BEAM BASED OPTIMIZATION OF THE ESR LINEAR OPTICS MODEL

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Abstract

In this work, we present a beam-based method to improve the optics modeling of the ESR. Using Differential Evolution (DE), a population-based derivative-free optimizer, we determine correction factors for the quadrupole families by minimizing the discrepancy between simulated and measured betatron tunes. The optimized lattice reduces the tune discrepancy of ΔQ to the level of 0.01. The agreement in the beta functions and dispersion is preserved or slightly improved.

INTRODUCTION

The precise settings of the quadrupole strengths is crucial for the reliable operation of synchrotrons and storage rings. Quadrupoles determine the machine optics and directly control the horizontal and vertical betatron tunes, which should be set away from resonance lines. If the tunes are not well controlled, resonant beam loss, reduced beam lifetime, and limited operational flexibility may occur. For the Experimental Storage Ring (ESR) at GSI, an accurate determination of the effective k-values of the quadrupole families is therefore essential. By refining these parameters, the machine can be operated more efficiently, with faster setup procedures, improved beam quality, and a more reliable agreement between simulation and experiment. Such improvements not only benefit daily operation but also provide a solid ground for future machine studies, for example in the investigations of beam loss during deceleration.

PROBLEM STATEMENT

The issue addressed in this work is the deviation between simulated and measured tunes at the ESR. As an illustrative example, a measurement yielded horizontal and vertical tunes of $Q_x = 2.2972$ and $Q_y = 2.2260$, whereas the corresponding simulation predicted $Q_x = 2.3511$ and $Q_y = 2.3073$. This results in deviations of $\Delta Q_x = 0.0539$ and $\Delta Q_y = 0.0813$, which roughly correspond to 3 %. The goal of this work is to determine effective quadrupole strengths that reduce these tune deviations to values on the order of 0.01 or below.

THEORETICAL BACKGROUND

A proper understanding of the present problem requires the relation between the betatron tune and the quadrupole focusing strength. The machine tune is $Q = 1/(2\pi)\oint_C \beta^{-1}(s)\,ds$ (see [1]), and in the linear (small-perturbation) approximation, a change of a quadrupole strength of Δk produces a tune shift

$$\Delta Q = \frac{1}{4\pi}\,\beta\,\Delta k\,L, \tag{1}$$

where β is the beta function at the quadrupole position and L is the quadrupole length [2]. The quadrupole strength is $k = 1/(B\rho)(\partial B_y/\partial x)$, with $B\rho = p/q$ the magnetic rigidity.

CONSTRAINTS

The quadrupole configuration of the ESR has several unique characteristics. In total, the machine contains 20 quadrupoles, which are grouped into ten families. Within each family, the quadrupoles share the same power supply, hence are controlled simultaneously. This symmetry significantly reduces the number of independent parameters, and thus restricts the freedom with which the optics of the machine can be adjusted. In addition, the families are arranged symmetrically: for example, the first and last family, the second and second-to-last family, and so forth, always have identical strengths.

MEASUREMENTS

From Eq. 1 one can derive a straightforward method to optimize the quadrupole strengths. By performing tune measurements for different quadrupole settings, it is possible to relate the measured tune shifts to the corresponding changes in quadrupole strength, and thereby determine the effective beta function at the quadrupole location. The quadrupole strength is $k = k_0 + \Delta k$ and the tune is $Q = Q_0 + \Delta Q$, where Q_0 and k_0 are the tune and quadrupole strength at the initial setting. Here Δk is the applied change in quadrupole strength, while ΔQ is obtained from the tune measurement. The corresponding β-values can be directly compared with those from the lattice simulation. For the present measurement campaign, each quadrupole family was individually varied by approximately ±2 % around its initial strength. For each family, ten different k-values within this range were tested and three tune measurements were performed per setting, resulting in a total of roughly 300 measurements. This systematic variation allows a direct comparison between measurement and simulation, enabling the determination of effective scaling factors for the quadrupole strengths. As an example, Fig. 1 shows the comparison between measured and simulated tunes for quadrupole family E01QS0D, which we call for convenience “Family 0”. This method ignores the possible differences of quadrupoles within the same family.

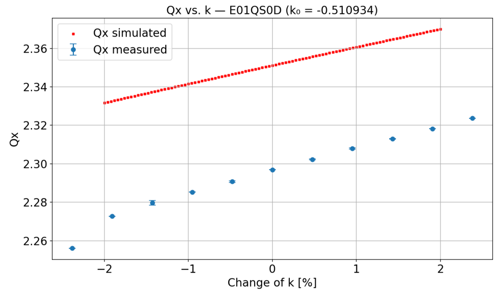

Figure 1: Comparison between measured and simulated tunes for quadrupole Family 0.

DIFFERENTIAL EVOLUTION

In the accelerator community, several methods exist for retrieving optics model of particle accelerators, as reviewed in Ref. [3]. The first methods [4] were gradient-based and later gradient-free methods were employed; however, in this work we employ a stochastic algorithm belonging to the class of genetic algorithms.

