

BETATRON RADIATION STUDIES AS A PATH TO PLASMA UNDULATORS

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Abstract

There is strong interest in developing compact user facilities, with plasma technology emerging as a promising solution not only for acceleration modules but also for ancillary components such as free-electron lasers. While substantial progress has been achieved in miniaturizing diagnostics, detectors, and transfer lines, conventional undulators remain bulky and costly. This has motivated the exploration of advanced undulator concepts, including plasma ion-channel undulators based on electron betatron motion. We present a feasibility study for plasma-based undulators at SPARC_LAB within the EuPRAXIA@SPARC_LAB project, providing quantitative predictions of gain and radiation performance under realistic experimental conditions.

INTRODUCTION

The manuscript investigates the feasibility of plasma-based undulators at the SPARC_LAB facility [1] within the EuPRAXIA [2] and EuPRAXIA@SPARC_LAB [3] projects, aimed at developing compact plasma-based user facilities for free-electron lasers (FELs). Since conventional magnetic undulators remain bulky and expensive, the study explores advanced concepts based on the betatron motion of electrons in plasma ion channels to emulate undulator behavior. In particular, the work focuses on a wakeless ion-column configuration, which enables compact FEL amplification with larger energy-spread tolerance and shorter gain lengths compared to conventional undulators. Using realistic electron beam parameters from SPARC_LAB and numerical simulations, the formation of a stable ion channel is described and the onset of FEL radiation is shown in a 10 cm plasma undulator, providing a proof-of-principle framework for future compact X-ray sources.

THEORETICAL BACKGROUND

Ion Channel Formation

Ion column (IC) formation requires that the thin cylindrical volume, after ionization, is depleted of all electrons for a length much longer than the plasma wavelength. In this paper, we will consider depletion by use of an electron bunch (driver) preceding the bunch intended to emit radiation (witness). The ionization technique might rely on the

propagation of ultrashort, high-intensity laser pulses in gases that is governed by the interplay between Kerr self-focusing and plasma-induced defocusing arising from multi-photon or tunnel ionization. This dynamic balance leads to intensity clamping and the formation of extended plasma channels, known as filaments [4,5]. This technique enables the generation of plasma channels with tunable transverse dimensions and density profiles. In particular, plasma channels can be produced with radii comparable with the transverse size of the driving bunch, creating the conditions for the formation of a fully evacuated ion channel. In this regime, the strong space-charge fields of the driver expel nearly all plasma electrons from the channel volume, leaving behind a stationary column of ions. Because the expelled electrons cannot return quickly enough to restore charge neutrality, the ion column persists over the interaction length and provides a stable linear focusing force for the propagating beam.

Such a configuration is especially attractive for plasma-undulator applications, where the controlled betatron oscillations of relativistic electrons inside the ion channel can emulate the behavior of a conventional magnetic undulator in a much more compact setup. Figure 1 shows an example of well-formed ion channel, as considered in the simulations. The plasma is assumed to be ionized through laser filamentation.

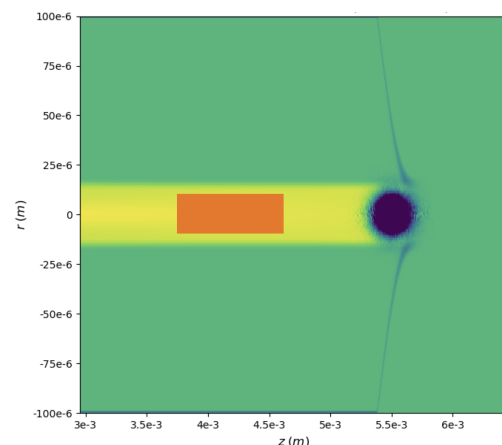


Figure 1: Example of a well-formed ion channel, assuming the following parameters: $n_0 = 5 \cdot 10^{15} \text{ cm}^{-3}$, $R_{ch} = 15 \text{ } \mu\text{m}$, $Q_b = 600 \text{ pC}$, $\sigma_z = 90 \text{ } \mu\text{m}$.

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Table 1: Electron beam and ion-channel parameters for betatron undulator-based FEL emission.

Parameter	Symbol	Unit	Value
Beam energy	E_{beam}	MeV	223
Beam charge	Q	pC	200
Norm. slice emittance	$\varepsilon_{n,rms}$	mm mrad	1
RMS longitudinal size	σ_z	μm	<30
Ion-channel radius	R_{ch}	μm	15
Plasma density	n_0	cm^{-3}	$5 \cdot 10^{15}$

