SENSITIVITY OF FCC-EE BEAM PERFORMANCE TO RESONANCE DRIVING TERMS IN THE PRESENCE OF BEAM–BEAM INTERACTIONS

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Abstract

The control of nonlinear beam dynamics is essential for achieving the luminosity targets of FCC-ee, particularly in the presence of strong beam–beam interactions and machine imperfections. Resonance Driving Terms (RDTs) provide a systematic framework to characterize nonlinear dynamics and quantify the strength of resonances excited by nonlinear magnetic elements in the lattice. This contribution presents a sensitivity study of individual RDTs and their impact on beam losses, vertical emittance, and luminosity in FCC-ee, evaluated using tracking simulations including beam–beam interactions. The results establish a ranking of the relative importance of individual RDTs on the performance, providing guidance for future RDT-based correction strategies.

INTRODUCTION

The Future Circular electron-positron Collider (FCC-ee) [1–3] is a proposed high-luminosity lepton collider designed to operate at multiple center-of-mass energies for precision studies of the Z, W, Higgs, and top quark. Achieving the demanding luminosity targets requires tight control of nonlinear beam dynamics in the presence of both machine imperfections and strong beam–beam interactions [4–6]. Lattice nonlinearities and magnetic field imperfections can degrade beam lifetime and luminosity through the reduction in Dynamic Aperture (DA) and Momentum Acceptance (MA). In addition, the strong beam-beam interactions at FCC-ee modify the linear and nonlinear properties of the lattice which can further reduce MA/DA as well as the effectiveness of standard optics corrections designed without beam-beam. Resonance Driving Terms (RDTs) provide a framework to characterize nonlinear dynamics and quantify the strength of resonances excited by magnetic errors and nonlinear elements in the lattice. Controlling the most harmful RDTs can therefore help mitigate the impact of nonlinearities on beam performance, especially in the presence of beam-beam effects. This paper presents a systematic sensitivity study to identify which RDTs most strongly affect beam performance in the presence of beam–beam interactions. Individual RDTs are varied in a controlled manner using a response-matrix and Singular-Value-Decomposition (SVD) approach, and tracking simulations are used to evaluate the impact on beam losses, emittance, and luminosity. The study focuses on the Local Chromaticity Correction (LCC) optics [7] at Z energy.

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SIMULATION TOOLS

The studies are performed using the Xsuite simulation framework [8], which provides particle tracking, beam–beam modeling, and optics correction capabilities, as well as interfaces to external optics tools such as MAD-NG [9]. RDTs are evaluated using two methods. The first relies on analytical calculations in MAD-NG, allowing direct computation of RDTs from the lattice model. Figure 1 shows the third and fourth order normal RDTs around Interaction Point 2 (IP2) for the ideal lattice without beam–beam effects.

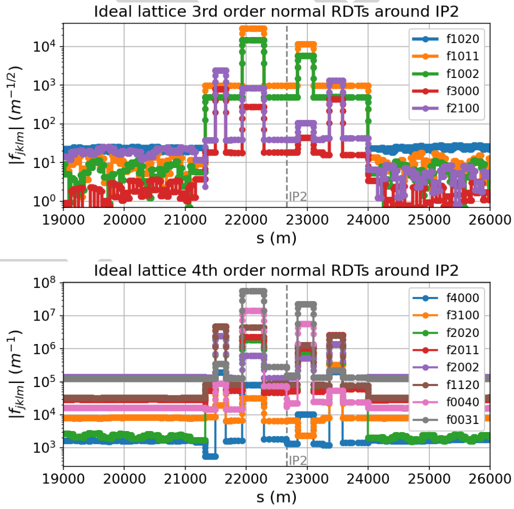

Figure 1: Calculation of third-order (top) and fourth-order (bottom) normal RDTs of the ideal lattice around IP2 using MAD-NG. f_{1020} (blue) overlays f_{1002} (green) around the IP.

The second method extracts RDTs from tracking simulations using Xsuite and OMC3 [10]. A single particle with an initial transverse offset is tracked for a few thousand turns (without synchrotron radiation), and turn-by-turn data at selected locations (e.g. BPMs) are analyzed in the frequency domain. The amplitudes of specific spectral lines are used to reconstruct the corresponding RDTs [11]. The advantage of this method is that it can incorporate the contribution of any lattice element (including beam-beam interactions) without requiring an explicit analytical model. The trade-off is that it is more computationally expensive as it requires tracking, storing, and analyzing turn-by-turn data for each of the studied configurations. A comparison between the two methods for the f_{1020} term in the ideal lattice (as an example) is shown in Fig. 2. The good agreement between these two independent approaches validates the MAD-NG calculations, which are used throughout this paper for RDT evaluation.

