THE ESSENCE FACTOR TO DETERIORATE THE CIRCULAR POLARIZATION RADIATION PERFORMANCE IN APPLE-KNOT UNDULATOR*

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Abstract

The APPLE-KNOT undulator forms composite magnetic fields by superimposing APPLE and KNOT fields with different period lengths. In this configuration, the APPLE field serves as the dominant component to approximate the target photon energy, while the KNOT field acts as an additional component to transversely deflect the electron beam off-axis. Although variable polarization modes can be realized with a low on-axis heat load, previous studies have observed a sharp reduction in flux and significant degradation of the polarization degree in the circular polarization (CP) mode. This paper discusses this phenomenon in detail from a theoretical perspective. The analysis reveals that the presence of an additional field with a longer period is the essence factor that inherently suppresses the radiation performance of CP mode. Theoretical findings are highly consistent with simulation results, demonstrating that selecting the KNOT field as the dominant component can effectively improve CP characteristics without significantly compromising the linear polarization performance.

INTRODUCTION

Various types of undulators have been developed to meet different experimental requirements. The APPLE-type undulator is well known as the light source for generating variable polarization. The most conventional polarization-tunable undulator is the APPLE-II undulator [1]. However, low photon energy applications in high-energy storage rings lead to substantial on-axis heat load in linear polarization modes. To address this issue, the APPLE-KNOT(AK) undulator combines APPLE and KNOT arrays with a period ratio of 2:3 to achieve variable polarization while maintaining low on-axis heat load [2]. Subsequent developments including four-row merged configurations have been implemented at various facilities [3,4]. Despite these advances, a common observation across AK undulator designs is that the circular polarization (CP) mode exhibits a lower photon flux than the linear polarization modes, usually together with a reduced polarization degree. This phenomenon has been noted in both simulation studies and practical implementations [5]. A recent APPLE X KNOT approach for SLS 2.0 has also demonstrated that knot-type magnetic configurations can be adapted to advanced APPLE undulator platforms to reduce on-axis power while preserving full polarization control [6]. These developments further motivate a clear understanding of the physical mechanism that governs the photon-flux performance of APPLE-KNOT undulators, especially in circular polarization mode.

This paper further investigates the essential factor that deteriorates the CP-mode performance in AK undulators. Theoretical findings are validated through numerical simulations based on Hefei Advanced Light Facility (HALF) parameters [7] and further verified using High Energy Photon Source (HEPS) parameters [8]. The basic theoretical framework is related to our previously published study [9]. Here we focus on additional results including the field-strength-ratio dependence, the short-period high energy case, and the preliminary end design with field-integral analysis. These new results are used to further clarify the design implications of selecting the longer-period KNOT field as the dominant component.

PRINCIPLES

In theory, the composite magnetic field of AK undulator at circular polarization mode can be simplified as

$$\begin{aligned} B_y &= -B_{\text{strong}} \sin \frac{2\pi}{\lambda_1} z - B_{\text{weak}} \cos \frac{2\pi}{\lambda_2} z \\ B_x &= B_{\text{strong}} \cos \frac{2\pi}{\lambda_1} z - B_{\text{weak}} \sin \frac{2\pi}{\lambda_2} z \end{aligned} \tag{1}$$

where B_{strong} and B_{weak} are the peak fields corresponding to the period lengths of λ_1 and λ_2, respectively. With $B_{\text{strong}} = \alpha B_{\text{weak}} = \alpha B_0$, and $\lambda_1 = m\lambda_2 = m\lambda_0 (\alpha > 1, m > 0$, and $m \neq 1)$, we have

$$\begin{aligned} B_y &= -B_0 \left(\alpha \sin \frac{2\pi}{m\lambda_0} z + \cos \frac{2\pi}{\lambda_0} z \right) \\ B_x &= -B_0 \left(-\alpha \cos \frac{2\pi}{m\lambda_0} z + \sin \frac{2\pi}{\lambda_0} z \right). \end{aligned} \tag{2}$$

In the traditional AK undulator (Case A), the APPLE array (shorter period) serves as the strong field and the KNOT array (longer period) as the weak field, corresponding to $m = 2/3$. In contrast, the new proposed configuration (Case B) selects the KNOT array as the strong field, yielding $m = 3/2$. The following analysis compares the two cases with the same period and field-strength ratio to identify the origin of the CP photon-flux reduction, using the parameters listed in Table 1. The photon flux is evaluated from the standard far-field undulator-radiation formalism.

