TOWARDS GENERATION OF ORBITAL ANGULAR MOMENTUM THZ RADIATION VIA DIELECTRIC WAKEFIELD ACCELERATION

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Abstract

Dielectric wakefield acceleration driven by high-energy electron beams can generate high power, narrowband terahertz radiation via coherent Cherenkov emission. While conventional dielectric-lined waveguides have been studied extensively for fundamental-mode excitation, recent theory suggests that higher-order modes carrying orbital angular momentum (OAM) can be deliberately excited in modified structures. We present progress toward the generation of OAM-carrying THz radiation using tailored drive beam distributions and novel dielectric geometries at the UCLA MITHRA facility. OAM modes introduce transverse field components with helical wavefronts, which offer opportunities to enable simultaneous longitudinal acceleration and transverse focusing of witness beams. This dual functionality may simplify beamline design and mitigate beam breakup instabilities that currently limit efficiency in wakefield accelerators. We discuss the theoretical framework, particle-in-cell simulation results, experimental configurations, and newly developed diagnostics for OAM mode characterization.

INTRODUCTION

Dielectric wakefield acceleration (DWA) is a highly efficient technique for driving the next generation of compact particle accelerators and generating high-power, narrowband terahertz (THz) radiation [1]. In standard DWA schemes, a relativistic electron drive beam will pass through a dielectric waveguide, leading to the excitation of a wakefield generated via coherent Cherenkov radiation emission [2–4] (Shown in Fig.1). Both theoretical and experimental research have focused on generating fundamental waveguide modes, such as TM01 modes, to accelerate a trailing witness electron bunch. Fundamental modes thus far have proven highly effective at converting the drive beam's kinetic energy into high-gradient longitudinal accelerating fields, allowing for a trailing witness beam to accelerate off of the drive beam's wake [5]. This has established DWA as a core area of interest for the generation of ultra-compact advanced accelerators.

Despite accelerating gradients reaching 1 GV/m or above achieved in conventional DWA experiments [5], the technology faces significant challenges regarding beam stability. While fundamental accelerating modes generate high gradients for acceleration, partial coupling to hybrid modes can lead to beam instabilities. These hybrid modes generate transverse wakefields that induce beam breakup (BBU) instabilities, kicking the beam off axis [6, 7]. This is commonly resolved by using complex external magnets to transversely focus the beam and precise alignment to prevent off-axis instabilities. Therefore, a key limitation of fundamental modes is that they cannot simultaneously suppress BBU and accelerate the electron beam, limiting the efficiency of DWA.

This paper presents our progress toward specialized excitation of higher-order modes carrying orbital angular momentum (OAM) at the UCLA MITHRA accelerator facility. We simulate OAM carrying THz radiation within the dielectric-lined waveguide by employing a tailored drive beam distribution and intentionally coupling the hybrid mode within the drive beam in a specialized dielectric geometry [8]. This presents a novel high-power THz OAM source that generates structured-light THz light by using the coherent emission of DWA. By generating these structured OAM modes, transverse field components with helical wavefronts are present within the accelerating wakefield. Through these transverse components, we could enable the simultaneous longitudinal acceleration and transverse focusing of witness beams. This enhanced DWA will simplify beamline design while reducing BBU instabilities.

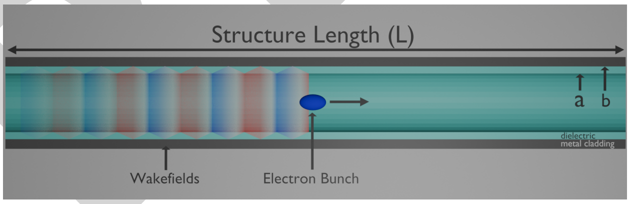

Figure 1: Schematic of a standard DWA scheme in a cylindrical waveguide. A relativistic electron bunch (the blue ellipse) propagates through a vacuum channel of inner radius a and outer radius b, surrounded by a dielectric liner (ε_r) of thickness $b-a$ with a metal coating. Coherent Cherenkov radiation emitted by the drive bunch excites wakefields that follow behind.

GENERATING $\mathrm{LG_{01}}$ FROM $\mathrm{HEM_{11}}$

In DWA, a relativistic electron beam propagating on axis couples predominantly to the azimuthally symmetric TM_{01} mode. When the electron beam is displaced off-axis by some distance Δr, the resulting azimuthal asymmetry leads to the excitation of the hybrid HEM_{11} mode [7], which carries the first-order azimuthal dependence $e^{\pm i\varphi}$. This mode is responsible for the transverse deflecting wakefields that drive the BBU instability in modern dielectric wakefield structures [7].

To convert this to an alternative mode to reduce BBU, the HEM_{11} mode can be decomposed into a superposition of two opposing OAM eigenstates [8, 9]:

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$$\mathbf{E}_{\mathrm{HEM}_{11}} \propto \cos(\varphi) = \frac{1}{2}\left[e^{+i\varphi} + e^{-i\varphi}\right] \tag{1}$$

Each eigenstate carries a topological charge $(\ell) = \pm 1$. When these modes are in a superposition, such as in the HEM_{11} mode, they cancel, resulting in a non-OAM beam. Despite each of these eigenstates being an inherently OAM mode. Using this characteristic, a structure can be designed such that a singular OAM mode is enhanced, leading to a Laguerre-Gaussian LG_{01} beam. The LG_{01} beam has a topological charge $\ell = +1$ with similar field dependence to the HEM_{11} of $e^{+i\varphi}$. This results in a helical wavefront that carries the OAM of $\ell\hbar$ per photon within the beam.

