STUDIES ON OPTIMIZATION OF A TILTED-SOLENOID MUON IONIZATION COOLING CHANNEL IN G4BEAMLINE

C.-H. Nee*[1], R. Al-Harthy[1], P. Rao[1], J. Eldred[2], D. Stratakis[2],
K. Yonehara[2], M. Hedges[2], S. Jindariani[2], S. Dasu[1]
[1]University of Wisconsin–Madison, Madison, WI, USA
[2]Fermi National Accelerator Laboratory, Batavia, IL, USA

Abstract

Ionization cooling is essential for future muon colliders. We investigate a cooling-channel demonstrator using tilted solenoids to modify the lattice optics and introduce transverse-longitudinal coupling without dipole magnets. G4Beamline simulations are used to compare tilted-solenoid configurations with an IMCC reference lattice. Among the tested tilted cases, 0.75°–1° provides the best balance of six-dimensional emittance reduction, longitudinal stability, and transmission. Matching diagnostics show that the initial transverse-emittance decrease includes both ionization cooling and loss of large-amplitude particles from the unmatched input beam. Using a survival beam, the 1° tilted-solenoid lattice gives transverse cooling comparable to the IMCC reference. These results indicate that tilted-solenoid lattices are a promising cooling-channel option, with further optimization needed for longitudinal matching and transmission.

INTRODUCTION

Muon colliders are an attractive option for future high-energy lepton collisions because they can reach multi-TeV center-of-mass energies without the severe synchrotron-radiation limitations of electron machines. Their feasibility, however, depends critically on rapid reduction of the large initial muon phase space within the short muon lifetime [1]. Ionization cooling, in which momentum is reduced in absorbers and restored by RF cavities, remains one of the practical method for achieving this emittance reduction on the required timescale [2].

Recent studies within the International Muon Collider Collaboration (IMCC) have considered cooling-channel concepts that use dipole fields to generate dispersion and enable emittance exchange [1]. In this study, we investigate an alternative approach in which the lattice optics are modified through controlled tilting of solenoidal elements. The goal is to assess whether this tilted-solenoid configuration can provide useful transverse-longitudinal coupling while preserving the cooling performance of a periodic channel composed of solenoids, RF cavities, and wedge absorbers.

The study is intentionally restricted to the cooling channel itself; upstream production, capture, and matching sections are not included. This allows the intrinsic response of the lattice to be evaluated in a controlled manner. Using G4Beamline [3], we examine the dependence of beam cooling, longitudinal stability, and transmission on both the solenoid tilt angle and the RF phase.

SIMULATION SETUP

A low-energy μ^+ beam with mean momentum 200 MeV/c is injected directly into the cooling channel. The reference momentum and beam spreads are chosen to represent the entrance of an IMCC-inspired cooling channel after the upstream production, capture, and front-end sections for the demonstrater [1]. The beam is generated with a Gaussian distribution, with a time spread of 0.140 ns and a longitudinal momentum spread of 15 MeV/c. The upstream matching section is not included in this study, so the input distribution should be interpreted as a representative cooling-channel entrance beam rather than a fully matched beam for each tilted-solenoid configuration.

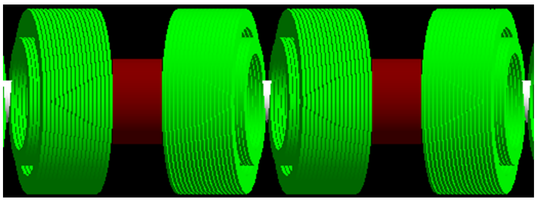

Figure 1: Visualization of one cooling-channel cell including solenoidal focusing elements (green), RF cavities (red), and wedge absorbers (white). Equal and opposite tilt angles about the horizontal (x) axis (vertical in this view), with an exaggerated 10°, are applied to the solenoid groups.

Table 1: Key Parameters of the Simulated Cooling Channel

Parameter	**Value**
Cell length	2000 mm
Number of cells	50
Reference momentum	200 MeV/c
RF frequency	704 MHz
RF gradient	30 MV/m
RF phase	20 deg
Large solenoid inner/outer radius	285 mm and 355 mm
Small solenoid inner/outer radius	185 mm and 245 mm
Large/small solenoid pancakes	17 / 5
Large/small solenoid current	328 A and 300 A
Tilt angle	scan: 0.25°–2°
Absorber material	LiH
Wedge angle	10 deg

The simulations are performed with G4Beamline [3], based on Geant4 [4], using the QGSP_BERT physics list.

