UPDATED DESIGNS OF 6D AND FINAL COOLING CHANNELS FOR A MUON COLLIDER

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Abstract

The muon collider is regarded as a strong candidate for probing physics beyond the current energy frontier, combining the advantages of lepton collisions with the potential to reach multi-TeV center-of-mass energies. To obtain the required luminosity, the six-dimensional (6D) emittance of the muon beam must be reduced by several orders of magnitude within the short muon lifetime. This reduction relies on ionization cooling and is realized through two major stages: 6D cooling and final transverse cooling. In this work, we present updated lattice designs for both the rectilinear 6D cooling channel and the final cooling channel. The new design achieves a twofold decrease in the final transverse emittance, representing a significant step toward satisfying the beam-quality demands of a future muon collider.

INTRODUCTION

Electron–positron colliders are attractive because they provide clean collision environments, making events simpler to analyze than those produced in hadron machines. A major limitation for circular electron–positron colliders at high energy is synchrotron radiation, which is already significant for multi-GeV electron beams and becomes increasingly severe toward the multi-TeV regime because of the low electron mass. Muons, by contrast, are much heavier and therefore suffer far less from synchrotron radiation, while still remaining elementary leptons. For this reason, muon colliders are considered a promising option for future high-energy physics research [1]. A central challenge for the entire muon-collider complex is the short muon lifetime, which places stringent constraints on beam production, cooling, acceleration, and collision. In addition, muon beams generated from pion decay have emittances far larger than the acceptance of downstream acceleration systems. As a result, dedicated cooling is required before acceleration. The full cooling channel contains two parts: 6D cooling and final cooling. The 6D cooling system itself is divided into a pre-merge section, which reduces the emittance to the level required for bunch merging [2] ($\varepsilon_T \leq 1.3$ mm, $\varepsilon_L \leq 1.7$ mm), and a post-merge section, which cools the beam again after the emittance growth introduced by bunch merging and further reduces it to ease the demands on final cooling. The final cooling channel must then decrease the transverse emittance to 22.5 μm [3] in order to meet the luminosity target. In this paper, we report updated designs for both the 6D and final cooling channels. Relative to earlier studies [4, 5], G4beamline v3.08 simulations [6] show that the revised designs reduce the output transverse emittance by about a factor of two, while the longitudinal emittance at the end of final cooling remains below the target value of 64 mm [3].

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PRINCIPLE OF MUON COOLING

Because the muon lifetime is only 2.2 μs in its rest frame, ionization cooling is the only practical cooling approach for muon beams. In an ionization-cooling channel, muons lose both transverse and longitudinal momentum while passing through an absorber via ionization energy loss. The lost longitudinal momentum is then restored by RF cavities, while the lattice is arranged so that the transverse momentum is not restored, resulting in a net decrease of the transverse emittance. The evolution of the normalized transverse emittance is given by [7]:

$$\frac{d\varepsilon_T}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\varepsilon_T}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_T E_s^2}{2E_\mu m_\mu c^2 L_R}. \tag{1}$$

where ε_T denotes the normalized transverse emittance, E_μ is the muon energy in GeV, m_μ is the muon mass, β is the particle velocity normalized to the speed of light c, β_T is the transverse beta function, dE_μ/ds is the energy loss per unit length, E_s is the characteristic scattering energy, and L_R is the radiation length of the absorber. The first term describes cooling, while the second represents heating caused by multiple scattering.

To achieve longitudinal cooling, dispersion must be introduced at the absorber location, typically by using a wedge absorber. Under dispersion, particles with higher momentum pass through a thicker region of the wedge and therefore lose more energy, which reduces the longitudinal emittance. In this situation, the equilibrium transverse and longitudinal emittances can be written as [7]:

$$\varepsilon_T^{eq} = \frac{\beta_T E_s^2}{2\left|\frac{dE_\mu}{ds}\right|\beta g_T m_\mu c^2 L_R}. \tag{2}$$

$$\varepsilon_L^{eq} = \frac{\beta_L m_e c^2 \gamma^2 (1 - \frac{\beta^2}{2})}{2 g_L \beta m_\mu c^2 [\frac{\ln(K\gamma^2\beta^2)}{\beta^2} - 1]}. \tag{3}$$

