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UNCERTAINTY ANALYSIS OF TIME CONSTANT MEASUREMENTS IN SRF CAVITY TESTING

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Abstract

At the start of superconducting radio frequency (SRF) cavity testing, cavity time constant measurements are made. From the measured time constant the field probe external quality factor is calculated. This value is then used for the calculation of unloaded quality factor and accelerating gradient for the remainder of the testing process. In previous work the uncertainty of the time constant measurement has been given as 3%, but this is dependent on the measurement equipment and method used to estimate the time constant from the measured cavity decay curve. In the vertical test facility (VTF) at STFC's Daresbury Laboratory the decay curve is measured using different hardware to other facilities. We present efforts to estimate the uncertainty of time constant measurements at our VTF to compare our measurement method with those used elsewhere.

INTRODUCTION

Superconducting radio frequency (SRF) cavities are a well established technology and have been deployed in many large-scale accelerator facilities. A key stage in the production of these cavities is a test to ascertain the 2K performance of the cavity at Vertical Test Facilities (VTFs). The testing, known as "Q vs E" testing, generates a plot of the unloaded quality factor (Q_0) against accelerating gradient (E_{acc}) for a range of gradients. These values are not directly measured, and instead calculated from measurements of power incident to the cavity (forward power, P_f), reflected from the cavity (reflected power, P_r) and coupled out of the cavity via the field probe (transmitted power, P_t). Throughout most of the process in the Superconducting Radio Frequency Lab (SuRF Lab) at STFC's Daresbury Laboratory the following calculations are used:

$$Q_0 = \frac{Q_t P_t}{P_c}$$

$$E_{acc} = \sqrt{Q_t P_t \frac{r/Q}{L}}$$

Where L is the cavity length, and r/Q is the design ratio of shunt impedance to quality factor.

The field probe quality factor, Q_t, is a measure of the ratio of stored energy in the cavity to the energy dissipated externally to the cavity through the field probe in an RF period. This Q_t value is calculated early in the process from measurements of power decay, and is used in the above calculations for the remainder of the testing. The decay measurement process initially requires capturing the forward, reflected and transmitted power in steady state operation, and then switching off forward power to the SRF cavity. The stored energy is then dissipated through loss mechanisms in the cavity walls and the two couplers. With no forward power to the cavity, the total power lost (P_{tot}) over time (t) is given via:

$$P_{tot} \propto e^{\frac{-t}{\tau_L}}$$

This relationship allows for the estimation of the power decay constant of the cavity, τ_L, by tracking the field probe power following incident power switch off and using calculations to estimate the value of τ_L. The derivations required to calculate Q_t, Q_o and E_{acc} following these measurements can be found in [1]. The work discussed here focuses on the calculation of standard uncertainty that can be applied to each decay constant measurement in the SuRF Lab. The motivation of the work was to understand if the measurement method used in the SuRF Lab provides a different uncertainty than the 3-4% cited by other facilites [1–3].

DECAY CONSTANT MEASUREMENT AT DARESBURY LABORATORY

The decay measurement in the SuRF Lab is performed in a different way to other SRF cavity test facilities. The decay is measured via the envelope output of the Analog Devices ADL5511 Envelope and TruPwr RMS Detector [4], rather than a power detector [3, 5], spectrum analyser [2], or oscilloscope [6]. Between the cavity output and envelope detector there is a signal chain of switches, digital step attenuators and amplifiers that give the operator control over the power level at the input of the detector. This ensures the full dynamic range of the envelope detection system can be used at as much as practicable. The ADL5511 outputs a voltage that is proportional to the voltage envelope of the original signal. The minimum voltage output is ~1.15V, which is taken to be the noise floor.

The envelope output voltage is then digitised via a National Instruments CompactRIO with the sample rate set by the operator. A regression analysis is performed on the data by a LabVIEW program to determine the time constant of the cavity. The digitised voltage still contains the ~1.15V DC offset, so the curve never decays to zero. This necessitates the use of non-linear least squares regression methods. The testing software makes use of the LabVIEW Exponential Fit Virtual Instrument (VI), which implements the Levenburg-Marquardt algorithm [7].

Note that the envelope output of the detector is proportional to the voltage envelope of the cavity, so any calculations give the voltage time constant, which is double the power time constant used in the calculations of Q_0 and E_{acc}.

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UNCERTAINTY IN DECAY DATA

Calculating the uncertainty of the parameters of a fitted model requires the uncertainty of the data the model is fitted to. In this case the data is ADC samples over time. The uncertainty of these ADC samples can be attributed to the noise in the system, meaning that a measurement of the noise will provide an estimate of the uncertainty of a single sample.

Method

To measure the system uncertainty two approaches were taken. The first was to place a 50Ω termination in place of the cavity pickup signal, and to use the ADC to measure the envelope detector voltage with the standard software settings. The hardware settings were varied as they would in a normal test, changing attenuation and amplification between the input port and the envelope detector. This was to provide information on system noise as a function of hardware settings.

For the second set of measurements the termination was replaced with a signal generator producing a CW signal at 650 MHz. In this case the signal generator power was varied whilst hardware settings remained fixed, as well as varying the hardware settings whilst the signal generator power was fixed. The purpose of this was to investigate the potential of a relationship between input power level and measurement uncertainty.

In each case, the standard uncertainty of an individual sample was taken to be the standard deviation of the set of 1000 samples taken of the fixed input signal.

Results

Most system settings did not have an impact on the standard deviation of the voltage, aside from a known increase in system noise when using a high amplification chain. There is a relationship between the standard deviation of the voltage and mean voltage. This was modelled using the following function:

$$y = \begin{cases} ma + c & x < a \\ mx + c & x \geq a \end{cases}$$

The data and fitted model can be seen in Fig. 1.

