OPTIMISATION OF BEAM TRANSFER BETWEEN RCS IN THE MUON COLLIDER WITH SYNCHRONOUS PHASE

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ABSTRACT

The Muon Collider requires fast acceleration to preserve the intensity of the decaying muons. The current design uses a chain of four rapid cycling synchrotrons (RCS), where tens of turns are used per ring to accelerate from 63 GeV to 5000 GeV. Because of the short muon lifetime, the design uses injection and extraction while the magnets are ramping (accelerating buckets). Due to the different machine parameters and injection on accelerating buckets, there are longitudinal mismatches at transfer between the synchrotrons. By carefully controlling the synchronous phase at transfer, the mismatch can be reduced, which is essential for preserving beam quality. This contribution details the optimisation study and its predicted effect on emittance growth through the RCS chain.

INTRODUCTION

The Muon Collider provides the opportunity for higher precision than typical hadron colliders with higher energy than typical lepton colliders. The preliminary design, however, comes with unprecedented challenges in accelerator physics and engineering. Different options for the siting of the complex are being studied at CERN, Fermilab, and a greenfield project, with no constraints on site location or existing tunnel dimensions. The current baseline greenfield design [1] includes four rapid-cycling synchrotrons (RCS) with counter-rotating μ^+ and μ^- beams. Each accelerator operates with total radio-frequency (RF) voltages on the order of gigavolts, significantly higher than the megavolt-range voltages of the current energy frontier collider, the Large Hadron Collider. In order to achieve this, the baseline assumption is to have on the order of thousands of 1.3 GHz TESLA [2] cavities per ring.

To minimise muon decay by maximising the acceleration rate, injection and extraction for each accelerator are performed on-ramp (during acceleration), reducing the required cycle time. Parameter differences from one RCS to the next mean that the RF buckets will have different shapes. This will cause filamentation, as the bunch matched to the first bucket will not be matched to the second one. The filamentation will lead to longitudinal emittance growth, degrading beam quality. This problem is addressed in this paper, by optimising one machine parameter, the synchronous phase, ϕ_s.

For this study, the longitudinal beam dynamics code BLonD [3] was used. BLonD is a 2D tracking code that breaks the ring into one or more kick and drift sections. All terms that would change the particle energy (RF, wakefields, ...) are lumped into the kick, and the rest of the lattice is treated as a drift, which changes only the time coordinate. This enables faster computation than full 6D codes such as Xsuite [4]. During 2025–2026, BLonD has undergone a major rewrite, enabling extensions to make the code suitable for future accelerators, including the Muon Collider RCS chain [5].

The accelerators in the RCS chain differ from conventional synchrotrons in several ways. Particularly noteworthy in this context is the very high RF voltage, which is distributed across multiple RF stations. These high voltages lead to synchrotron tunes in the order of 10^{-1}, much higher than in the order of 10^{-3} seen in most synchrotrons. Relevant accelerator parameters are listed in Table 1. The optics used in this study is a 28 section version of the arcs from the greenfield study [1].

Table 1: RCS Parameters from [1]

RCS	Turns	Energy [GeV]	Circumference [m]
RCS 1	17	63-313	5990
RCS 2	55	313-750	5990
RCS 3	66	750-1500	10700
RCS 4	55	1500-5000	35000

MODIFICATIONS TO TRACKING CODE

Asynchronous Ramping

Typically, for longitudinal tracking, it is assumed that the beam and magnetic field are constant within one section. This approximation is not valid for the RCS chain due to the fast acceleration rate. To more accurately account for the changing field, the beam reference coordinates can be updated between RF stations.

The standard tracking equations are given by

$$\Delta E_{n+1} = \Delta E_n + g(\Delta t_n) - E_\mathrm{s},$$
$$\Delta t_{n+1} = \Delta t_n + f(\Delta E_{n+1}).$$

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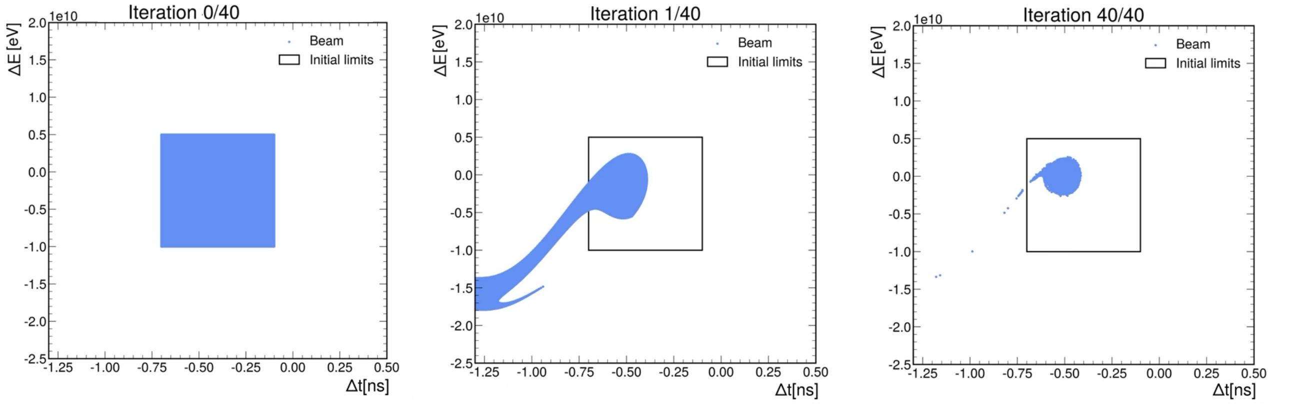