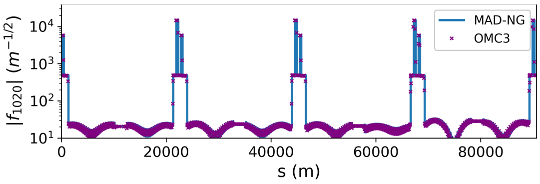

Figure 2: Estimation of f_{1020} for the ideal LCC lattice using MAD-NG (blue) and OMC3 (purple).

The tracking simulations in Xsuite include tapering, synchrotron radiation with quantum excitations, beamstrahlung, and beam–beam interactions. Beam–beam effects are modeled using the weak-strong approximation with a 3D beam–beam element that accounts for the longitudinal variation of the beam–beam force via longitudinal slicing of the strong beam (includes hourglass effect and crossing angle).

CONTROLLED RDT VARIATION

Controlled RDT variations are generated by constructing magnet configurations via a response matrix approach. A response matrix R is constructed by applying small strength variations to a large number of independently powered correctors (e.g. normal and skew sextupole components added to each sextupole) and evaluating the induced RDT changes at selected observation points (e.g. BPMs and/or IPs). The resulting system is inverted using Singular Value Decomposition (SVD), with truncation of small singular values to improve numerical stability, and Tikhonov regularization [12] to suppress the amplification of noise in weak singular modes. While such a response matrix can target a specific RDT, it generally introduces unwanted changes in other RDTs of the same order, since any corrector change generally affects multiple RDTs simultaneously. To isolate the variation of a single target term, a weighted least-squares formulation is used, where non-target RDTs are included in the cost function with a penalty factor. This numerical approach suppresses the cross-talk between RDTs while preserving the targeted change. When the penalty factor is set to zero, the solution reduces to the standard (unweighted) case. However, coupling to higher-order RDTs is not yet accounted for in the cost function and will be addressed in future work. At each step, the tunes and linear dispersion remain unaffected, while linear chromaticity is restored to its nominal value. Higher-order quantities such as second-order chromaticity and second-order dispersion are not explicitly controlled and may vary together with the targeted RDT, although these variations remain relatively small. Figure 3 shows an example of the controlled enhancement of f_{3000} around IP2. The change is not localized to the final-focusing region but to the arc cells as well. All other third-order normal RDTs remain approximately unchanged within an certain tolerance, demonstrating isolation of the target term.

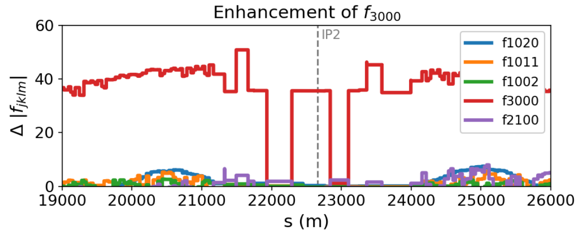

Figure 3: Change $\Delta|f_{jklm}|$ of the third-order normal RDTs around IP2 when f_{3000} is selectively enhanced. All other terms remain at their nominal values.

SENSITIVITY STUDY WITH BEAM-BEAM

To determine which RDTs most critically affect beam performance, individual third and fourth order normal RDTs are scanned globally using the framework described in Section . For each scan, a single RDT is enhanced while all others are kept at their nominal values, and the impact on beam losses, vertical emittance, and luminosity is evaluated through tracking simulations, including beam-beam effects, as described in Section . A total of 10^4 particles are tracked for 10^4 turns. Transverse emittances are monitored at a dispersion-free region, an artificial aperture is imposed at 20σ in both transverse planes, and particles exceeding the RF bucket length longitudinally are considered lost. Figure 4 shows the effect of varying individual third-order normal RDTs without (left) and with (right) beam-beam interactions, in terms of relative intensity (top), relative vertical emittance (middle) and relative luminosity (bottom). Horizontal and longitudinal emittances showed smaller variations and are not shown. Without beam-beam, f_{1011} and f_{3000} resulted in the largest beam degradation, primarily through beam losses and consequently luminosity reduction. The vertical emittance remains largely unaffected in all cases. For example, scaling f_{3000} to twice its nominal value results in approximately 5% beam losses over 10^4 turns. The picture changes significantly when beam-beam interactions are included. As before, f_{1011} and f_{3000} dominate, but at much weaker scalings and with a considerably stronger impact. Increasing f_{1011} leads to a simultaneous growth in beam losses and vertical emittance, while f_{3000} produces significant losses with negligible effect on the emittance. The remaining terms, f_{1020}, f_{1002}, and f_{2100}, show smaller sensitivity, though the coupling resonance terms f_{1002} and f_{1020}, which mix the horizontal and vertical planes, do produce a visible vertical emittance blow-up and some luminosity reduction.

