PRELIMINARY ANALYSIS OF TRANSIENT BEAM-LOADING EFFECTS IN ELECTRON STORAGE RINGS WITH A 3RD HARMONIC CAVITY

T. Y. Lee*, E.-S. Kim†, Korea University Sejong Campus, Sejong, South Korea
K. H. Kim, Grand Accélérateur National d'Ions Lourds, Caen, France

Abstract

In a 4th generation electron storage ring, harmonic cavities are often employed for increasing bunch length and beam lifetime. Effective bunch lengthening can be achieved when the first derivative of total accelerating voltage is near zero. However, the accelerating voltage fluctuation due to transient beam-loading (TBL) effect can affect the bunch lengthening in the harmonic cavity system. Therefore, we utilize both a Haissinski equation solver and a macroparticle tracking code to predict equilibrium beam distribution under TBL effects. The equilibrium longitudinal beam distributions obtained from both approaches are presented and cross-validated for different fill patterns.

INTRODUCTION

Korea-4GSR, the first fourth-generation electron storage ring project in South Korea, has been planned to be constructed in Ochang [1]. To increase the beam lifetime and the threshold current of longitudinal beam instabilities, third-harmonic cavity (3HC) systems will be adopted in Korea-4GSR project. The additional accelerating voltage provided by 3HC, flattens the longitudinal potential and increases the rms bunch length.

However, when a non-uniform fill pattern is applied, the additional voltage given by 3HC can deviate from the designed value due to beam-cavity interaction. It leads to the bunch-to-bunch variations in the beam induced voltage. Consequently, since the total accelerating voltage is no longer identical across the bunch train, the bunch profile (i.e., the bunch centroid and rms length) also becomes asymmetric. This phenomenon is known as *transient beam loading* (TBL) effects, which is a major concern in the fourth-generation light sources adopting the non-uniform fill pattern.

There are two methods to predict the bunch profile under TBL effects. One is a semi-analytical method, which numerically solves the Haissinski equation. The Haissinski equation describes the equilibrium beam distribution, and a fixed-point iteration method is commonly used to obtain its solution. The other is the macroparticle tracking code method, which requires more computational resources than semi-analytical method. In contrast to semi-analytical method, it can predict not only steady state, also dynamic behavior.

For a cross-validation of two methods, the different fill patterns are considered. The specific configurations of fill pattern are shown in Table 1.

* satori1103@korea.ac.kr
† eskim1@korea.ac.kr

Table 1: Fill Patterns Used for Simulations

	Train	Number of Train
Uniform	1332f	1
Single train	1024f + 308g	1
Multi train	250f + 83g	4
Hybrid	901f + 215g + 1f + 215g	1

The total average beam current I_0 is fixed to 400 mA, is the target nominal current of Korea-4GSR. However, excluding the single train case, these fill patterns are hypothetical and are not currently considered for the Korea-4GSR storage ring.

LONGITUDINAL MOTION WITH 3HC

Equation of Motion w/o Beam Effects

In an electron storage ring, the equation of longitudinal motion with a 3HC can be written as follows:

$$\begin{aligned} \frac{d\tau}{dt} &= \alpha_c \delta \\ \frac{d\delta}{dt} &= \frac{1}{E_0 T_0}\Big(V_1 \sin(h\omega_0\tau + \phi_{s1}) \\ &\quad + V_2 \sin(3h\omega_0\tau + \phi_{s2}) - U_0\Big) \end{aligned} \tag{1}$$

where α_c is a momentum compaction of a ring, h is RF harmonics, E_0 is a beam energy [eV], $T_0 = 2\pi/\omega_0$ is a revolution period, and U_0 is a radiation loss per turn [eV]. (V_1, ϕ_{s1}) and (V_2, ϕ_{s2}) denote the amplitude and phase of a main RF voltage and a 3rd harmonic RF voltage, respectively.

