MULTI-PARAMETER MITIGATION OF CSR-INDUCED EMITTANCE GROWTH IN 3D BEAMS

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Abstract

Phase space degradation due to Coherent Synchrotron Radiation (CSR) presents a significant challenge in the generation of high-brightness electron beams. In this work, we use a new 3D CSR code, currently in development, to understand the combined effect of longitudinal profile shaping and shielding in beams of large transverse size. Our results indicate that longitudinal profile shaping alone produces modest improvements in the high-Derbenev regime, while the addition of shielding walls preserves the level of improvement achievable in the 1D limit, even at gap sizes significantly larger than the conventional shielding threshold for a Gaussian beam.

INTRODUCTION

Collective effects such as Coherent Synchrotron Radiation (CSR) within a compressed electron bunch place stringent limits on the achievable beam brightness in X-ray free-electron laser (FEL) facilities and linear colliders. As a short, high-intensity bunch propagates through the bends of a magnetic bunch compressor, the tail-head CSR interaction induces correlated energy changes along the bunch, leading to increased projected transverse emittance [1] and the seeding of microbunching instabilities [2].

Several mitigation strategies have been explored. Shielding the bend magnets with parallel conducting plates reduces the long-range CSR interaction and has been demonstrated experimentally to suppress CSR-induced energy spread [3]. Tuning the optical lattice can cancel the emittance growth through careful design of the dispersion function [4]. Longitudinal profile shaping — preparing the initial current profile so that the integrated CSR wake is uniform along the bunch — was proposed in [5] and demonstrated to reduce emittance growth by more than an order of magnitude in the 1D limit. This result was further updated in [6] through a multi-parameter optimization study that also included shielding walls placed at gap sizes larger than $(R\sigma_z^2)^{1/3}$, the threshold at which shielding can independently mitigate energy spread for a Gaussian bunch.

The accuracy of 1D CSR models depends critically on the Derbenev parameter

$$\kappa = \frac{\sigma_x}{\sigma_z}\left(\frac{\sigma_z}{R}\right)^{1/3}, \tag{1}$$

which characterizes the relative importance of the transverse beam structure on the CSR interaction. For $\kappa \ll 1$ the 1D approximation holds; for $\kappa \gtrsim 1$ the transverse variation of the wake plays a significant role and cannot be compensated by longitudinal shaping alone. As a result, there is a gap in the literature in understanding the effects of profile shaping, particularly in conjunction with shielding for beams of large transverse size. In the rest of this work, we show preliminary profile shaping results for beams of high κ and explore its effectiveness in minimizing CSR induced energy spread.

NUMERICAL FORMULATION

The radiated electric and magnetic fields at any point in space and time are computed following Jefimenko's equations:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int d\mathbf{r}'\left[\frac{\mathbf{R}}{|\mathbf{R}|^3}\rho_r + \frac{\mathbf{R}}{c|\mathbf{R}|^2}\dot{\rho}_r - \frac{\dot{\mathbf{J}}_r}{c^2|\mathbf{R}|}\right], \tag{2}$$

$$\mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi}\int d\mathbf{r}'\left[\frac{\mathbf{R}\times\mathbf{J}_r}{|\mathbf{R}|^3} + \frac{\mathbf{R}\times\dot{\mathbf{J}}_r}{c|\mathbf{R}|^2}\right], \tag{3}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, subscript r denotes evaluation at the retarded time t_r, and the retardation condition $c(t - t_r) = |\mathbf{R}|$ is solved using the algebraic form derived by Cai and Ding [7]:

$$\xi = \alpha - \frac{\beta}{2}\sqrt{\chi^2 + \zeta^2 + 4(1+\chi)\sin^2\alpha}, \tag{4}$$

with $\chi = x/R$, $\zeta = y/R$, $\xi = -\ell/2R$, and α the retarded half-angle. The resulting longitudinal and transverse wake kernels are integrated over α from the magnet entrance to the observation point. Near $\alpha = 0$ the kernel is singular; this is handled by a change of variables $\rho = \alpha^2$ in a small *bubble* region around the singularity, after which standard Gauss quadrature applies throughout. The fields are computed on a moving mesh that follows the bunch, with the fields interpolated onto the particle coordinates at each step. The charge and current densities entering Jefimenko's equations are represented as sums of smooth 3D Gaussian shape functions (although any smooth function will suffice):

$$\rho(\mathbf{r}',t) = Q_b \sum_i w_i S_i(\mathbf{r}'(t)), \tag{5}$$

$$\mathbf{J}(\mathbf{r}',t) = Q_b \frac{\partial \mathbf{r}'(t)}{\partial t}\sum_i w_i S_i(\mathbf{r}'(t)), \tag{6}$$

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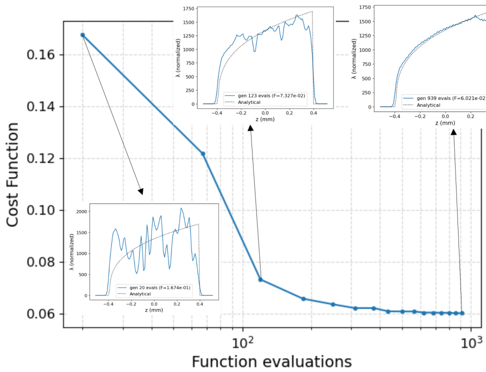


Figure 1: Convergence of the cost function F vs. number of function evaluations. In the first ~100 evaluations the optimizer primarily reduces the wake spread; the roughness penalty then progressively smooths the profile.

where Q_b is the bunch charge and S_i is a Gaussian shape function in $\mathbb{R}^3$. Our method effectively extends the 1D technique of Mayes and Hoffstaetter [8] to transversely staggered integration lines.