FEL Formalism in Plasma Wave Undulators

A one-dimensional (1D) FEL model has been developed starting from the electron dynamics in an ion-channel-based plasma undulator and from the wave equation describing the radiation field [6, 7]. Under the assumptions of forward-propagating radiation and a slowly varying envelope, the resulting set of equations governing power growth, bunching, and saturation in the 1D plasma-undulator FEL model is obtained [8]:

$$\begin{cases}
 \frac{dE}{dz} = -\frac{n_e e}{4\varepsilon_0 \gamma_0} \left[\frac{K_{\beta j}}{1 + \delta_j} \left[J_0 \left(\frac{K_{\beta j}^2}{4 + 2K_{\beta j}^2} \right) + J_1 \left(\frac{K_{\beta j}^2}{4 + 2K_{\beta j}^2} \right) \right] e^{-i\psi_j} \right], \\
 \frac{d\psi_j}{dz} = \frac{4\pi}{\lambda_\beta} \left[\delta_j + \frac{\delta_j K_{\beta j}^2}{2} - \frac{K_{\beta j}^2}{4} \right], \\
 \frac{d\delta_j}{dz} = -\frac{eK_{\beta j}}{4\gamma_0^2 m c^2 (1 + \delta_j)} (E e^{i\psi_j} + E^* e^{-i\psi_j}),
 \end{cases} \quad (1)$$

where ψ_j denotes the ponderomotive phase, δ_j the relative energy deviation, $K_{\beta j}$ the undulator strength parameter, n_e the electron density within the FEL bucket, and λ_β the betatron wavelength.

The model accounts for the betatron motion in a fully developed ion channel, including the proper dependence of the coupling coefficients on the betatron strength parameter and the associated Bessel-function reduction factor. This approach enables an accurate description of FEL-like amplification processes in plasma-based undulators.

Results

The numerical investigation of the plasma-based undulator FEL described in this work relies on solving the self-consistent coupled evolution of the radiation field, the ponderomotive phase, and the relative energy deviation for a large ensemble of macro-particles ($\sim 10^6$) interacting with the emitted radiation field.

Assuming a pre-formed, uniform ion channel with constant background plasma density of $5 \times 10^{15} \text{ cm}^{-3}$. The witness electron beam parameters are chosen to be compatible with the SPARC_LAB photo-injector operating conditions as listed in Table 1.

For the case of parameters listed in Table 1, considering a witness beam with a $10 \mu\text{m}$ rms matched transverse size,

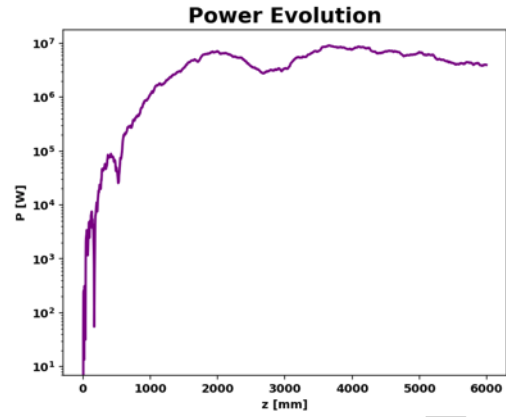


Figure 2: Evolution of the radiated power along the plasma undulator for a 200 pC matched witness beam ($10 \mu\text{m}$ rms transverse size). Exponential growth and saturation is achievable with a meter-scale channel.

Table 2: Radiation Parameters

Parameter	Symbol	Unit	Value
Radiation wavelength	λ_r	nm	223
Radiation peak power	P	MW	80
Number of photons	N_{ph}		$1.3 \cdot 10^7$

the resulting power evolution along the plasma channel is shown in Fig. 2, displaying clear exponential growth along the interaction region and approaching saturation in less than 2 m, demonstrating that collective FEL amplification can be achieved under suitable beam conditions and in a relatively compact undulator length.

The radiation parameters are listed in Table 2.

CONCLUSION

In this work, we investigated the feasibility of realizing a compact free-electron laser based on betatron radiation emitted in a plasma ion channel acting as an effective plasma undulator. Motivated by the development of ultra-compact radiation sources for plasma-based user facilities, we considered experimentally realistic beam and plasma parameters relevant to the SPARC_LAB test facility and the EuPRAXIA@SPARC_LAB project.

The present analysis is consistent with the unified description of laser-plasma radiation sources discussed in [9]. As highlighted therein, coherent amplification in plasma undulators requires stringent constraints on energy spread, transverse emittance, and uniformity of the effective undulator parameter. The scaling laws derived in that framework were employed here to assess the expected radiation brightness and coherence properties relative to conventional magnetic undulators. Building on this approach, we performed time-dependent numerical simulations of FEL interactions in a 10 cm long plasma undulator.

A systematic parametric study of beam charge and normalized transverse emittance shows that significant microbunching and detectable radiation power can be achieved within

such a compact interaction length, provided that the electron beam is properly matched to the ion channel. The simulations demonstrate the strong sensitivity of FEL gain to transverse beam quality and confirm the key role of beam matching in maximizing the beam–radiation coupling strength.

For beam parameters compatible with present-day SPARC_LAB operation, the simulations predict FEL emission in the ultraviolet spectral range, around 223 nm, with gain lengths sufficiently short to enable the observation of exponential amplification within a meter-scale plasma device.

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