Flat Potential Condition

The term inside the round bracket in the second expression of Eq. 1 represents a one turn energy gain or *accelerating voltage* $V_{acc}(\tau; V_1, V_2, \phi_{s1}, \phi_{s2})$. For effective bunch lengthening, $V_{acc}(\tau)$ should satisfy the following *flat potential condition* [2]:

$$\begin{aligned} \sin(\phi_{s1}) &= \frac{m^2}{m^2-1}\sin(\phi_{s0}) \quad (\frac{\pi}{2} < \phi_{s1} < \pi) \\ V_2 &= V_1\sqrt{\frac{1}{m^2} - \frac{1}{m^2-1}\sin^2(\phi_{s0})}, \quad (V_2 > 0) \\ \tan(\phi_{s2}) &= -\frac{m\sin(\phi_{s0})}{\sqrt{(m^2-1)^2 - m^4\sin^2(\phi_{s0})}} \quad (-\frac{\pi}{2} < \phi_{s2} < 0) \end{aligned} \tag{2}$$

where $\sin(\phi_{s0}) = U_0/V_1$, $m = 3$ since a 3HC is considered.

Changes in Synchrotron Motion

From Eq. 1 and replacing $\phi = h\omega_0\tau$, the Hamiltonian $H(\phi, \delta)$ is given by:

$$H(\phi,\delta) = \frac{\alpha_c}{2}\delta^2 + U(\phi)$$
$$U(\phi) = \frac{1}{h\omega_0 E_0 T_0}[V_1\cos(\phi+\phi_{s1}) \tag{3}$$
$$+\frac{V_2}{3}\cos(3\phi+\phi_{s2}) + U_0\phi - C]$$

where $C = V_1\cos(\phi_{s1}) + \frac{V_2}{3}\cos(\phi_{s2})$ denotes the integration constant ensuring that $U(0) = 0$.

Using action-angle variables (J, θ), the oscillation period can be expressed as:

$$H(\phi,\delta) = E$$
$$J(E) = \frac{1}{\pi}\int_{\phi_{\min}}^{\phi_{\max}}\sqrt{\frac{2(E-U(\phi))}{\alpha_c}}d\phi \tag{4}$$
$$\omega_s(E) = \dot{\theta} = \frac{\partial H}{\partial J} = \left(\frac{dJ}{dE}\right)^{-1}$$

where $\phi_{\max}$, $\phi_{\min}$, and ω_s are tuning points and an angular frequency of the synchrotron motion, respectively. The energy constant E is a dimensionless quantity, corresponds to the energy offset relative to the design energy E_0.

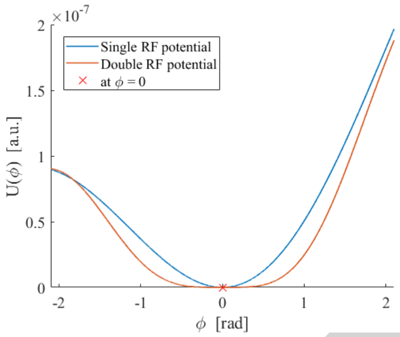


Figure 1: Comparison of potential well $U(\phi)$ between single RF vs. double RF system.

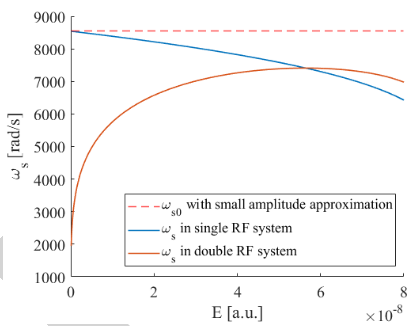


Figure 2: Comparison of energy-dependent synchrotron frequency using numerical integration of Eq. 4.

With a small amplitude approximation ($E \ll 1$), the rms bunch length σ_τ is $\alpha_c\sigma_\delta/\omega_s$, where σ_δ is a natural rms energy spread. Thus, it shows that the reduced longitudinal RF focusing $(-dU/d\phi)$ by 3HC leads to decreases in ω_s. Consequently, the rms bunch length is increased.

