IMPROVED RESISTIVE WALL EFFECTIVE RADIUS TAKING INTO ACCOUNT YOKOYA FACTORS

A. Gamelin*, V. Gubaidulin, Synchrotron SOLEIL, Saint-Aubin, France
S. Thakur, Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Abstract

When computing the resistive wall impedance, one must account for variations in beam pipe aperture, conductivity, and beta function. This can be done by either summing individual contributions or using an "effective radius" in an analytic formula. However, the standard "effective radius" formula has limitations: it ignores cross-plane influences and lacks formulas for quadrupolar or monopolar wakes. This paper introduces an improved "effective radius" definition using Yokoya factors, addressing these limitations. This updated formula was used to develop SOLEIL's new impedance model, significantly improving the agreement between simulated and measured instability thresholds.

INTRODUCTION

The resistive wall impedance, describing the way the beam interacts with the resistive part of the beam pipe walls, is a key component present in all impedance models made to study collective effects in synchrotrons. Most of the time, the resistive wall impedance is computed using various analytic expressions, either simple ones like the thick wall formula [1] or more complex ones like the multi-layered cylindrical formula implemented in codes like IW2D [2].

Because of its distributed nature over the full synchrotron circumference, the variation of beam pipe aperture, conductivity, and beta function along the accelerator must be taken into account. It can be achieved by splitting the synchrotron in different sections with the same properties and then summing the different contributions. Another way is to compute an "effective radius" [3], which can then be used in analytic expressions to get the resistive wall impedance for the full accelerator.

Both methods are mathematically equivalent, so if the input data is the same, it results in the exact same impedance for the overall accelerator. But the "effective radius" formalism can be more convenient to use and generally allows to be more precise in the description of the accelerator apertures (for example, to compute the resistive wall of a tapered chamber).

RESISTIVE WALL EFFECTIVE RADIUS

The resistive wall effect can be split into its short-range part (single bunch effect) and long-range part (multi-bunch and multi-turn effect). The short-range and long-range parts exhibit different scaling with respect to beam pipe radius a [3]. In the longitudinal plane, the resistive wall wake function is inversely proportional to a^2 for short-range and to a for the long-range regime. In the transverse plane, it is inversely proportional to a^4 for short-range and to a^3 for the long-range regime.

The resistive wall effective radius $a_{\text{eff}}^{(n)}$ for power n [3, Eq. (27)], is defined as:

$$a_{\text{eff}}^{(n)} = \left(\frac{\sqrt{\sigma^*}}{L\beta^*} \sum_i \frac{l_i \beta_i}{a_i^n \sqrt{\sigma_i}} \right)^{-1/n} \tag{1}$$

where the summation on the index i corresponds to the different beam pipe components in the ring, l_i is the component length, a_i its aperture, β_i the beta function at its location, σ_i its conductivity and $L = \sum l_i$ is the total length. The average beta β^* and average conductivity σ^* are defined by:

$$\beta^* = \frac{1}{L} \sum_i l_i \beta_i \quad \text{and} \quad \sigma^* = \frac{1}{L} \sum_i l_i \sigma_i \tag{2}$$

This formula can be applied to either horizontal or vertical planes using a_i as the corresponding aperture and different power of n to get effective radius for long and short range regimes.

Nevertheless, there are several issues with the present definition. It does not clearly define how the longitudinal effective radius should be computed. More generally, both for the longitudinal and transverse effective radius, the influence of one dimension on the other is neglected (for example the impact of the horizontal aperture on the effective vertical radius). Finally, the present definition does not take into account quadrupolar or monopolar wakes.

MODIFIED FORMULA USING YOKOYA FACTORS

We propose a new definition of the effective radius aiming to solve the issues raised above. The idea of the modified effective radius formula is that, to define each section of the accelerator, instead of using the horizontal or vertical aperture only, the smallest dimension r_{min} (e.g. semi-minor axis for an elliptical pipe) is used in addition to Yokoya factors ($Y_{\text{long}}, Y_{\text{xdip}}, Y_{\text{ydip}}, Y_{\text{xquad}}, Y_{\text{yquad}}$) [4].

