# RECIRCULATION MODELLING FOR ENERGY-RECOVERY LINACS*

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*Abstract*

A flexible simulation scheme has been developed as a high-level Python wrapper around RF-Track to enable fully self-consistent simulations of recirculating and energy-recovery linear accelerators (ERLs). This framework dynamically routes multi-bunch beams through multiple beamlines based on their evolving phase-space parameters and is integrated directly into the RF-Track tracking workflow. Within this scheme, a wakefield model has been implemented that enables higher-order modes to be self-consistently excited and evaluated over successive recirculation passes. This approach enables realistic modelling of multi-pass beam dynamics, including the accumulation of wakefields from previously transported bunches. Two independent benchmarking campaigns have been carried out. The recirculation scheme has been benchmarked against MAD-X and Bmad/Tao, demonstrating consistent beam transport and routing behaviour across multiple passes. Separately, the wakefield model was validated against Xwakes, showing close agreement and confirming the correct implementation of wakefield effects. These results demonstrate the robustness of the approach and provide a pathway toward comprehensive start-to-end simulations of ERL operation.

## INTRODUCTION

Energy-recovery linear accelerators (ERLs) [1, 2] represent an important advancement in modern accelerator physics, offering high beam power and excellent beam quality while significantly reducing overall power consumption. Simulating these machines presents unique computational challenges, as the beam is recirculated through the same Radio-Frequency (RF) structures over multiple passes to enable energy recovery, thereby coupling the dynamics of accelerating and decelerating bunches. To fully capture these complex interactions, a modern and flexible computational approach is required to simulate recirculating lattices effectively. The primary motivation for this work is to develop a highly adaptable tool for seamless start-to-end simulations of recirculating accelerators, enabling advanced beam dynamics studies and providing a flexible platform for the design and optimisation of future ERLs.

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## MODELLING METHODS

### *Recirculation Model*

A flexible recirculation scheme has been developed and integrated as an extension of RF-Track [3], introducing a high-level machine controller that enables self-consistent tracking of recirculating beams through arbitrary lattice configurations. The core of the method is a dynamic routing and tracking algorithm that operates on individual bunches while preserving full six-dimensional phase space information and allowing collective effects such as wakefields to accumulate across multiple passes.

The machine is initialised from a user-defined beam distribution and a lattice description, provided as a structured configuration of seperate beamlines. During initialisation, this description is parsed and converted into RF-Track-compatible tracking objects, with each element mapped to its corresponding representation, including RF cavities, drifts, and multipole magnets. RF cavities are constructed from either analytical models or external electromagnetic field maps, with automatic scaling to match the desired accelerating voltage and phase.

A routing table defines the connectivity between beamlines, specifying the next beamline to be traversed after each pass of an individual bunch. This routing can be static or conditional. In the latter case, it selects beamlines dynamically based on desired bunch properties such as average beam energy, enabling energy-dependent routing that is essential for multi-pass recirculating lattices common in ERLs.

The framework additionally supports the generation and transport of arbitrary bunch trains, where individual bunch properties such as charge, timing, phase-space coordinates, and initial offsets can be independently specified. This allows realistic modelling of non-uniform filling patterns and injection scenarios.

Tracking is performed using an event-driven queue. Each bunch is associated with a global time coordinate and is inserted into a priority queue that ensures chronological processing. At each step, the earliest bunch is extracted, tracked through the current beamline, and then reinserted into the queue with an updated global time coordinate and phase-space. The simulation proceeds until all bunches have exited the machine or no further routing is defined (Fig. 1).

The final output is assembled as a combination of all individual tracking steps. In addition, the complete traversal history of each bunch is retained, enabling detailed diagnostics, beam evolution studies, and analysis of recirculation dynamics.

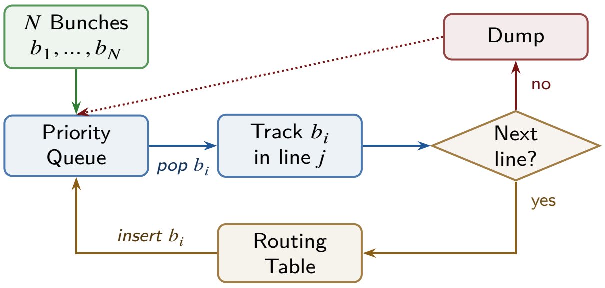


Figure 1: Schematic overview of the recirculation tracking scheme.

### *Wakefield Model*

The wakefield formulation for standing-wave structures is based on a modal decomposition, where the wake response of an element is represented as a sum of resonant modes characterised by their angular frequency $\omega_k$, shunt impedance $R_k$, and quality factor $Q_k$,

$$W(t) = \sum_k W_k(t;\ \omega_k, R_k, Q_k). \tag{1}$$

Each mode is treated as a complex phasor with damped oscillatory behaviour, analogous to an RLC resonator [4].