ESTIMATION OF BUNCH DISTRIBUTION UNDER TBL EFFECTS

In the previous section, it was assumed that $V_{acc}(\tau)$ is purely generated by an external driven source. However, in general, the beam is another driven source. Especially in the electron storage ring, since the beam is stored and circulating, this beam loading effect contributes significantly $V_{acc}(\tau)$.

If the uniform fill pattern (i.e., All RF buckets are equally occupied) is used, the equilibrium beam loading voltage is as follows [3]:

$$V_{b,eq}(\tau) \approx -2I_0 R_s\cos(\psi)\cos(mh\omega_0\tau - \psi) \tag{5}$$

where R_s denotes a cavity shunt impedance, m is harmonic number of cavity (e.g., $m = 1$ means main cavity), the detuning angle is defined as $\psi = \mathrm{atan}(2Q(\omega_r - mh\omega_0)/\omega_r)$. Q is a cavity quality factor, ω_r is cavity resonant angular frequency.

As shown in Eq. 5, the beam loading voltage is in sinusoidal form; thus, it can be easily incorporated into the Eq. 1 case. However, the non-uniform fill pattern that contains gaps (i.e., empty RF bucket) or high charge bunch is generally used in actual operation of the storage ring. With a non-uniform pattern, each bunch experiences a different accelerating voltage since the beam loading voltage is no longer identical across the bunch train.

This TBL effect causes the variations in the bunch profile and may even lead to the longitudinal beam instabilities. In this paper, we consider a 'weak' beam loading regime in which the TBL effect modifies only the equilibrium bunch distribution without inducing longitudinal beam instabilities.

Haissinski Equation

Haissinski equation, which describes the relationship between the accelerating voltage and the equilibrium bunch distribution $\rho(\tau)$, can be written as [4]:

$$\rho(\tau) = \rho_0\exp\left(-\frac{\Phi(\tau;\rho(\tau))}{\alpha_c\sigma_\delta^2}\right) \tag{6}$$

where $\Phi(\tau;\rho(\tau))$ is equivalent to the potential $U(\phi = h\omega_0\tau)$. The difference of the symbol from U to Φ is used to emphasize that beam-induced effects are considered in the potential. ρ_0 is normalizing factor. The potential $\Phi(\tau)$ with beam-induced effects can be expressed as:

$$\Phi(\tau) = -\frac{1}{E_0 T_0}\int_{\tau_s}^{\tau} V_{acc}(\tau';\rho(\tau'))d\tau' \tag{7}$$

τ_s is the synchronous position satisfying $V_{acc}(\tau_s) = 0$. The total accelerating voltage consists of two parts, the beam-independent part $V_{\mathrm{ext}}(\tau)$ and the beam-dependent part $V_{\mathrm{beam}}(\tau;\rho(\tau))$. To solve this Haissinski equation, the fixed-point iteration method [5] or Newton's method [6] is typically used to solve Haissinski equation numerically.

Macroparticle Tracking

In the macroparticle tracking code, the bunch distribution is approximated by an ensemble of macroparticles with an individual phase space coordinate (τ_{ij}, δ_{ij}), where i denotes the macroparticle index in a bunch, j is the bunch index.

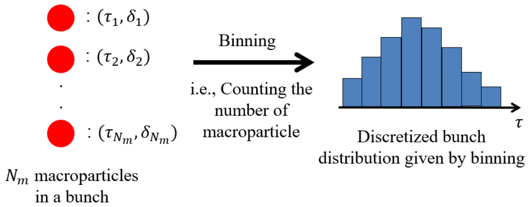


Figure 3: How to represent the time evolution of bunch distribution in the macroparticle tracking

From Eq. 1, we can update (τ_{ij}, δ_{ij}) of each macroparticle as follows:

$$\begin{aligned} \tau_{(n+1)} &\approx \tau_{(n)} + T_0 \frac{d\tau}{dt} = \tau_{(n)} + \alpha_c T_0 \delta_{(n)} \\ \delta_{(n+1)} &\approx \delta_{(n)} + T_0 \frac{d\delta}{dt} = \delta_{(n)} + \frac{1}{E_0} V_{acc}(\tau_{(n)}) \end{aligned} \tag{8}$$

where $_{(n)}$ means the turn number.