For the longitudinal effective radius, it is defined as:

$$r_{\text{long}}^{(n)} = \left(\frac{\sqrt{\sigma^*}}{L} \sum_i Y_{\text{long}} \frac{l_i}{r_{\text{min}\,i}^n \sqrt{\sigma_i}} \right)^{-1/n} \tag{3}$$

Where $n = 1$ for long range and $n = 2$ for short range regime. For the horizontal and vertical dipolar effective radius, it is defined as:

$$r_{\text{xdip}}^{(n)} = \left(\frac{\sqrt{\sigma^*}}{L\beta_x^*} \sum_i Y_{\text{xdip},i} \frac{l_i \beta_{x\,i}}{r_{\text{min}\,i}^n \sqrt{\sigma_i}} \right)^{-1/n} \tag{4}$$

* alexis.gamelin@synchrotron-soleil.fr

$$r_{\text{ydip}}^{(n)} = \left(\frac{\sqrt{\sigma^*}}{L\beta_y^*} \sum_i Y_{\text{ydip},i} \frac{l_i \beta_{y\,i}}{r_{\min i}^n \sqrt{\sigma_i}} \right)^{-1/n} \tag{5}$$

For the horizontal and vertical quadrupolar effective radius, it is defined as:

$$r_{\text{xquad}}^{(n)} = \pm \left| \frac{\sqrt{\sigma^*}}{L\beta_x^*} \sum_i Y_{\text{xquad},i} \frac{l_i \beta_{x\,i}}{r_{\min i}^n \sqrt{\sigma_i}} \right|^{-1/n} \tag{6}$$

$$r_{\text{yquad}}^{(n)} = \pm \left| \frac{\sqrt{\sigma^*}}{L\beta_y^*} \sum_i Y_{\text{yquad},i} \frac{l_i \beta_{y\,i}}{r_{\min i}^n \sqrt{\sigma_i}} \right|^{-1/n} \tag{7}$$

With $\pm = +$ if the sum is positive, $\pm = -$ otherwise. For both dipolar and quadrupolar wakes, $n = 3$ for the long-range and $n = 4$ for short range regimes.

Thanks to the use of Yokoya factors, the modified definition is unambiguous for the longitudinal effective radius, considers the influence of one dimension on the other and provides formula for quadrupolar wakes.

The limitations of the effective radius obtained with this new definition are, in fact, the same ones as the Yokoya factors themselves [4]. It will work with ordinary metals, even if a metallic coating is applied, but it will not work for dielectric or ferromagnetic materials.

FORMULA FOR MONOPOLAR WAKE

When a beam is going off-center in a vacuum chamber, or when the chamber is right/left or top/down asymmetric, then a monopolar wake component appears [5–7].

Monopolar wake from an off-center device

The general wake function in the horizontal plane W_x can be written as [7]:

$$\begin{aligned} W_x(z, x_S, x_T) = W_{\text{mono}}(z) + W_{\text{dip}}(z)x_S \\ + W_{\text{quad}}(z)x_T + \dots \end{aligned} \tag{8}$$

Where x_S is the source particle, x_T is the test particle, z is the longitudinal distance between the two and W_{mono}, W_{dip}, W_{quad} are the monopolar, dipolar and quadrupolar wake function components. If the device is right/left symmetric then $W_{\text{mono}}(z) = 0$. In the same device, if the beam is off-center by an offset x_1, then we can write:

$$\begin{aligned} W_x(z, x_S, x_T) &= W_{\text{dip}}(z)x_1 + W_{\text{quad}}(z)x_1 \\ &+ W_{\text{dip}}(z)(x_S - x_1) \\ &+ W_{\text{quad}}(z)(x_T - x_1) + \dots \\ &\approx W_{\text{mono}}(z) + W_{\text{dip}}(z)x'_S \\ &+ W_{\text{quad}}(z)x'_T \end{aligned} \tag{9}$$

Where x'_S and x'_T are the new source and test positions in the referential translated by x_1 to be centered on the beam. This makes it clear that $W_{\text{mono}}(z) = W_{\text{dip}}(z)x_1 + W_{\text{quad}}(z)x_1$ for an off-center beam. Physically, moving the beam in one direction is similar to moving the chamber in the other direction. So, in first approximation, for a beam "on-center" in an asymmetric chamber, with right boundary x^+ and left boundary x^-, the offset of the chamber with respect to the beam Δ is given by:

$$\Delta = \frac{|x^+| + |x^-|}{2} - x^+ \tag{10}$$

Which hold only if the chamber asymmetry is mostly in the x (or y) axis plane (like, for example, a photon beam absorber). Then, the monopolar wake is given by:

$$W_{\text{mono}}(z) \approx W_{\text{dip}}(z)\Delta + W_{\text{quad}}(z)\Delta \tag{11}$$