The temporal evolution of a mode is governed by a complex propagation factor

$$B_k = i\omega_k - \frac{\omega_k}{2Q_k}, \tag{2}$$

where the first term describes oscillation at frequency $\omega_k$ and the second describes exponential decay on a timescale $2Q_k/\omega_k$. Each bunch or bunch slice acts simultaneously as a source term and a test term. As a source term traverses the structure, it excites the electromagnetic modes according to its charge and transverse distribution moments. The resulting modal amplitudes are retained and propagated in time, allowing subsequent test terms to experience the residual wakefields generated by earlier source terms. Both longitudinal and transverse wakefield components are included, with higher-order modes (HOMs) incorporated through a multipole expansion. The inclusion of longitudinal higher-order mode wakefields naturally accounts for HOM-driven energy loss, consistent with the fundamental theorem of beam loading [4]. Additionally, the formulation supports both symmetric and asymmetric mode descriptions, enabling realistic representation of wakefields up to the n-th order.

The wakefield contribution of a given mode is written by separating source excitation from test evaluation. Denoting the transverse and longitudinal components by subscripts $u$ and $z$ respectively,

$$W_{k,u}(t) = \Im\left\{A_k\, e^{B_k t} \sum_j C_j\, e^{-B_k t_j}\right\}, \tag{3}$$

$$W_{k,z}(t) = \Re\left\{D_k\, e^{B_k t} \sum_j E_j\, e^{-B_k t_j}\right\}. \tag{4}$$

In Eqs. (3) and (4), the summation represents the source term, where each slice or bunch $j$, arriving at time $t_j$, excites the mode with strengths $C_j$ and $E_j$ determined by its charge, a combination of its transverse distribution moments, and the shunt impedance of the mode. By causality, only previously transported slices contribute to the complex sum, ensuring that wakefields of a given slice or bunch act exclusively on subsequent particles. This accumulated excitation is stored as a single complex phasor per mode, so the full distribution history of prior bunches need not be retained. In this way, both short-range (intra-bunch) and long-range (multi-bunch) effects are preserved. The factor $e^{B_k t}$ corresponds to the evaluation of the test particle. It describes how the stored modal amplitude evolves and is sampled by a particle arriving at time $t$. The coefficients $A_k$ and $D_k$ encode the transverse and longitudinal coupling strengths of the mode, including the appropriate multipole scaling [5].

Wakefield forces are applied directly to macroparticles during tracking and scale with the local charge and effective interaction length. An optional history mechanism records the evolution of modal amplitudes at all excitation times, enabling post-processing of the cavity response, including voltage reconstruction and instability analysis.

The implementation was carried out within the `RF-Track` framework through the `UserEffect` interface, which allows users to define custom collective effects as a `Python` or `Octave` class. In this work, the interface was used to implement wakefield effects through a `Python` class interacting directly with the `RF-Track` C++ core, enabling transparent integration into the particle-tracking workflow while preserving compatibility with the general lattice and recirculation framework.

Together with the recirculation formalism, this approach provides a unified, time-domain description of wakefield effects suitable for studies of Beam Breakup (BBU) instabilities, transient and steady-state beam loading, and other collective phenomena relevant to the operation and optimisation of energy-recovery linacs.

## BENCHMARK RESULTS

### *Recirculation Benchmark*

The recirculation scheme has been benchmarked against `MAD-X` [6] and `Bmad/Tao` [7] using the PERLE [8] 250 MeV lattice (version 2.4.250). The same lattice description and optical configurations were reproduced across the three frameworks to ensure consistency of the comparison.

Beam transport was first validated on a pass-by-pass basis by comparing key optical functions and trajectory parameters. Good agreement was observed in the evolution of beam energy, transverse optics, and path length across all passes. In addition, the routing logic of the recirculation model was systematically verified by tracking multiple bunches through successive turns and confirming that beamline transitions occurred as expected.