The DA and MA have also been evaluated at a fixed scaling of $|f_{jklm}| \approx 1.4$, for each third-order RDT term, with and without beam-beam, and are shown in Fig. 5. Without beam-beam, the DA remains mostly unaffected across all terms, while the MA is reduced for f_{3000} and f_{1011}, indicating a coupling between transverse nonlinearities and off-momentum

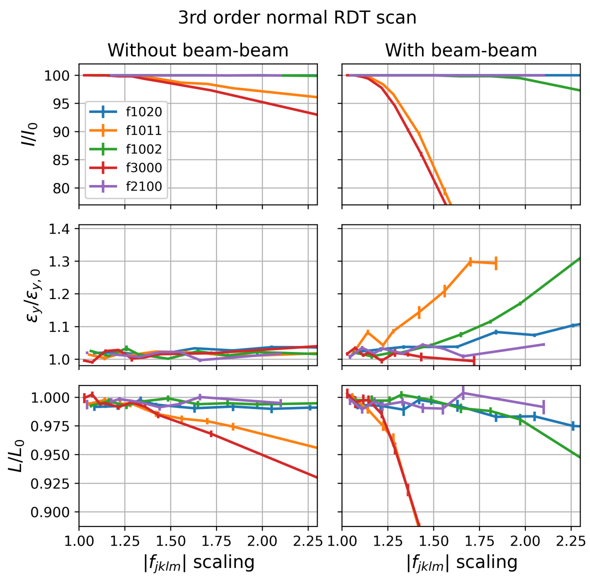

Figure 4: Impact of individual third-order normal RDTs on beam losses (top), vertical emittance (middle), and luminosity (bottom), without beam-beam (left) and with beam-beam (right). Each curve corresponds to a single RDT varied globally while all others are kept at their nominal values.

dynamics, even in the absence of beam-beam. With beam-beam, both DA and MA are further reduced for all terms, but most strongly for f_{3000} and f_{1011}, which is consistent with the beam losses observed in tracking.

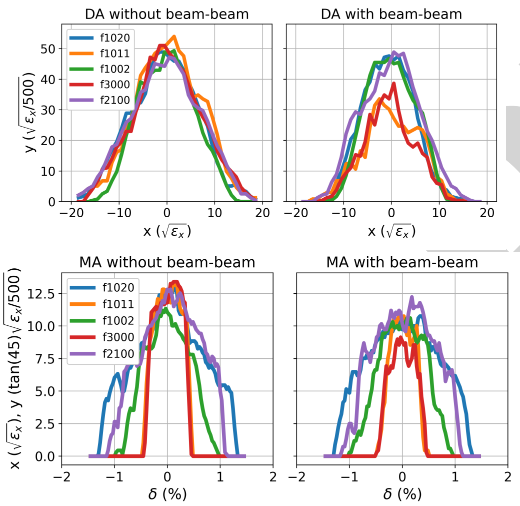

Figure 5: DA (top) and MA (bottom) for individual third-order normal RDTs scaled to approximately $|f_{jklm}| \approx 1.4\times$ their nominal values, without beam-beam (left) and with beam-beam (right).

For fourth-order normal RDTs, which are excited by normal octupolar field components, the sensitivity is considerably weaker. Figure 6 shows the luminosity as a function of the scaling factor for all fourth-order normal terms with beam-beam. No individual term produces a significant effect within the scanned range, even at scaling factors far beyond those of the third-order study.

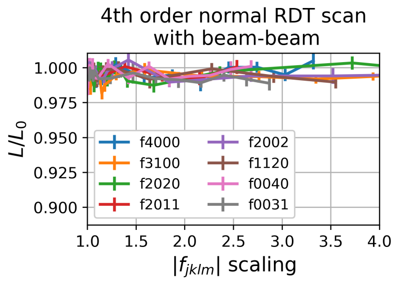

Figure 6: Luminosity as a function of the scaling factor for individual fourth-order normal RDTs, evaluated with beam-beam. No significant beam degradation is observed across the scanned range.

These results help identify the relative importance of individual RDTs in the presence of beam-beam effects in the LCC lattice at Z energy. Third-order RDTs should be prioritized more in any correction strategy, and in particular f_{1011} and f_{3000}. Within the explored parameter range, fourth-order normal RDTs show significantly weaker impact. In addition, the strong sensitivity observed in the presence of beam-beam underlines the importance of including beam-beam in the correction framework. This study is limited to normal sextupolar and octupolar RDTs in an ideal lattice. Skew sextupole components, which can be present by magnet misalignments and field errors in realistic lattices, have not yet been evaluated and will be part of future work.

SUMMARY AND OUTLOOK

A systematic sensitivity study of individual third and fourth order normal RDTs has been performed for the FCC-ee LCC lattice at Z energy, using tracking simulations including weak-strong beam-beam. The results give a first indication of the most critical RDTs to be targeted for correction in the presence of beam-beam, with f_{1011} and f_{3000} identified as the most dominant. The response-matrix and SVD framework presented here provides a basis for constructing dedicated RDT correction knobs, which will be part of future work, together with a deeper investigation of the mechanisms driving beam losses and emittance growth, and an extension of the sensitivity study to skew RDTs excited by magnet misalignements and field errors.

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