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Table 1: Design parameters of the electron beam in the middle of a long straight section of HALF.

Parameter	**Specification**
Beam energy (GeV)	2.2
Average current (mA)	350
Horizontal / Vertical emittance (pm·rad)	176.4/13.2
Horizontal / vertical beta function (m)	6.834/2.536
Horizontal dispersion function	0.0005

Figure 1 compares the normalized photon flux as a function of photon energy for three field strength ratios (α=1.5, 2 and 4). The photon energy is scanned by changing the field strength. For all α values, Case B consistently yields higher flux than Case A across the entire energy range, with the advantage becoming more pronounced as α increases.

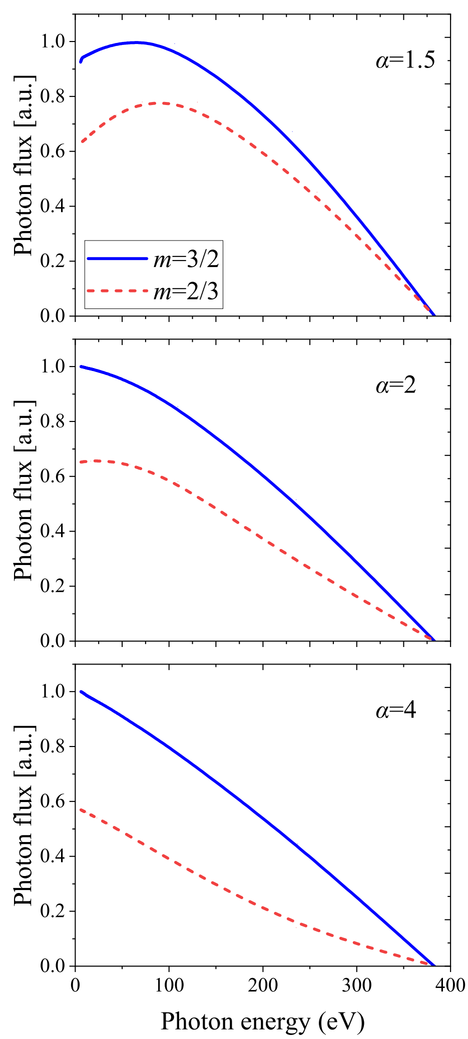

Figure 1: Normalized photon flux at different resonance photon energy for $m = 3/2$ (solid) and $m = 2/3$ (dashed), with $\lambda_u = 240$ mm and the electron beam energy of 2.2 GeV.

The heat load suppression capability of an AK undulator relies on its long period to steer radiation power away from the axis. In the short-period regime, the heat load gradually converges toward the undulator axis. This diminishes the suppression effectiveness and reduces the photon flux. Moreover, in the short-period regime the AK structure may introduce on-axis heat load in CP mode, a feature intrinsically absent in APPLE-II undulators.

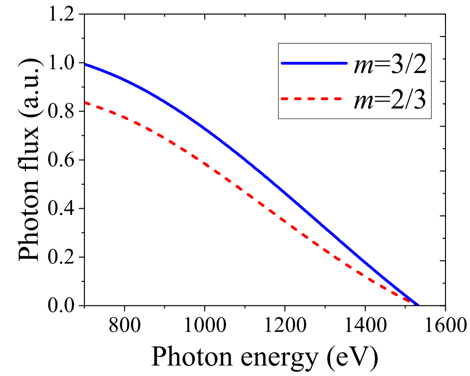

Figure 2: Normalized photon flux at different resonance photon energy for $m = 3/2$ (solid) and $m = 2/3$ (dashed), with $\lambda_u = 60$ mm and the electron beam energy of 2.2 GeV.

Nevertheless, short-period AK undulators retain a degree of heat load suppression in linear polarization modes. This makes them potentially applicable for a high-energy beamline with stringent on-axis heat load constraints, where a moderate reduction in photon flux is acceptable. To quantitatively evaluate this high-energy applicability, we maintain the field strength ratio $\alpha = 1.5$ and the electron beam energy of 2.2 GeV, but reduce the period to $\lambda_u = 60$ mm. As illustrated in Fig. 2, although AK undulators are not originally developed for short-period or high-energy applications, this discussion extends their potential operational scope. When an AK configuration is considered in such regimes, the ratio between the dominant and additional field components should be evaluated to determine the optimal scheme.