Yokoya factor for monopolar wakes

In a similar manner as Yokoya, we can define a factor Y_{mono} which relates monopolar wake of chamber of arbitrary cross section W_{mono} to the dipolar wake of a circular chamber $W_{\text{dip,circ}}$:

$$W_{\text{mono}}(z) = Y_{\text{mono}} W_{\text{dip,circ}}(z) \tag{12}$$

Expressing the monopolar wake W_{mono} using previous expression, we get:

$$\begin{aligned} W_{\text{mono}}(z) &\approx W_{\text{dip}}(z)\Delta + W_{\text{quad}}(z)\Delta \\ &\approx Y_{\text{dip}} W_{\text{dip,circ}}(z)\Delta + Y_{\text{quad}} W_{\text{dip,circ}}(z)\Delta \\ &\approx \Delta(Y_{\text{dip}} + Y_{\text{quad}}) W_{\text{dip,circ}}(z) \end{aligned} \tag{13}$$

Which gives:

$$Y_{\text{mono}} \approx \Delta(Y_{\text{dip}} + Y_{\text{quad}}) \tag{14}$$

This formula for Y_{mono} can be used for beam offset ($= \Delta$) or for simple asymmetric beam pipe ($\Delta = \frac{|x^+|+|x^-|}{2} - x^+$). For more complex asymmetric beam pipes, a computation of Y_{mono} from its definition should be done.

Effective monopolar radius formula

In the same way as for dipolar and quadrupolar wakes, we propose a definition for the monopolar effective radius $r_{\text{mono}}^{(n)}$ using the monopolar Yokoya factor Y_{mono} defined above:

$$r_{\text{xmono}}^{(n)} = \pm \left| \frac{1}{L} \sqrt{\frac{\sigma^*}{\beta_{1/2,x}^*}} \sum_i Y_{\text{xmono},i} \frac{l_i \sqrt{\beta_{x\,i}}}{r_{\min i}^n \sqrt{\sigma_i}} \right|^{-1/n} \tag{15}$$

$$r_{\text{ymono}}^{(n)} = \pm \left| \frac{1}{L} \sqrt{\frac{\sigma^*}{\beta_{1/2,y}^*}} \sum_i Y_{\text{ymono},i} \frac{l_i \sqrt{\beta_{y\,i}}}{r_{\min i}^n \sqrt{\sigma_i}} \right|^{-1/n} \tag{16}$$

With $\pm = +$ if the sum is positive, $\pm = -$ otherwise. Where $n = 3$ for long range wake and $n = 4$ for short range wake (as monopolar wakes derive from dipolar and quadrupolar wakes). As monopolar wakes are only weighted by square root of beta function [1], we introduce a $\beta_{1/2}^*$ parameter:

$$\sqrt{\beta_{1/2}^*} = \frac{1}{L} \sum_i l_i \sqrt{\beta_i} \tag{17}$$

[1] Contrary to dipolar or quardupolar wakes, the monopolar wake potential expression don't depend on the transverse offset, therefore the $\sqrt{\beta}$ dependency only comes from averaging the transverse kicks received at different locations.

Table 1: Resistive Wall Effective Radius for SOLEIL Storage Ring ($\beta_x^* = 7.91$ m, $\beta_y^* = 8.11$ m, $\rho^* = 3.33 \times 10^{-8}\ \Omega$ m)

Formula	ID configuration	Long Range (mm)					Short Range (mm)				
		$r^{(1)}_{\text{long}}$	$r^{(3)}_{\text{xdip}}$	$r^{(3)}_{\text{ydip}}$	$r^{(3)}_{\text{xquad}}$	$r^{(3)}_{\text{yquad}}$	$r^{(2)}_{\text{long}}$	$r^{(4)}_{\text{xdip}}$	$r^{(4)}_{\text{ydip}}$	$r^{(4)}_{\text{xquad}}$	$r^{(4)}_{\text{yquad}}$
Original	Open	5.19	28.8	7.93	/	/	7.41	28.82	7.81	/	/
Modified	Open	6.22	10.97	8.45	-11.14	10.75	7.48	9.46	8.19	-9.55	9.80
Original	Closed	4.98	28.80	6.34	/	/	6.68	28.82	5.45	/	/
Modified	Closed	5.49	6.22	6.71	-6.25	8.50	5.85	4.94	5.66	-4.95	6.75

APPLICATION TO SOLEIL

SOLEIL is a third-generation synchrotron light source and has been in operation since 2006. The last complete impedance model for the SOLEIL storage ring dates back to 2004 [8, 9]. Comparisons between this model and measurements from 2007 revealed discrepancies of up to a factor of two [10].

