, `B_angle`, `B_bending_radius` for the dipole (defining field strength, sector angle, radius, respectively). `Q_z`, `B_z` are also included, as they describe the distance to the initialized beam from the first quadrupole and the sector dipole, respectively. Quadrupole radii and dipole bending radius boundaries are determined based on the incoming beam size to maximize transmission.

The optimization proceeds in two stages: we first optimize the quadrupole parameters using a global Differential Evolution search [13], [14], followed by local refinement with the Nelder-Mead algorithm [15]. The resulting particle distribution is then fed into the dipole, where dipole parameters are optimized similarly. During this step, the beta penalties are weighted more heavily for quadrupoles, while the dispersion penalty receives a higher weight for the dipole. Table 2 shows the relevant Twiss parameters for the beam after going through the optimized QQB system. The transverse phase space evolution from the wedge to the end of the dispersion suppressor can be seen in Fig. 3, and the evolution of these parameters throughout the system is illustrated in Fig. 4. Table 3 shows the values of the optimized parameters for the QQB design.

Discussion and Future Work

Comparing the values in Table 2 with the target estimates discussed previously, we find that the target suppression of D_x , D_y has been achieved. However, this comes with a clear trade-off: while the initial particle distribution exhibits low beta functions, the stronger focusing required to reduce dispersion leads to an increase in β_x , β_y as well as an increase in transverse emittance in the x direction. We also note a trend of increasing beta function values with momentum spread δ , which suggests the presence of chromatic effects induced by the quadrupoles. The growing transverse emittance in the x direction for the dispersion correction design could point toward the growth of stochastic effects introduced by the wedge, which could be causing deviations from ideal linear optics. Although these effects are not explicitly quantified here, they may become relevant for downstream beam dynamics and warrant further investigation. Similar designs,

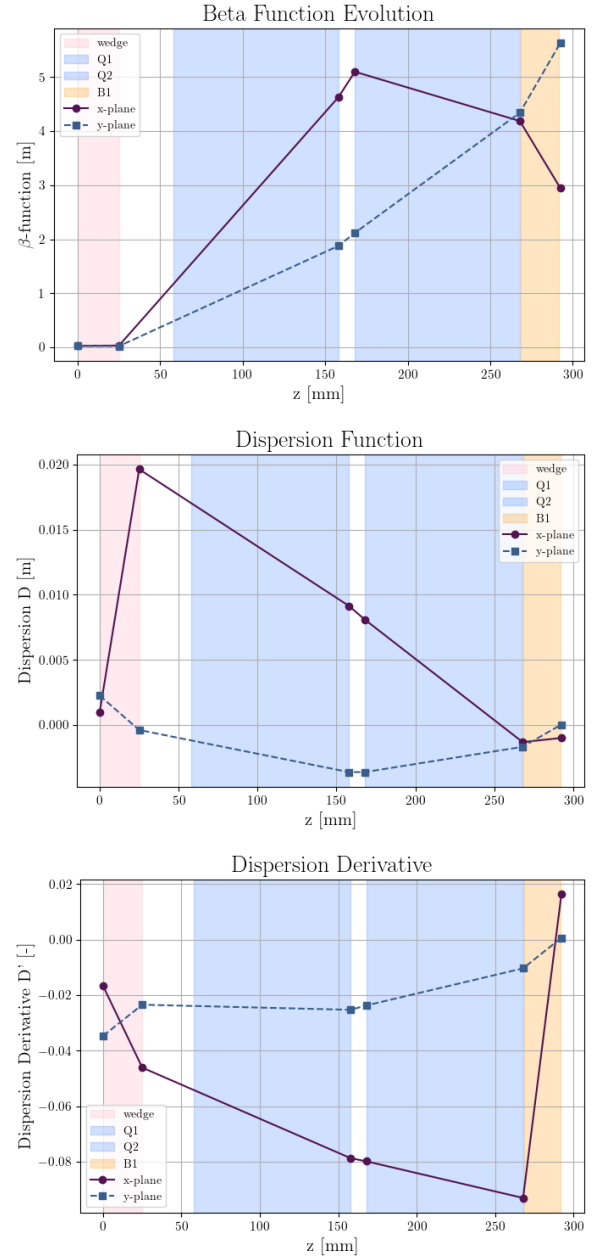


Figure 4: The evolution of Twiss parameters in the QQB dispersion suppressor with respect to the longitudinal beam axis.

such as BQQ or BQQB, prove to have a similar potential, so more work on exploring all potential designs would be needed. Future designs should look to combine this dispersion suppressor with the rest of the downstream elements of the final cooling channel to quantify the effect of dispersion on the final transverse emittance value.

CONCLUSION

We have designed and developed parameters for a dispersion suppressor in a wedge-based final cooling system for a Muon Collider. Directions for future development are discussed.

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