Figure 1: Different iterations of the numerical matching procedure, obtained by repeatedly tracking the first turn of the RCS. Starting from a uniform grid, the resulting distribution is reinjected at each step.

With one extra reference energy update, they become:

$$\begin{aligned}
\Delta E_{n+1} &= \Delta E_n + g(\Delta t_n) - E_\mathrm{s}, \\
\Delta t_{n+1} &= \Delta t_n + f(E_{n+1}), \\
\Delta E_{n+2} &= \Delta E_{n+1} - E_\mathrm{s}, \\
\Delta t_{n+2} &= \Delta t_{n+1} + f(\Delta E_{n+1}).
\end{aligned}$$

ΔE_n and Δt_n are the n^{th} energy and time coordinate, respectively, which are relative to the reference energy, E_s, which is the energy for the design orbit. Convergence is reached after approximately 4 extra steps between two RF stations in RCS1.

Higher-Order Momentum Compaction Factor

In addition to time-dependent effects, non-linearities in the lattice also play a significant role in shaping the longitudinal dynamics. The default drift equation implemented in BLonD (Eq. 1) uses the first-order phase-slip factor, which is sufficient for most cases.

$$\Delta t_{n+1} = \Delta t_n + \frac{T\,\eta_0}{\beta_s^2 E_s}\,\Delta E_n \qquad (1)$$

The momentum compaction factor, α_n, of the RCS chain was evaluated up to third order. Particles with a range of momentum deviations were tracked using Xsuite [4], and the longitudinal displacement was computed as a function of momentum deviation, $\zeta(\delta)$, with $\delta = \frac{\Delta E}{E_s \beta_s^2}$. Fitting a Taylor expansion to $\zeta(\delta)$ then allows α_n to be computed as

$$\zeta(\delta) = \alpha_0\delta + \alpha_1\delta^2 + \alpha_2\delta^3 + \dots . \qquad (2)$$

For the RCS1 arc, the momentum compaction factors obtained from the Taylor expansion are

$$|\alpha_0| = 0.0009, \quad |\alpha_1| = 0.0022, \quad |\alpha_2| = 0.0037. \qquad (3)$$

$|\alpha_0|$ is consistent with the value obtained with the Xsuite twiss calculation [4]. The higher-order term $|\alpha_1|$ is large, indicating that higher-order effects are non-negligible and motivating the use of a more complete drift model. The corresponding drift equation is

$$\Delta t_{n+1} = \Delta t_n + T\left(\frac{P(\delta_n)\left(1+\frac{\Delta E_n}{E_s}\right)}{1+\delta_n} - 1\right), \qquad (4)$$

where $P(\delta_n)$ is $1 + \alpha_0\delta + \dots$ up to the desired order. T is the revolution period of the reference particle. This is more computationally expensive than Eq. 1, but includes the non-linear contributions, which is necessary with large α_1 and α_2 terms.

Matching Routine

Due to the high synchrotron tune (Q_s) and accelerating bucket, existing matching routines within BLonD were not suitable, as they relied on a Hamiltonian based on a low Q_s approximation. To enable matching to arbitrary conditions, we developed a purely numerical matching routine. A grid of particles is injected and the first turn is tracked. The distribution is then re-injected into the first turn and tracked again. This process is repeated iteratively until the distribution is stable, an example can be seen in Fig. 1.

OPTIMISATION OF BUCKET TRANSFER

The RF bucket shape in each RCS is dependent on the accelerator parameters. In the previous iteration of the design, the synchronous phase was specified to be 135° in each RCS, using the convention that the stable phase is between 90° − 180°. This was chosen without considering transfer from one RCS to the next. To compare the bucket shapes, we first fill the bucket at injection to RCS1 and track the distribution to extraction. This distribution can be compared to a full bucket at RCS2 injection, as shown in Fig. 2.

The aspect ratios of the two bunches are different. This difference will cause the beam to filament at injection to RCS2. To mitigate this effect, ϕ_s, of each synchrotron is optimised to improve bucket-to-bucket matching between extraction and injection throughout the whole chain.