A new impedance model for SOLEIL was developed to address previous discrepancies [11]. Key improvements include using the modified effective radius (this paper) for resistive wall calculations, the dominant impedance contributor, and adding quadrupolar resistive wall wakes via quadrupolar effective radius. The model's frequency range was extended from 30 GHz to 100 GHz using a finer mesh, and new machine components added over the past 20 years were incorporated.

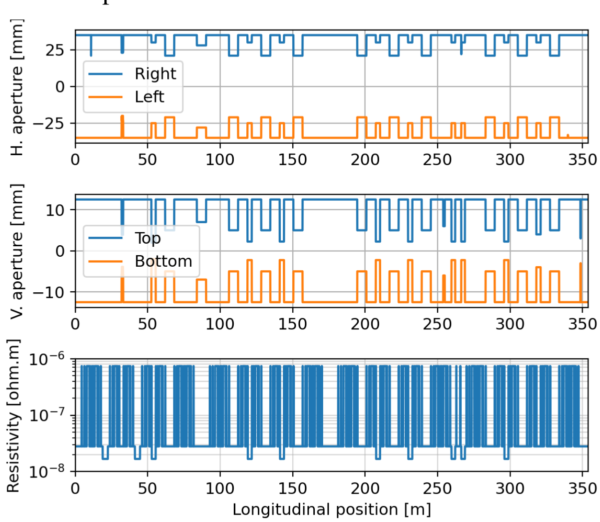

Figure 1: Vacuum chamber parameters in SOLEIL (with closed IDs). Top and center: horizontal and vertical aperture. Bottom: resistivity.

Figure 1 displays the physical aperture and resistivity values along SOLEIL's storage ring, used to compute the effective radii in Table 1. Yokoya factors for most chambers were derived using the elliptic beam pipe formula [12, 13].

The longitudinal effective radii $r^{(n)}_{\text{long}}$ remain similar between the original and modified formulas. However, the modified formula yields slightly larger vertical effective radii $r^{(n)}_{\text{ydip}}$ due to the larger horizontal aperture ($Y_{\text{ydip}} \approx 0.823$). For open IDs, the horizontal effective radii $r^{(n)}_{\text{xdip}}$ are significantly smaller in the modified formula, dominated by the smaller vertical aperture ($Y_{\text{xdip}} \approx 0.411$). In closed IDs, $r^{(n)}_{\text{xdip}}$ is even lower than the vertical radii, as the horizontal beta functions in low-gap sections are much higher, reducing the horizontal effective radius and increasing the beta-weighted transverse impedance in the horizontal plane—an effect also observed at ESRF [14].

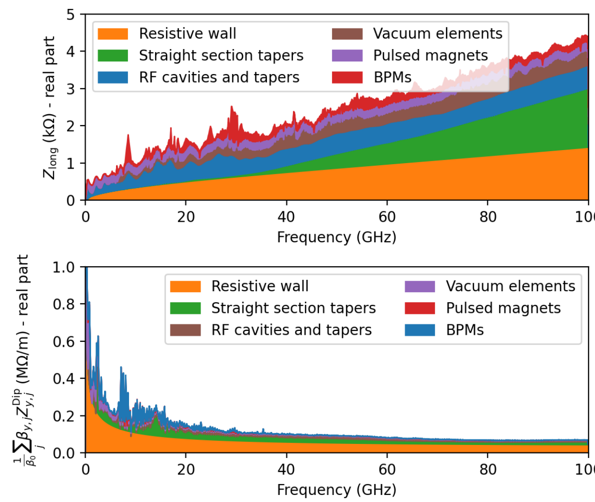

Figure 2: Impedance model of SOLEIL and its different components. Top: real part of longitudinal impedance. Bottom: real part of vertical dipolar impedance.

Both the usual and this improved resistive wall effective radius formalism have been implemented in the `ResistiveWallModel` class of mbtrack2 [15]. The resistive wall impedance is computed using `IW2D` [2] using the short-range effective radius, including a 0.5 µm layer of NEG (ρ_{DC} =1.25 × $10^{-6}\ \Omega$ m). Figure 2 shows the new impedance model of SOLEIL, with its real part of the longitudinal and vertical dipolar impedance. This new impedance model has been used in tracking simulation, using `mbtrack2` [15, 16], and allow to reach a much better agreement with respect to beam measurement compared to the 2004 model [17], in particular in the vertical plane, mainly due to the addition of quadrupolar wakes.