* cnee@wisc.edu

Particle distributions are recorded at regularly spaced longitudinal positions and processed with ECALC9 to extract derived beam quantities, including transverse, longitudinal, and six-dimensional emittances, average longitudinal momentum, and transmission. The results are presented in two stages: first, a scan of solenoid tilt angles is performed; second, an RF-phase scan is used to evaluate the sensitivity of the preferred tilt configurations to longitudinal tuning.

RESULTS

Dependence on Solenoid Tilt Angle

The performance of the tilted-solenoid cooling channel is evaluated as a function of the solenoid tilt angle, while all RF parameters are fixed to the baseline configuration given in Table 1. For reference, the same input beam is also transported through the IMCC reference lattice. This comparison is intended to show the relative response of the tilted-solenoid optics under a common input condition, rather than a fully optimized comparison between independently matched lattices.

Figures 2–5 show the evolution of transverse emittance ($\epsilon_\perp$), longitudinal emittance (ϵ_L), six-dimensional emittance ($\varepsilon_{6\mathrm{D}}$), and particle transmission. All configurations show a rapid initial decrease in $\epsilon_\perp$ within the first $\sim$10–20 m, followed by a slower cooling regime, as shown in Fig. 2. Since the input beam is not separately matched to each tilted-solenoid configuration, this early decrease should not be interpreted as purely faster ionization cooling. It also includes preferential loss of large-amplitude particles during the initial mismatch transient. After this transient region, the $\epsilon_\perp$ depends only weakly on the tilt angle. The IMCC reference and the tilted-solenoid cases show broadly similar $\epsilon_\perp$ reduction, indicating that transverse cooling is relatively robust across the tested configurations. However, the ϵ_L in Fig. 3 shows a stronger tilt dependence. Smaller tilt angles perturb the optics less and therefore preserve the longitudinal phase space more effectively, while larger tilts introduce stronger optics distortions and larger longitudinal-emittance fluctuations.

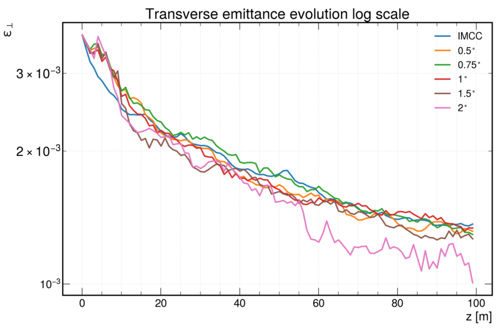

Figure 2: $\epsilon_\perp$ evolution for the IMCC reference and tilted-solenoid angle scan.

The combined effect is shown by the $\varepsilon_{6\mathrm{D}}$ in Fig. 4. Moderate tilt angles provide strong $\varepsilon_{6\mathrm{D}}$ reduction, but the transverse and $\varepsilon_{6\mathrm{D}}$ curves must be interpreted together with the transmission shown in Fig. 5. In particular, very large tilts can produce apparently low final $\epsilon_\perp$, but this occurs with substantially lower particle survival. The IMCC reference maintains the highest transmission for the same input beam, while the tilted-solenoid cases show stronger losses, especially for larger tilt angles. These results indicate that the optimal tilted-solenoid configuration is not selected by transverse cooling alone. The current best result is at moderate tilt angles, especially near $0.75°$–$1°$, where the channel gives strong $\varepsilon_{6\mathrm{D}}$ reduction while maintaining better longitudinal stability and transmission than the larger-tilt cases. The lower transmission relative to the IMCC reference indicates that the present tilted-solenoid lattice is not yet optimally matched, motivating the matching diagnostics discussed below.

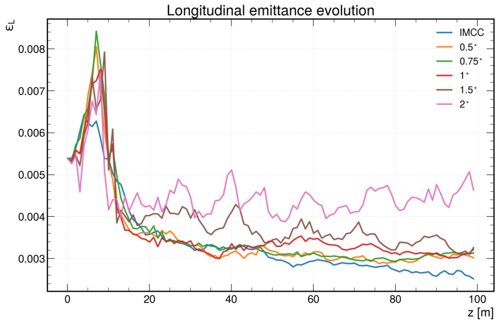

Figure 3: ϵ_L evolution for the IMCC reference and tilted-solenoid angle scan.