where g_T and g_L are the transverse and longitudinal partition numbers, respectively, defined as

$$g_T = 1 - \frac{D}{2w}. \quad (4)$$

$$g_L = \frac{2\gamma^2 - 2\ln[K(\gamma^2 - 1)]}{\gamma^2 \ln[K(\gamma^2 - 1)] - (\gamma^2 - 1)} + \frac{D}{2w}. \quad (5)$$

with D the dispersion, w the distance from the beam centroid to the wedge apex, γ the Lorentz factor, and $K = 2m_e c^2/I$, where I is the mean excitation energy.

6D COOLING CHANNEL DESIGN

Figure 1 presents the overall configuration of the 6D cooling channel. The channel is divided into pre-merge and post-merge cooling sections, each composed of multiple stages labeled A1, A2, …for the pre-merge part and B1, B2, …for the post-merge part. Every stage consists of a sequence of identical cooling cells. The lattice of a representative cell used in this study is shown in Fig. 2, and follows the general concept of the earlier design [4]. In the present version, however, the dipole field is no longer produced by tilting the solenoids. Instead, separate dipole magnets are introduced, which makes the dipole component easier to adjust, for example during commissioning. As illustrated in Fig. 2, each cooling cell contains solenoids of alternating polarity, dipole magnets to generate dispersion, RF cavities to compensate the ionization energy loss, and liquid-hydrogen (LH_2) wedge absorbers.

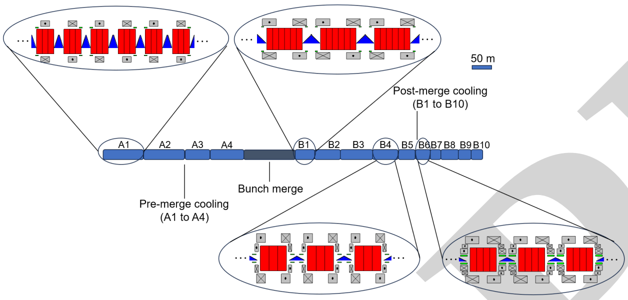

Figure 1: Overview of the rectilinear cooling channel (diagrams in the ellipses are zoomed-in schematics illustrating the layout of one specific cooling stage)

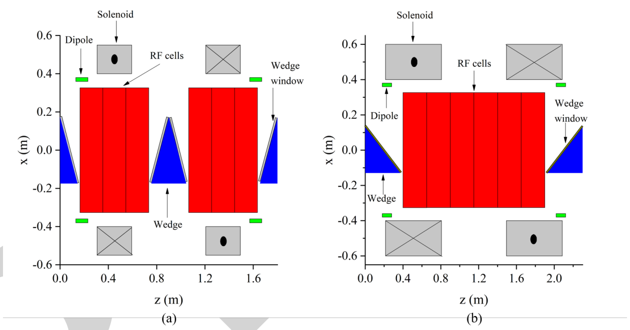

Figure 2: Schematics of two cooling cell layouts in 6D cooling: (a) A1; (b) B1

Using an optimization procedure based on the differential-evolution algorithm, we obtained an updated design for the

Table 1: Simulated normalized emittance and transmission at the end of each stage in 6D cooling channel

Stage	Length [m]	ε_T [mm]	ε_L [mm]	ε_{6D} [mm^3]	Trans-mission
Start	–	16.96	45.53	13 500	–
A1	104.4	5.165	18.31	492.6	75.2%
A2	106.8	2.473	7.113	44.03	84.4%
A3	64.8	1.556	3.880	9.594	85.6%
A4	86.8	1.239	1.741	2.861	91.3%
Start	–	5.129	9.991	262.5	–
B1	50.6	2.881	9.085	76.0	85.2%
B2	66.6	1.991	6.579	26.7	89.4%
B3	84.0	1.271	4.050	6.734	87.5%
B4	66.0	0.9353	3.161	2.828	89.8%
B5	44.0	0.7034	2.511	1.322	89.4%
B6	38.5	0.4823	2.291	0.5527	88.4%
B7	28.0	0.3855	2.064	0.312	92.8%
B8	46.15	0.2649	1.861	0.1307	87.9%
B9	33.8	0.190	1.716	0.0613	85.2%
B10	29.61	0.1396	1.558	0.030 09	87.1%