The coefficients w_i are binned and computed and stored at each timestep, and as a result, the evolution of the bunch is captured in the wake computation, resulting in self-consistency between radiated fields and evolving profile shape. Shielding by parallel conducting plates at gap H is implemented via the method of images. Validation of this approach against 1D results is reported in [6, 9].

OPTIMIZATION SETUP

To derive a current profile that minimizes the CSR-induced energy spread in a single dipole (and thus emittance growth at the end of a chicane), we tune the amplitudes $\{w_i\}$ of $N_G = 32$ longitudinal Gaussian shape functions to minimize

$$F(\mathbf{x}) = F_{\text{wake}}(\mathbf{x}) + \alpha\, F_{\text{rough}}(\mathbf{x}), \tag{7}$$

where the wake term is the weighted RMS energy deviation,

$$F_{\text{wake}}(\mathbf{x}) = \sqrt{\iint [\Delta E(\mathbf{r}) - \langle \Delta E \rangle]^2 \rho(\mathbf{r})\, d\mathbf{r}}, \tag{8}$$

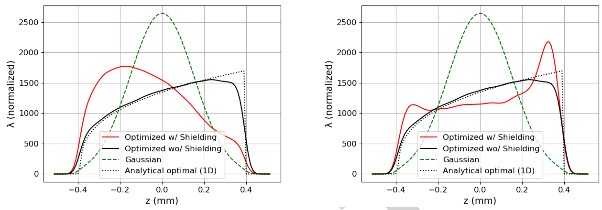
and the roughness penalty suppresses noisy solutions by minimizing the curvature (second derivative of the longitudinal profile $\lambda(z)$):

$$F_{\text{rough}}(\mathbf{x}) = \frac{1}{\max_z \lambda} \sqrt{\int \left[\frac{d^2\lambda(z;\mathbf{x})}{dz^2}\right]^2 dz}. \tag{9}$$

The weight α is tuned so that the roughness term contributes approximately half of the wake term at convergence. The transverse profile is assumed to be a fixed bi-Gaussian with RMS width σ_x, σ_y. Optimization uses the PatternSearch algorithm from PyMoo [10], And the final solution is smoothed further by increasing the widths of each Gaussian by 15%. The convergence behavior is illustrated in Fig. 1. The optimizer first reduces F_{wake} in the initial ~100 evaluations, finding the macroscopic profile shape; the roughness penalty then dominates, smoothing out high-frequency fluctuations while preserving the gross profile structure. The beam and lattice parameters used in this study are summarized in Table 1. All simulations were performed at 1 GeV to isolate the dependence on the Derbenev parameter independently of energy scaling.

Table 1: Beam and Lattice Parameters

Parameter	**Value**
Bending angle	25°
Bending radius	2.19 m
Beam energy	1 GeV
Bunch length (RMS)	150 μm
Horizontal size (RMS)	0.1–5 mm
Vertical size (RMS)	10 μm
Derbenev criterion κ	0.05–2.5



(a) Low Derbenev ($\kappa \approx 0.05$) (b) High Derbenev ($\kappa \approx 2.5$)

Figure 2: Optimized current profiles with and without a 1 cm shielding gap, compared to the analytical solution of [5] and a Gaussian reference, for (a) low- and (b) high-Derbenev regimes.

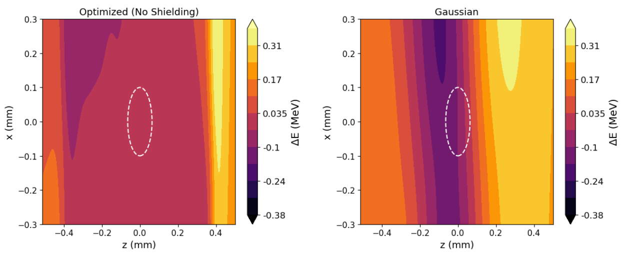
ANALYSIS OF OPTIMAL PROFILES

Low-Derbenev Regime ($\kappa \approx 0.05$)