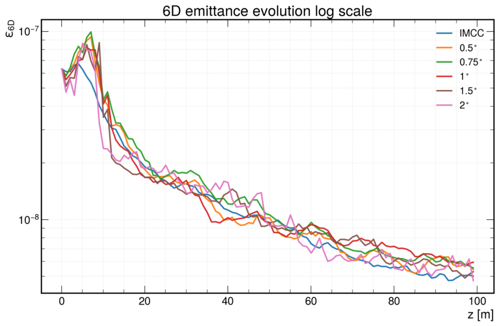

Figure 4: $\varepsilon_{6\mathrm{D}}$ evolution for the IMCC reference and tilted-solenoid angle scan.

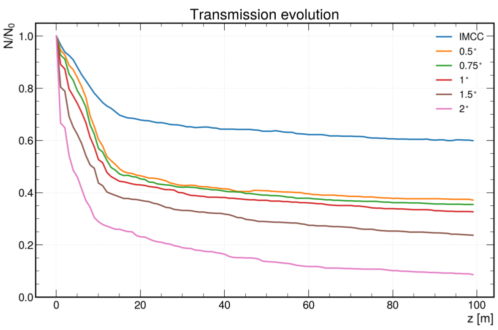

Figure 5: Particle transmission for the IMCC reference and tilted-solenoid angle scan.

Transmission and Matching Diagnostics

To separate cooling behavior from acceptance and particle-loss effects, an additional phase-space diagnostic is performed for the IMCC reference and the 1° tilted-solenoid case. Figure 6 compares the initial and final x–x' phase-space distributions at the z = 0.05 m and z = 99.05 m planes. The same input beam and wedge configuration are used for both lattices. Both lattices reduce the occupied transverse phase space for the surviving beam. However, as shown by the transmission comparison in Fig. 5, the survival is different: the IMCC reference retains about 60% of the particles to the final plane, while the 1° tilted-solenoid case retains about 33%. The lower survival in the tilted-solenoid case indicates that the input distribution is not yet well matched to the tilted-solenoid acceptance. This explains why part of the early $\epsilon_\perp$ reduction in Fig. 2 is caused by the mismatched particles rather than by ionization cooling alone. No dedicated matching section was included in this study so that the IMCC reference and tilted-solenoid lattices could be compared under identical input-beam conditions, allowing a direct comparison of the lattice response. Dedicated matching section, together with optimization of the wedge orientation and longitudinal acceptance, is therefore an important next step.

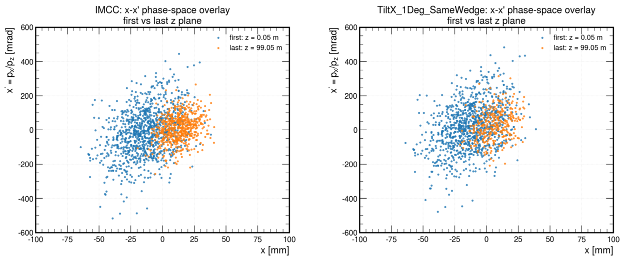

Figure 6: Initial and final x–x' phase-space distributions for the IMCC reference and 1° tilted-solenoid lattices using the same input beam.

Survival Beam Comparison

As a complementary diagnostic, a survival beam is constructed from particles that survive to a downstream reference plane, using their corresponding initial coordinates as the input distribution. The same survival beam is then transported through both the IMCC reference lattice and the 1° tilted-solenoid lattice. This comparison reduces the influence of early acceptance losses and provides a clearer test of the cooling behavior for the surviving beam core. Figure 7 shows that both lattices provide clear transverse cooling. The $\epsilon_\perp$ evolution of the 1° tilted-solenoid case closely follows that of the IMCC reference, indicating that the tilted-solenoid lattice can achieve comparable transverse cooling once the early acceptance-loss effect is removed. The ϵ_L evolution, however, remains better controlled in the IMCC reference toward the end of the channel. This suggests that the tilted-solenoid concept can reproduce the main transverse cooling effect of the reference lattice, while further optimization is still needed for longitudinal matching, emittance exchange, and transmission.