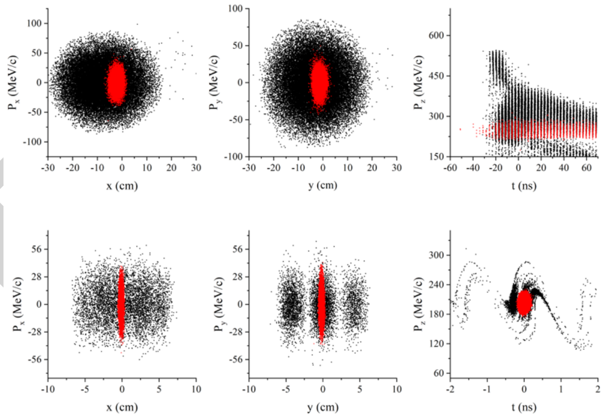

Figure 3: Particle distribution in phase space at the start and end of the 6D cooling channel (top: pre-merge, bottom: post-merge).

6D cooling channel. The simulated normalized emittances and transmission efficiencies at the end of each stage are listed in Table 1. The pre-merge section contains four stages with a total length of 362.8 m. Over this section, the transverse emittance decreases from 16.96 mm to 1.239 mm, and the longitudinal emittance decreases from 45.53 mm to 1.741 mm, with an overall transmission of 49.6%, including muon decay. The post-merge section contains ten stages and has a total length of 487.26 m. In this part, the transverse and longitudinal emittances are reduced from 5.129 mm and 9.991 mm to 0.1396 mm and 1.558 mm, respectively, while the overall transmission is 28.5%, again including muon decay. Compared with the earlier design [4], the revised channel lowers the final transverse emittance by approximately a factor of two. This smaller transverse emittance at the exit of 6D cooling is especially beneficial for the downstream final

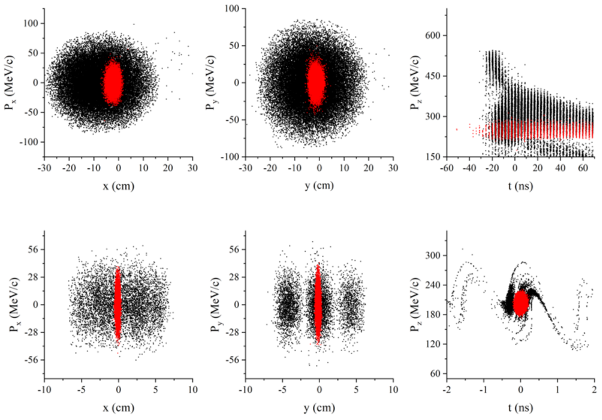

Figure 4: Particle distribution in phase space at the start and end of the 6D cooling channel (top: pre-merge, bottom: post-merge)

cooling channel. More detailed information on the optics and hardware parameters of the 6D cooling channel can be found in Ref. [8]. Figure 4 shows the beam phase-space distributions at the entrance and exit of the 6D cooling channel, demonstrating a clear shrinkage of the phase-space volume.

FINAL COOLING CHANNEL DESIGN

The layout of a cooling cell in the final cooling channel is shown in Fig. 5. Each cell includes one high-field solenoid together with several matching solenoids that help limit emittance growth. The solenoid polarity alternates from one stage to the next in order to suppress the buildup of angular momentum. Stage 0 is different from the later stages because it contains no RF cavities. In contrast to previous studies [5], we also removed the RF cavities operated at zero accelerating phase that had been introduced for phase rotation. Our simulations show that eliminating these cavities does not worsen the cooling performance.