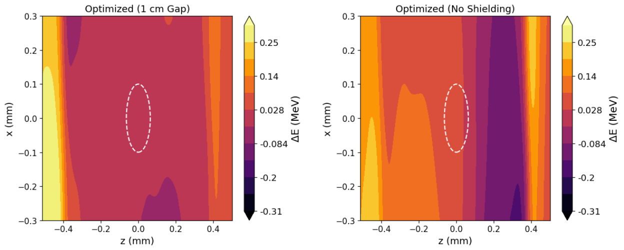
For a beam with $\sigma_x = 0.1$ mm ($\kappa \approx 0.05$), the 3D wake is well-modeled by a 1D calculation. Without shielding, the optimizer recovers a profile close to the analytical $(s-a)^{1/3}$ solution of Mitchell et al., as shown in Fig. 2 (Left). The corresponding 2D energy kick map in Fig. 3(a) shows a substantially uniform kick across the beam core (marked by the dashed ellipse). With 1 cm shielding, the free-space optimal profile overcompensates, creating a drop in energy loss near the bunch tail and increased spread. The optimizer corrects for this by shifting charge toward the tail, recovering a flat wake in Fig. 3(b). This solution is similar to the results reported in our earlier work [6], where we performed a similar study for low Derbenev beams.

High-Derbenev Regime ($\kappa \approx 2.5$)

For a beam with $\sigma_x = 5$ mm ($\kappa \approx 2.5$), the behavior changes substantially. Without shielding, the optimizer finds profiles, shown in Fig. 2 (Right), that differ markedly from both the Gaussian and the analytical 1D solution, yet achieve only modest reduction in F. The 2D energy kick map in Fig. 4(a) retains significant structure across the transverse di-

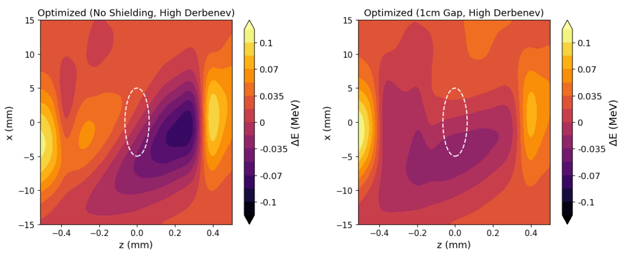


(a) Optimized, no shielding (b) Gaussian, no shielding



(c) Optimized, 1 cm shielding (d) Free-space optimal, 1 cm gap

Figure 3: Deviation from mean CSR-induced energy kick $\Delta E - \langle \Delta E \rangle$ on the (z, x) plane. Top row: optimized and Gaussian profiles without shielding. Bottom row: optimized profile with 1 cm shielding, and the free-space optimal profile evaluated under a 1 cm gap. The dashed ellipse indicates the 1σ beam boundary.



(a) No shielding (b) 1 cm shielding gap

Figure 4: Deviation from mean CSR-induced energy kick on the (z, x) plane for the high-Derbenev optimized profiles.

mension even after optimization, which is driven entirely by the large transverse size of the bunch. In essence, this result implies that profile shaping on its own is likely insufficient to minimize energy spread to the same extent in 1D. We postulate that shaping both transverse and longitudinal profiles together would result in better improvement. With shielding, the situation is quite different. The corresponding optimal profile in Fig. 2 (Right) perturbs the 1D result, creating two small modulations near the back and front of the bunch. Looking at the 2D wake map in Fig. 4(b), we observe a substantially flatter distribution across the beam cross-section (though some transverse variation still remains). The improvement with shielding is significantly better than using longitudinal profile shaping alone, as shown in the improvement in cost in Fig. 5. The likely physical mechanism here is that the image bunches are as wide as the original bunch, thus flattening the wake at transverse offsets that are other-

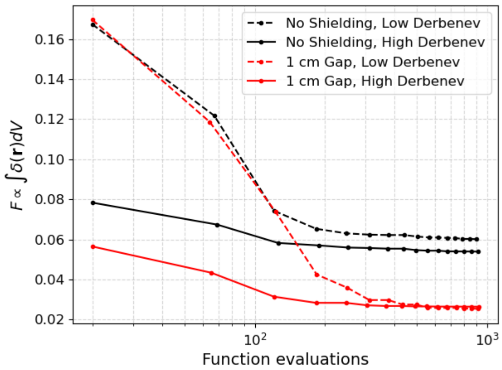


Figure 5: Cost function F as a function of number of function evaluations for the low and high Derbenev cases. Note that the shielded optimizer yields better improvement in comparison to profile shaping alone.

wise unaffected by purely changing the longitudinal current profile.

CONCLUSION AND FUTURE PLANS

In this work, we use a 3D CSR simulation tool, currently in development, to obtain longitudinal current profile shapes that minimize energy spread caused by CSR. We improve on the results presented in our earlier work [6], by considering beams of high transverse size. Our results seem to indicate that in the high-Derbenev regime, longitudinal profile shaping alone produces only modest improvement. Including shielding walls, even at gap sizes significantly larger than $(R\sigma_z^2)^{1/3}$ restores a level of improvement comparable to profile shaping for low-Derbenev beams.

We postulate that shaping the transverse profile for high Derbenev beams would result in improvements without shielding, which we plan to explore in future work. We also plan to explore the use of more sophisticated optimization techniques (Bayesian optimization etc.) to more effectively traverse the search space. This work is part of a larger effort to understand and mitigate CSR, including ongoing experimental study at the Argonne Wakefield Accelerator facility.

ACKNOWLEDGEMENTS

This research was supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under Award DE-SC0024445.

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