Figure 2 shows the longitudinal evolution of $\beta_x$, $\beta_y$, $\sigma_x$, and $\sigma_y$ across all six passes alongside the relative differences with respect to `MAD-X` and `Bmad/Tao`. The results are consistent throughout the lattice, with relative deviations

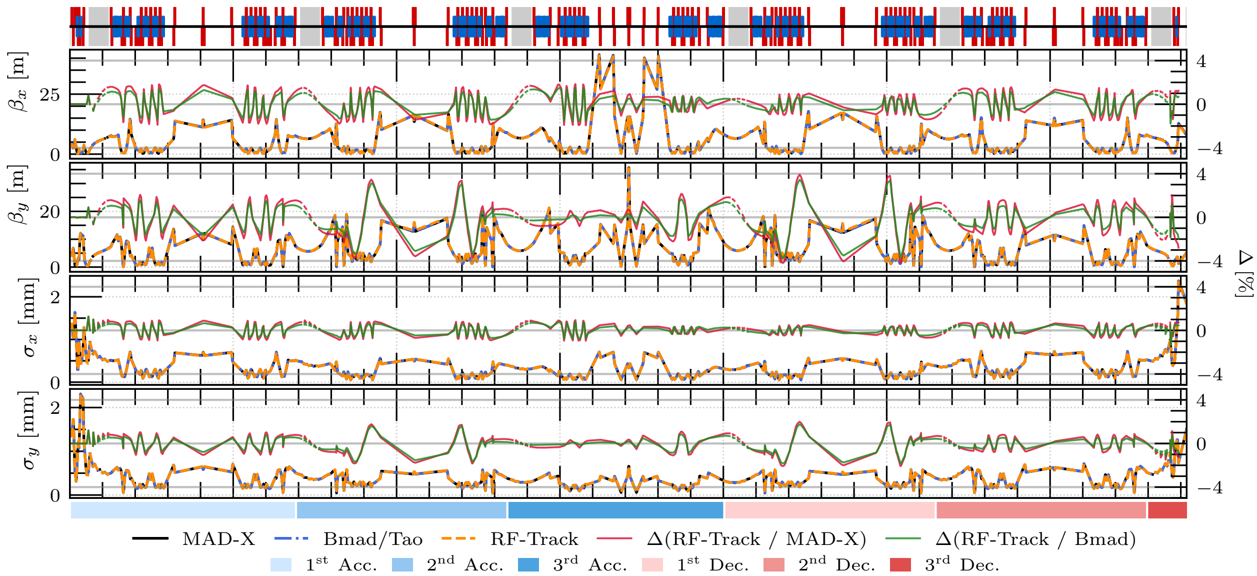


Figure 2: Optics comparison along the PERLE 250 MeV lattice (version 2.4.250). The top strip shows the machine layout: dipoles (blue), quadrupoles (red), and RF cavities (grey). The next four panels show the statistical horizontal and vertical beta functions ($\beta_x$, $\beta_y$) and the corresponding RMS beam sizes ($\sigma_x$, $\sigma_y$) as functions of the longitudinal coordinate $s$. The right-hand axis of each panel shows the relative difference between codes. The coloured band at the bottom identifies the six passes through the recirculating linac.

remaining below 2 % for the transverse beta functions and beam sizes. The residual discrepancies originate inside the RF cavities and are attributed to differences in the RF cavity models adopted by each code, in particular in the treatment of the fringe fields.

This benchmark confirms that the implementation accurately reproduces the expected beam transport and routing in a realistic multi-pass ERL lattice, validating its use for more advanced recirculating beam dynamics studies.

### *Wakefield Benchmark*

The wakefield implementation has been benchmarked against the `Xwakes` module of `Xsuite` [9, 10], which provides a validated reference for the application of wakefields in time-domain beam dynamics simulations. The comparison focused on both wake functions and the resulting transverse and longitudinal kicks for the monopole, dipole, and quadrupole components.

A series of test cases were constructed using identical modal parameters and beam configurations in both frameworks. The resulting wake functions and kicks were compared directly between the two implementations, with close agreement observed across all cases. Deviations in the wake functions were not greater than $\mathcal{O}(10^{-15})$, consistent with the limits of double-precision numerical precision. The corresponding differences in kicks were slightly larger, at the level of $\mathcal{O}(10^{-12})$–$\mathcal{O}(10^{-13})$, reflecting accumulated floating-point rounding errors during tracking [5].

These results confirm the accuracy of the modal formulation and its integration within the tracking scheme, demonstrating that both the excitation and the time evolution of higher-order modes are accurately reproduced.

## CONCLUSION

A flexible and self-consistent simulation framework for recirculating and energy-recovery linacs has been developed as a high-level extension to `RF-Track`. The approach combines dynamic bunch routing with a modal wakefield model, enabling multi-pass tracking with wakefield effects accumulated across successive recirculation turns. Two benchmarking campaigns demonstrate the validity of both the recirculation scheme and the wakefield implementation, establishing a reliable foundation for accurate start-to-end simulations of ERL operation.

Future work will focus on further optimisation of the framework and its integration directly into the core `RF-Track` codebase to improve performance and scalability. This development will enable efficient large-scale simulations of multi-bunch operation in complex recirculating machines. The tool provides a foundation for detailed studies of beam breakup instabilities, transient and steady-state beam loading, and general beam dynamics in ERL and recirculating lattice configurations.

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