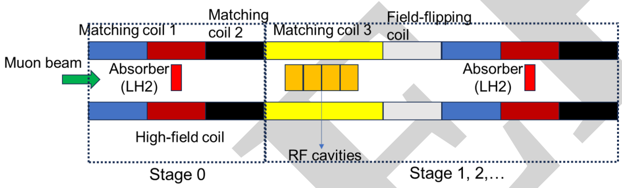

Figure 5: Cooling cell layout in final cooling

We study two possible input conditions for final cooling: a smaller-emittance beam taken from the end of stage B10 and a larger-emittance beam taken from the end of stage B8 of the 6D cooling channel. For each case, a dedicated final cooling channel was designed using a differential-evolution optimization procedure, with the goal of minimizing the final transverse emittance while preserving acceptable transmission. The resulting performance is summarized in Tables 2 and 3. In both cases, the transverse emittance reaches the target value of 22.5 μm. However, the case with the smaller input emittance ends with a longitudinal emittance that is nearly half that of the larger-input case, mainly because the required channel is shorter and contains fewer stages. Along the channel, the average longitudinal momentum is gradually decreased so that smaller transverse beta functions can be achieved in the absorbers, allowing lower equilibrium emittances. The peak solenoidal field is kept near 40 T, consistent with present high-field-solenoid concepts [9]. The beam phase-space distributions at the entrance and exit of the final cooling channel are shown in Fig. 6.

Table 2: Simulated normalized emittance and transmission at the end of each stage in final cooling channel (small input emittance case)

Stage	Length [m]	ε_T [μm]	ε_L [mm]	Transmission	Average p_z [MeV/c]	Peak B_z [T]
Start	–	140	1.52	–	95	–
0	2.04	125.1	1.916	99.5%	79.2	40
1	4.67	79.75	5.237	91.1%	46.8	–29.3
2	4.63	53.46	7.695	85.9%	37.1	39.4
3	3.89	40.63	10.88	92.4%	31.5	–41
4	4.12	28.79	15.69	92.8%	28.3	39.6
5	5.07	22.82	22.07	92%	26.5	–42.7

Table 3: Simulated Normalized Emittance and Transmission at the End of Each Stage in the Final Cooling Channel (large input emittance case)

Stage	Length [m]	ε_T [μm]	ε_L [mm]	Transmission	Average p_z [MeV/c]	Peak B_z [T]
Start	–	260.4	1.832	–	135	–
0	2.76	210.8	2.547	99.6%	95.4	33.6
1	5.2	160.3	5.171	90.5%	65	–36
2	5.4	121.7	8.688	88.5%	53	35.5
3	4.27	94.85	10.34	90.9%	46.2	–41.8
4	5.2	62.89	15.07	90.3%	36.1	40.9
5	6.84	40.95	22.74	84.7%	31	–41.3
6	5.17	32.17	31.97	93.7%	30	38.4
7	5.57	22.43	42.68	82.2%	28	–43.4

CONCLUSION AND OUTLOOK

In this work, we have presented updated designs for both the 6D cooling channel and the final cooling channel of a muon collider. Simulation results indicate that the final cooling channel can reduce the transverse emittance to 22.5 μm while maintaining the longitudinal emittance below the present target. In the future, we plan to implement new space-charge and other collective-effect modules in G4beamline in order to assess their influence on channel performance more realistically. In parallel, we will continue to work with hardware experts to develop more realistic models for solenoids, RF cavities, absorbers, and other essential components.

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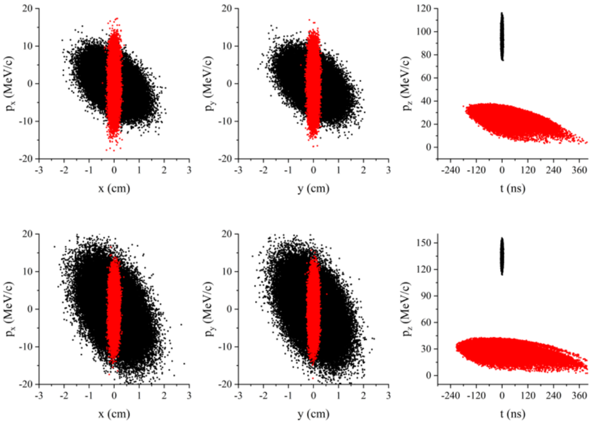

Figure 6: Particle distribution in phase space at the start and end of the final cooling channel (top: small input emittance case, bottom: large input emittance case)

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