Dependence on RF Phase

After identifying 0.75°–1° as the preferred tilt range, an RF-phase scan is performed with all other lattice parameters fixed to evaluate the sensitivity of the channel to longitudinal matching. Figure 8 shows the evolution of the $\varepsilon_{6\mathrm{D}}$ and average longitudinal momentum. For phases between 10° and 30°, the $\varepsilon_{6\mathrm{D}}$ evolution is similar, indicating that the cooling performance is relatively insensitive to RF phase within this range. The corresponding $\langle p_z \rangle$ oscillations remain stable, showing that energy loss in the absorbers is adequately compensated by RF re-acceleration. At larger phase offsets, the momentum oscillations become less stable and the emittance reduction degrades, consistent with poorer longitudinal matching. These results indicate a useful RF-phase tolerance for the tilted-solenoid channel. A phase near 20°–25° is therefore used as a representative operating region for further optimization.

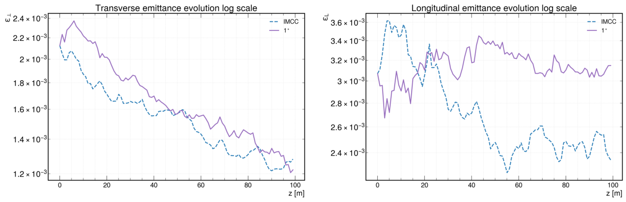

Figure 7: Transverse and ϵ_L evolution for the IMCC reference and 1° tilted-solenoid lattice using the survival input beam.

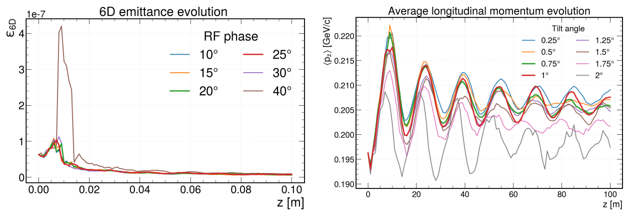

Figure 8: Dependence of cooling performance on RF phase. Left: evolution of the $\varepsilon_{6\mathrm{D}}$. Right: evolution of the average longitudinal momentum $\langle p_z \rangle$.

CONCLUSION

A tilted-solenoid muon cooling channel has been studied with G4Beamline and ECALC9. The tilt-angle scan shows robust transverse cooling, while longitudinal stability and transmission are more sensitive to optics perturbations. Among the tested tilted-solenoid cases, 0.75°–1° provides the best compromise between $\varepsilon_{6\mathrm{D}}$ reduction and particle survival. The RF-phase scan also indicates useful tolerance over 10°–30°. Comparison with the IMCC reference shows that the present tilted-solenoid lattice needs a dedicated matched input beam, and the rapid initial $\epsilon_\perp$ reduction includes both ionization cooling and scraping of large-amplitude particles. The survival-beam comparison shows that the 1° tilted-solenoid lattice can produce transverse cooling comparable to the IMCC reference. This demonstrates that tilted solenoids can provide an effective cooling-channel concept, while future work should improve input matching, wedge orientation, longitudinal acceptance, and end-to-end channel optimization.

REFERENCES

[1] The International Muon Collider Collaboration, “The muon collider,” CERN, Geneva, Switzerland, Rep. CERN-EDMS-3284682, May 2025.
`https://edms.cern.ch/document/3284682/1`

[2] D. Neuffer, “Ionization cooling of muon beams,” *Phys. Rev. ST Accel. Beams*, vol. 18, p. 031003, 2015. 10.1103/PhysRevSTAB.18.031003

[3] T. J. Roberts and D. M. Kaplan, “G4Beamline simulation program for matter-dominated beamlines,” in *Proc. PAC’07*, Albuquerque, NM, USA, Jun. 2007, pp. 3468–3470.
`https://jacow.org/p07/papers/thpan103.pdf`

[4] S. Agostinelli *et al.*, “Geant4—a simulation toolkit,” *Nucl. Instrum. Methods Phys. Res. A*, vol. 506, pp. 250–303, 2003. 10.1016/S0168-9002(03)01368-8