CONCLUSION

An improved "effective radius" resistive wall formula using Yokoya factors has been introduced, addressing several shortcomings with the usual definition. This formula has been used to make a new impedance model for SOLEIL and has helped to get a better agreement between measurement and simulation.

REFERENCES

[1] A. Chao, *Physics of collective beam instabilities in high energy accelerators*. Wiley, 1993.

[2] N. Mounet and E. Métral, “Electromagnetic field created by a macroparticle in an infinitel long and axisymmetric multilayer beam pipe”, Rep. CERN-BE-2009-039, 2009.

[3] G. Skripka, R. Nagaoka, M. Klein, F. Cullinan, and P. F. Tavares, “Simultaneous computation of intrabunch and interbunch collective beam motions in storage rings”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 806, pp. 221–230, Jan. 2016. doi:10.1016/j.nima.2015.10.029

[4] K. Yokoya, “Resistive wall impedance of beam pipes of general cross section”, *Part. Accel.*, vol. 41, pp. 221–248, 1993.

[5] S. T. Wang, J. D. Perrin, S. Poprocki, W. Hartung, and D. L. Rubin, “Vertical-longitudinal coupling induced by wakefields at the Cornell Electron-Positron Storage Ring Test Accelerator”, *Phys. Rev. Accel. Beams*, vol. 21, no. 2, p. 024401, Feb. 2018. doi:10.1103/PhysRevAccelBeams.21.024401

[6] G. Stupakov and D. Zhou, “Transverse wakefields due to asymmetric protrusions into a vacuum chamber”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 764, pp. 378–382, Nov. 2014. doi:10.1016/j.nima.2014.08.006

[7] C. Zannini, K. Li, and G. Rumolo, “Effects of an Asymmetric Chamber on the Beam Coupling Impedance”, in *Proc. IPAC’12*, New Orleans, LA, USA, May 2012, paper WEPPR073, pp. 3099–3101.

[8] R. Nagaoka, “Numerical Evaluation of Geometric Impedance for SOLEIL”, in *Proc. EPAC’04*, Lucerne, Switzerland, paper WEPLT081, pp. 2041–2043, Sep. 2004.

[9] R. Nagaoka, “Instability Studies Using Evaluated Wake Fields and Comparison with Observations at SOLEIL”, in *Proc. EPAC’06*, Edinburgh, UK, paper THPCH032, pp. 2847–2849, Jul. 2006.

[10] R. Nagaoka *et al.*, “Beam Instability Observations and Analysis at SOLEIL”, in *Proc. PAC’07*, Albuquerque, NM, USA, paper WEOBC01, pp. 2019–2021, Aug. 2007.

[11] S. Thakur, “Beam coupling impedance study and validation of instability models for SOLEIL”, Master thesis, Université Paris-Saclay, 2025.

[12] R. L. Gluckstern, J. van Zeijts, and B. Zotter, “Coupling impedance of beam pipes of general cross section”, *Phys. Rev. E*, vol. 47, no. 1, p. 656, 1993.

[13] M. Migliorati, L. Palumbo, C. Zannini, N. Biancacci, and V. Vaccaro, “Resistive wall impedance in elliptical multilayer vacuum chambers”, *Phys. Rev. Accel. Beams*, vol. 22, no. 12, p. 121001, 2019. doi:10.1103/PhysRevAccelBeams.22.121001

[14] T. F. Günzel, “Transverse coupling impedance of the storage ring at the European Synchrotron Radiation Facility”, *Phys. Rev. ST Accel. Beams*, vol. 9, no. 11, p. 114402, Nov. 2006. doi:10.1103/PhysRevSTAB.9.114402

[15] A. Gamelin, W. Foosang, and R. Nagaoka, “mbtrack2, a Collective Effect Library in Python”, in *Proc. IPAC’21*, Campinas, Brazil, May 2021, pp. 282–285. doi:10.18429/JACoW-IPAC2021-MOPAB070

[16] A. Gamelin, W. Foosang, N. Yamamoto, V. Gubaidulin, and R. Nagaoka, MBTRACK2, Jul. 2025, doi:10.5281/zenodo.15847797

[17] V. Gubaidulin, M. El Ajjouri, A. Gamelin, R. Sreedharan, and S. Thakur, “Beam-based Instability and Bunch-by-Bunch Feedback Characterization at SOLEIL”, in *Proc. IPAC’26*, Deauville, France, May 2026, this conference, paper WEP5106.