UNCERTAINTY OF FIT PARAMETERS

The LabVIEW Exponential Fit VI does not provide uncertainties on the fit parameters, so a VI was created to perform the uncertainty calculation. The method was based upon the calculations shown in [8]. To offer a comparison to the estimations of uncertainty a copy of the fitting process was written in Python. Specifically, the SciPy optimize.curve_fit function [9] was used because it provides an estimate of parameter uncertainty. The Python implementation used the model below for the voltage decay whereas the LabVIEW implementation uses the standard form offered by the fit function and calculates tau from the "damping" parameter.

$$V = ae^{-\frac{t}{\tau_L}} + b$$

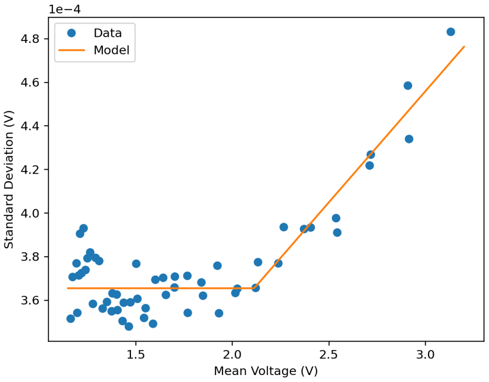

Figure 1: Plot showing the fit of the model to the standard deviation data.

Verification Method

Using the definition of standard uncertainty one can verify suitability of the uncertainty estimates. The standard uncertainty gives a confidence interval around each measurement point and by definition the confidence level is 68%. Therefore, it is expected that over repeated measurements the true value lies within 68% of the defined confidence intervals. By taking repeated measurements and calculating corresponding confidence intervals for each measurement, the percentage of intervals that contain the true value can be calculated. This can be used to estimate the confidence level for the calculated confidence interval. With this method, any interval can be verified, so it is also possible to compare the confidence intervals estimated by the software with the 3% uncertainty given by other facilities. In the case of a decay measurement of an SRF cavity it is difficult to have a known decay, so in lieu of a true value we use the mean τ_L. Since the fitting methods are slightly different, the mean τ_L was calculated for both the LabVIEW and Python curve fits.

In this test the assumption is made that are no other systematic errors present, and the mean gives the true value of the decay constant for this cavity. The authors acknowledge there are likely to be uncorrected errors caused by circulator mismatch [10] and low field Q-slope [11] within the system, but these errors are not expected to affect the results of this test.

Results

The measurement dataset comprises 50 decay measurements, which are displayed in Fig 2 along with the estimations of standard uncertainty. Table 1 shows the percentage of confidence intervals containing the mean value of the decay constant for both fit generation methods. The confidence intervals referenced are either calculated from the estimated uncertainty or by assuming an interval of ±3% around each measured point. The mean τ_L value calculated by LabVIEW was $4.81317s \pm 0.00047s$ with a confidence level of 68%, and the mean τ_L value calculated by the Python script was $4.82629s \pm 0.00050s$ with a confidence level of 68%.

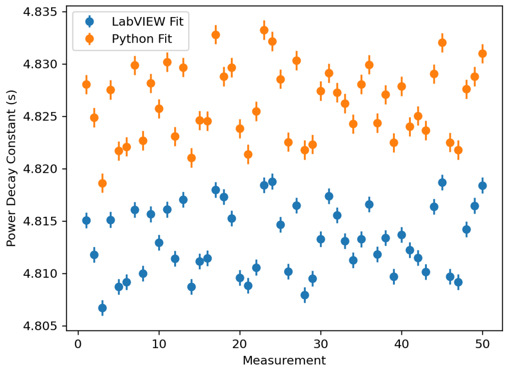

Figure 2: The dataset of 50 points, showing the two fits on a point-by-point basis. Error bars represent the estimate of standard uncertainty attached to these points.

Table 1: Percentage of Calculated Confidence Intervals Containing Mean Value

Fitting Software	Estimated Interval [%]	3% Interval [%]
LabVIEW	12	100
Python	8	100

The data suggests that assuming a standard uncertainty of 3% for this measuring system is an overestimate, since the mean value fell within all of the intervals created by this assumption. Conversely, the calculations of uncertainty implemented in both Python and LabVIEW have underestimated the standard uncertainty.

The difference in LabVIEW and Python estimates could arise from different implementations of the Levenberg-Marquardt algorithm or the use of slightly different fitting functions.

The inaccuracy in both LabVIEW and Python methods of estimating uncertainty presents an issue for analysis because this could either be due to an issue with the estimated uncertainties associated with the measured data, or an issue with the estimation of uncertainties from the fit. The estimation of parameter standard deviations relies on a linear approximation to the function, which is not likely to be appropriate for an exponential decay with a vertical offset [9]. Two potential solutions to this issue may be to reduce the amount of the decay curve captured so that the linear approximation is more valid, or change the measurement system so that there is no DC offset. This could be done in either software or hardware. Removing the DC offset would allow for the use of a simple linear regression upon the natural logarithm of the data. The simple linear regression would then be able to take advantage of common calculations of confidence intervals for the parameter estimates.

CONCLUSIONS

The results suggest that the measurement of decay constant in the SuRF Lab have a lower uncertainty than the usually assumed 3%. System noise has been measured and modelled in order to estimate uncertainties associated with the data used to calculate the decay constant. Efforts have been made to provide an estimate of the uncertainty associated with measurements of decay constant but they have not been successful. Possible explanations have been offered for the lack of success and alternative methods have been proposed for future investigations.

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