In order to accurately obtain the correction factors, we use the *Differential Evolution* (DE) method. This is a population-based global optimization algorithm. DE is derivative-free, robust against local minima, and straightforward to implement, which makes it well suited for the present problem. The algorithm starts with the initialization of a population of candidate solutions. Each candidate is represented by a vector $\vec{x}_i$, with, $i = 1, \ldots, \text{pop\_size}$ where i denotes the candidate index. In our case, each vector has $N_c = 10$ components corresponding to the scaling factors of the ten quadrupole families. In the mutation step, a mutant vector $\vec{v}_i$ is created by adding the scaled difference of two randomly chosen candidates to a third one: $\vec{v}_i = \vec{x}_{r_1} + F \cdot (\vec{x}_{r_2} - \vec{x}_{r_3})$ where r_1, r_2, r_3 are distinct random indices in [1,..,pop_size] and $F \in [0, 2]$ is the mutation factor controlling the amplitude of the variation (typically $F \approx 0.5$–0.9). The crossover step forms a trial vector $\vec{u}_i$ by mixing elements of the mutant vector $\vec{v}_i$ with the original candidate $\vec{x}_i$. The jth component of the new vector $\vec{u}_i$ is

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if } \text{rand}_j < CR \text{ or } j = j_{\text{rand}}, \\ x_{i,j}, & \text{otherwise}, \end{cases} \tag{2}$$

where $CR \in [0, 1]$ is the crossover rate and j_{rand} is a random integer $1 \leq j_{rand} \leq N_c$, with N_c the degree of freedom of the system. This ensures that at least one component is inherited from the mutant vector. In the selection step, the better solution between the current candidate $\vec{x}_i$ and the trial vector $\vec{u}_i$ is carried forward to the next generation:

$$\vec{x}_i^{\text{next}} = \begin{cases} \vec{u}_i, & \text{if } f(\vec{u}_i) \leq f(\vec{x}_i), \\ \vec{x}_i, & \text{otherwise.} \end{cases} \tag{3}$$

This process is applied to all the population, and repeated until convergence or until a predefined number of generations is reached. The quality of each candidate is assessed through the fitness function [6], which quantifies the discrepancy between simulation and measurement. For the present problem, it is defined as

$$\begin{aligned} f(\vec{x}_i) = \; & w_q \cdot \sqrt{(Q_x^{\text{sim}} - Q_x^{\text{meas}})^2 + (Q_y^{\text{sim}} - Q_y^{\text{meas}})^2} \\ & + w_\beta \cdot \sqrt{\left(\sum_{i=1}^{20} (\beta_{i,x}^{\text{sim}} - \beta_{i,x}^{\text{meas}})\right)^2 + \left(\sum_{i=1}^{20} (\beta_{i,y}^{\text{sim}} - \beta_{i,y}^{\text{meas}})\right)^2} \\ & + w_{\text{disp}} \cdot \sqrt{\left(\sum_{i=3}^{12} (D_i^{\text{sim}} - D_i^{\text{meas}})\right)^2}, \end{aligned} \tag{4}$$

where w_q, w_β, and w_{disp} are weights for the tune error, the beta-function error, and the dispersion error respectively. The weights were set to $w_q = 300$, $w_\beta = 1$, and $w_{\text{disp}} = 1$ respectively. This choice prioritizes tune matching while ensuring that the beta function and dispersion are not degraded. The dispersion used in the Eq. 4 is taken from Ref. [5]: note the index of the dispersion in Eq. 4 starts from $i = 3$. This is because the measured dispersion at BPMs $i = 1, 2$, appears suspiciously off as visible in Fig. 4 top. Since the initial discrepancies in β and D were already significantly smaller than those in the tunes, only a small weighting was required for these terms. Several test runs with different weight combinations were performed in order to verify that this choice leads to stable convergence and the desired balance between tune correction and optics preservation. In our implementation we used bounds $x_{i,j} \in [0.98, 1.02]$ for all family scalings, a population size of $N = 200$, $F = 0.5$, $CR = 0.5$, and up to 300 generations. Symmetry between paired families was enforced by tying the corresponding variables.

RESULTS

To test the robustness of the Differential Evolution method, 100 independent applications of this method with different random initializations were performed. The maximum spread of the obtained correction factors was below 0.003 for the first and last families, which is negligible compared to typical corrections of ~ 0.01. This demonstrates that DE yields stable and repeatable solutions (Fig. 2). The resulting

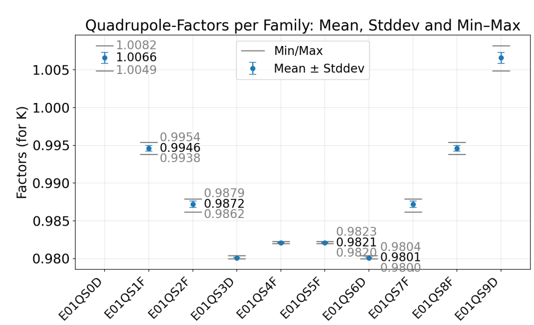

Figure 2: Quadrupole factors per family: mean, standard deviation, and min–max values.

mean correction factors for each quadrupole family are listed in Table 1. Applying these factors to the lattice significantly improves the agreement between simulation and measurement. As summarized in Table 2, the tune errors, initially of the order 0.05–0.08, are reduced to below 3×10^{-4}. The mean deviations in β and dispersion, which were already less critical, also show slight improvements.