RESULTS

The simulation parameters are presented in Tables 2 and 3, in which the superconducting 3HC is assumed.

Table 2: Korea-4GSR Ring Parameters with 9 IDs [1]

	Value	Unit
Beam energy, E_0	4	GeV
Circumference, C	799.297	m
Momentum compaction, α_c	7.775e-5	
Average beam current, I_0	400	mA
Radiation loss per turn, U_0	1449	keV
Initial rms bunch length, σ_τ	9.33	ps
Natural rms energy spread, σ_δ	0.104	%
Longitudinal damping time, τ_s	10.81	ms
Revolution frequency, f_0	375.07	kHz

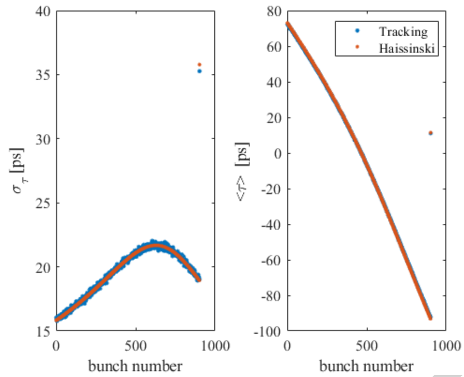
Table 3: Main and Harmonic RF Parameters [1, 7]

	Main	3rd	Unit
RF voltage, V_1	3.5	-(passive)	MV
Unloaded quality factor, Q_0	29909	3e8	
Total R/Q	1136.8	191.4	Ω
Detuning frequency Δf_r	-29.54	55	kHz
Coupling beta	4.5	1.0	

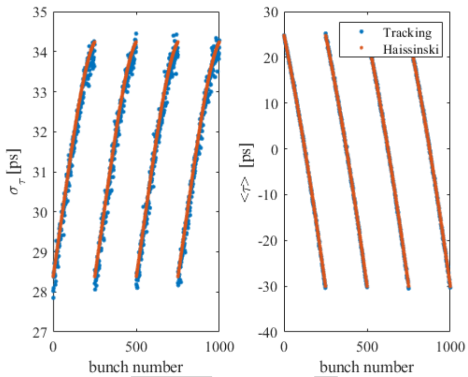
The simulation results with both the macroparticle tracking [8] and the semi-analytic Haissinski solver [6] as follows:

CONCLUSION

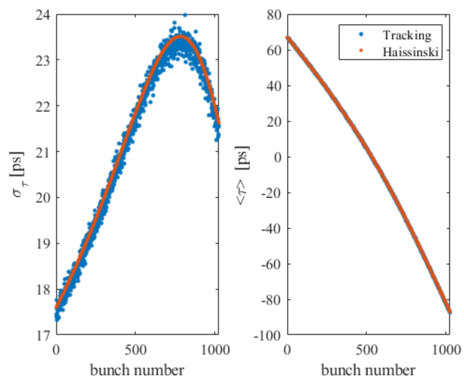
We investigate variations in the bunch profile caused by TBL effects for different non-uniform fill patterns. The effect



(a) Single train fill



(b) Multi train fill



(c) Hybrid fill

Figure 4: Comparison between macroparticle tracking and Haissinski eqaution solver for different non-uniform fill patterns

was evaluated using two independent methods: the Haissinski equation solver and the macroparticle tracking code, *STABLE*. The results are in good agreement. However, how to recover these undesirable variations has not been addressed and remains for future works.

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