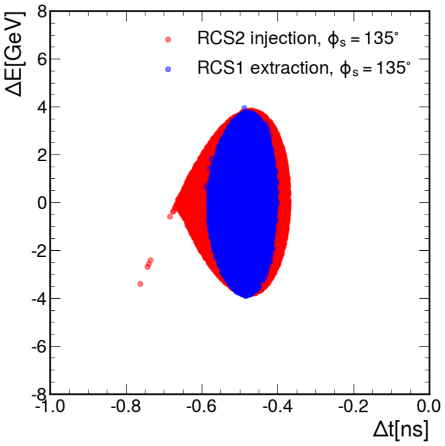

Figure 2: Bunch shapes at extraction from RCS1 and injection into RCS2 with the nominal ϕ_s of 135°.

Table 2: Synchronous phase, ϕ_s for each RCS for the greenfield siting and the optics from [1] with 28 arcs for all RCS.

RCS	**old ϕ_s [°]**	ϕ_s [°]
RCS 1	135	135
RCS 2	135	160
RCS 3	135	148
RCS 4	135	128

Optimal synchronous phases, which minimise longitudinal emittance growth have been found, without including intensity effects. First, end-to-end BLonD simulations of each RCS, for different ϕ_s values, were performed. In each case, the aspect ratio at injection and extraction was computed, plotted in Fig. 3.

Based on the computed aspect ratios, a numerical optimiser was used to minimise the difference across all accelerators within the range 115° – 160°. The resulting phases are shown in Table 2. The relative longitudinal emittances and bunch lengths throughout the RCS chain are plotted in Fig. 4 for ϕ_s = 135° in each RCS and in Fig. 5 the optimised ϕ_s, both with no intensity effects. In the future, this optimisation should be extended to include intensity effects and any updates to the accelerator optics.

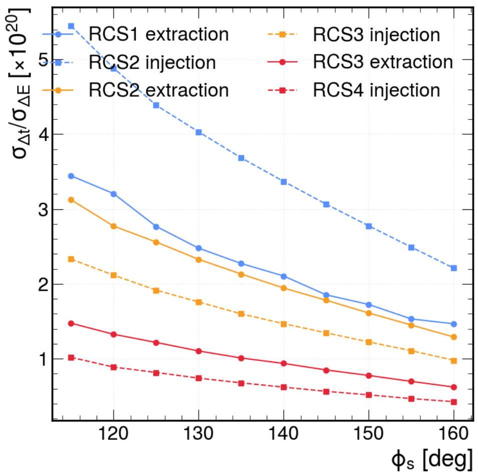

Figure 3: Aspect ratio of bunches as a function of ϕ_s at injection and extraction for RCS1-4.

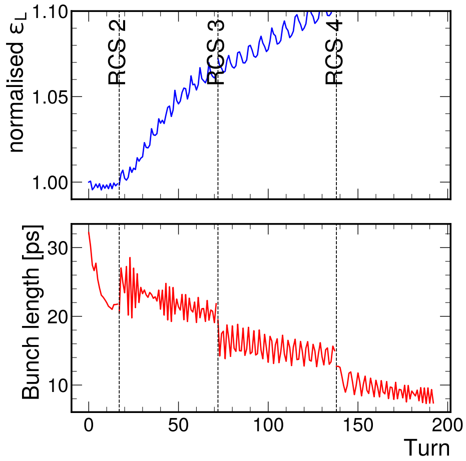

Figure 4: Emittance and bunch length evolution along the RCS chain, with the nominal ϕ_s. This assumes 28 RF stations with the optics from [1].

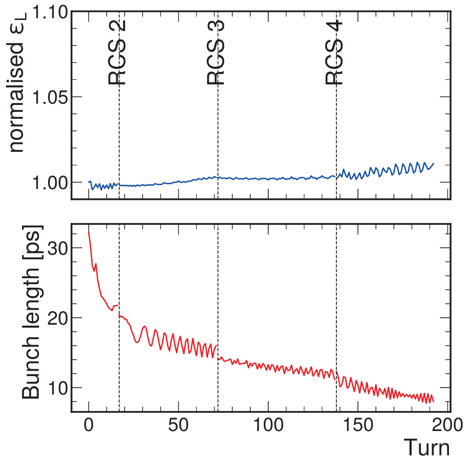

Figure 5: Emittance and bunch length evolution along the RCS chain, with the optimised ϕ_s. This assumes 28 RF stations with the optics from [1].

CONCLUSION

An optimised set of ϕ_s has been determined numerically for the RCS chain of the Muon Collider, based on a 28 RF station version of the optics, which reduces the emittance growth along the chain. To support this study, the longitudinal tracking code BLonD was extended during the comprehensive upgrade of BLonD v3 [5]. These developments enable more accurate modelling of the longitudinal beam dynamics in the Muon Collider RCS chain and provide a foundation for further studies as the parameters evolve.

ACKNOWLEDGEMENTS

This work is endorsed by the International Muon Collider Collaboration (IMCC).

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