UNDULATOR DESIGN

Based on this principle, a novel AK undulator configuration has been proposed in Ref. [9], where the longer-period KNOT field is enhanced to serve as the dominant component. In this section, we present additional design-related results which were not discussed before.

Table 2: Parameters for undulators in Fig. 3

Undulator	**HEPS AK**	**Novel AK**
Period $3\lambda_u$ (mm)	256.8	220.8
APPLE period λ_u (mm)	85.6	73.6
KNOT period $1.5\lambda_u$ (mm)	128.4	110.4
Number of periods	18	21
APPLE:Knot	5:2	3:10

To verify the generality of our theoretical conclusion beyond the specific parameters, we perform an independent validation based on HEPS. As shown in Table 2, another novel AK undulator is designed to match the same minimum photon energy as the HEPS traditional design. Figure 3 compares the photon flux of the two designs for different polarization states. The novel design consistently delivers higher flux across the entire energy range, confirming that the advantage of selecting the longer-period field as dominant is independent of storage ring parameters.

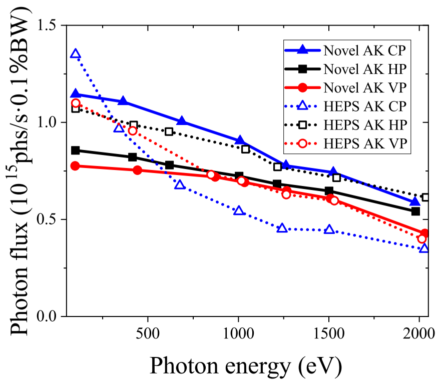

Figure 3: Comparison of the photon flux between the novel AK undulator and the HEPS AK undulator for different polarization states, with an electron beam energy of 6 GeV. For consistency, the acceptance angle at different photon energies is kept at two times the radiation divergence.

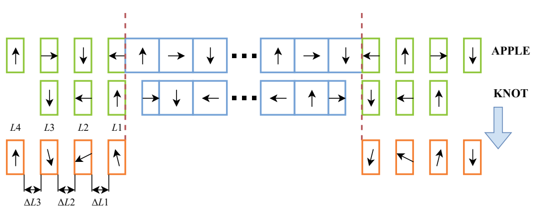

Figure 4: The concept of the end design.

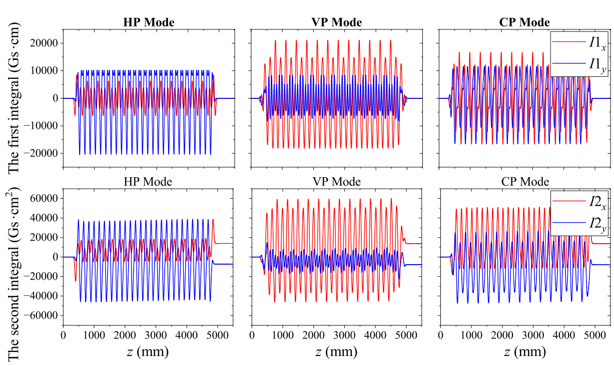

Figure 5: The corresponding first and second field integral distributions at the minimum working gap.

For practical implementation, proper termination design is essential to minimize field integrals. Taking the HALF-based novel AK undulator as an example, we adopted an antisymmetric end structure with optimized air gaps and end magnet dimensions, as shown in Fig. 4. By tuning the lengths of the air gaps and the magnetization directions of the end magnets, the first field integrals are naturally canceled for all three polarization modes. Figure 5 shows the corresponding first and second field integrals at the minimum working gap for horizontal, vertical and circular polarization modes, which remain within acceptable ranges.

The residual second field integrals exhibit a straight distribution along the undulator and can be further corrected by corrector coils if needed. These results demonstrate the engineering feasibility of the proposed configuration.

SUMMARY

This paper further discusses the essential factor that deteriorates the CP photon-flux performance in APPLE-KNOT undulators. The additional calculations presented here show that the advantage of selecting the longer-period KNOT field as the dominant component becomes more evident with increasing field-strength ratio and remains valid in a short-period high-photon-energy case. An independent HEPS-based validation further confirms that the conclusion is not limited to the HALF parameters. In addition, a preliminary end design and the corresponding field-integral analysis are presented as new engineering considerations for the proposed configuration. These results complement the previously published work and clarify the practical implications of the period-ratio selection in APPLE-KNOT undulators.

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