Table 1: Correction Factors for Each Quadrupole Family

Quadrupole Family	**Factor**
E01QS0D / E01QS9D	1.0066
E01QS1F / E01QS8F	0.9946
E01QS2F / E01QS7F	0.9872
E01QS3D / E01QS6D	0.9801
E01QS4F / E01QS5F	0.9821

Table 2: Comparison of Simulation and Measurement Before and After Applying Correction Factors. $|\Delta Q|$ is the Absolute Difference Between Measured and Simulated Tune

	Before Change	**After Change**
$\lvert\Delta Q_x\rvert$	0.0542	0.0002
$\lvert\Delta Q_y\rvert$	0.0810	0.0003

Figures 3–4 illustrate the effect on individual observables. The tune comparison for Family 0 (Fig. 3) shows that the previous deviation is essentially eliminated. The dispersion (Fig. 4) is largely unaffected, as it was already well described by the original model, while the β-functions (Fig. 4) show modest but consistent improvements. Overall, the corrections reduce tune discrepancies by nearly two orders of magnitude while preserving the good agreement in β and D.

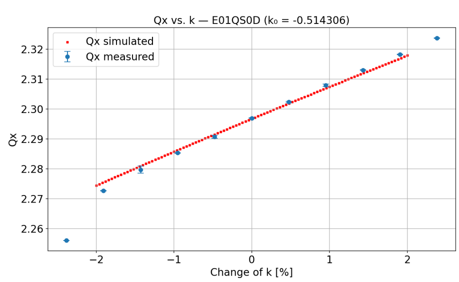

Figure 3: Measured vs. simulated tunes for quadrupole Family 0 after applying corrections.

VERIFICATION

To verify that the resulting correction factors do not only improve the lattice used in the initial measurement, a second measurement campaign was performed with a modified lattice configuration. See Table 3. Repeating the study for the modified lattice has yielded the results in Table 4. Looking at the tunes, a clear improvement in agreement is still observed (Table 4). The improvement is smaller than in the first case, but remains significant.

CONCLUSION AND OUTLOOK

In this work, an optimization of the ESR lattice was carried out by determining effective quadrupole strengths through a comparison of simulated and measured tunes. Using Differential Evolution, reliable correction factors were obtained, which significantly improved the agreement between simulation and measurement. The initial tune deviations of the order of 0.05–0.08 were reduced to below 3×10^{-4}, while the beta-function and dispersion deviations also showed slight improvements. A verification with an independent lattice configuration confirmed that the correction factors are robust and transferable, although the level of improvement depends on the specific machine settings. As with most model-based approaches, the accuracy of the corrections ultimately depends on the amount and quality of the measurement data. Future improvements can therefore be expected from extended measurement campaigns, including additional quadrupole variations and higher statistics.

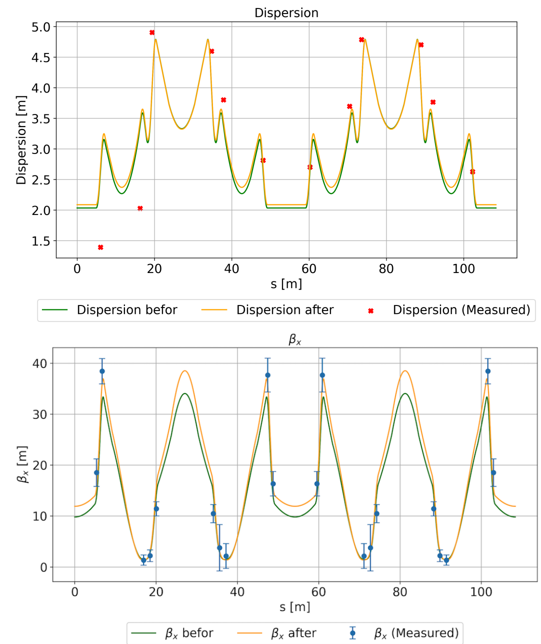

Figure 4: Dispersion and β_x function before and after corrections.

Table 3: Relative Difference in Quadrupole Strengths Between the First and Modified ESR Lattice

Quad Family	**%**	**Quad Family**	**%**
E01QS0D / E01QS9D	1.203	E01QS1F / E01QS8F	1.451
E01QS2F / E01QS7F	1.692	E01QS3D / E01QS6D	1.272
E01QS4F / E01QS5F	1.510		

Table 4: Comparison of Simulation and Measurement Before and After applying Correction Factors for the Modified Lattice. $|\Delta Q|$ is the Absolute Difference Between Measured and Simulated Tune

	Before Change	**After Change**
$\lvert\Delta Q_x\rvert$	0.0651	0.0136
$\lvert\Delta Q_y\rvert$	0.0852	0.0020

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