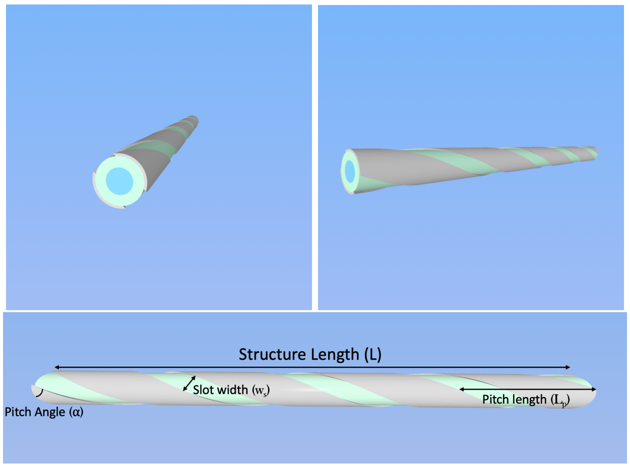

Figure 2: Helically slotted dielectric waveguide geometry for intrinsic OAM generation. A dielectric-lined waveguide (inner radius a, outer radius b, $\varepsilon_r = 3.8$) is coated with a metallic layer into which $N_s = \ell = 1$ helical slots of width w_s is carved at pitch angle α matched to the mode wavelength of the HEM_{11}, with a pitch length dependent on the topological charge $L_p = \lambda/\ell$. The slot acts as a selective radiative damping channel, which preferentially couples to an OAM mode.

To extract this single chirality OAM mode from the HEM_{11} mode, we propose a dielectric structure that selectively enhances an individual eigenstate of the OAM mode by helically damping counterpropagating states. This design uses a dielectric lined cylindrical waveguide coated with a thin layer of gold and then a layer of copper with $N_s = \ell = 1$ slots carved helically along the length of the dielectric waveguide at a pitch angle matched to the guided wavelength of the HEM_{11} mode, $\alpha = \arctan(L/2\pi r\ell)$, which is shown in Fig. 2. Simulations of this structure were tested with slots through the metal coating of varied slot widths of $w_s = 50\,\mu\mathrm{m}$ to $250\,\mu\mathrm{m}$ and pitch length $L_p = \lambda/\ell$.

The helically carved slots in this structure provide selective radiative damping. Because the slot pattern has a defined chirality, it couples asymmetrically with one of the two OAM eigenstates of HEM_{11}. This alone does not support a single-mode OAM beam, but rather a mixed-mode structure of HEM_{11}, TM_{01} and an LG_{01} like mode. To further select for the LG_{01} like waveguide mode, the drive electron beam must be brought off-axis helically matching the slots of the structure.

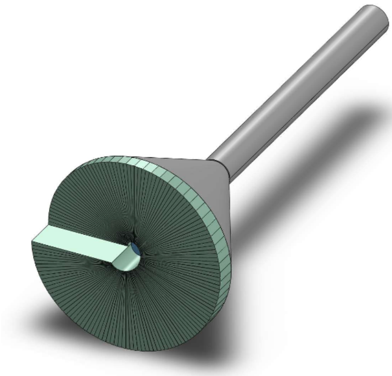

Figure 3: Spiral Phase Plate (SPP) structure with conical horn antenna and cylindrical dielectric waveguide. The SPP imprints an azimuthally varying phase $e^{i\ell\varphi}$ (where $\ell = 1$) onto the TM_{01} mode wakefield exiting the antenna. The plate is fabricated by 3D printing Cyclic Olefin Copolymer (TOPAS COC) using a discrete staircase approximation with 16 steps for waveguides with an operating frequency of 0.273 THz.

The OAM mode purity achieved in this experimental design depends on both the dielectric structure's geometry and the helical beam trajectory. For the structure alone, the relevant parameters are the slot width w_s, the pitch angle α, and the total interaction length where the differential damping between OAM eigenstates can accumulate.

The helical beam parameters must be optimised against the waveguide geometry and therefore the output wavelength. The orbital radius Δr and pitch controls the coupling strength to the $\ell = \pm 1$ eigenstates. For $\Delta r/a = 0.8$ and a helical pitch matched to the waveguide structure L_p, the beam current distribution will resonantly drive the $\ell = +1$ eigenstate while simultaneously suppressing the opposing chirality state, individually selecting a single LG_{01} like mode within the waveguide.

SIMULATION SETUP

Here we discuss the initial steps in generating coherent OAM THz radiation at the MITHRA facility [10], using the beam parameters in table 1.

To examine this experimental setup, we simulated two different structures for the generation of OAM from a THz source. These structures include a cylindrical dielectric waveguide capped with a spiral phase plate (Fig. 3) and a helically slotted waveguide structure (Fig. 2).

Spiral Phase Plate Structure The first structure we will test consists of a cylindrical dielectric waveguide with inner radius $a = 0.25$ mm, outer radius $b = 0.35$ mm, and dielectric constant $\varepsilon_r = 3.8$. This waveguide then passes into a conical horn antenna, which is capped by a spiral phase plate (SPP) at the end (Fig. 3). The SPP generates an

Table 1: UCLA MITHRA facility Beam Parameters and DWA Parameters

Parameter	Unit	Value
Charge	pC	250
Energy	MeV	30
γ_{Beam}	constant	64
ϵ_x, ϵ_y (initial)	mm mrad	1, 1
σ_x, σ_y (initial)	μm	20, 20
σ_z (initial)	μm	90

LG_{01} mode by adding an azimuthally varying optical path to imprint the helical phase profile onto a fundamental mode wakefield generated from the cylindrical waveguide [11, 12]. This converts the beam into an OAM carrying beam with topological charge $\ell = 1$ (see Fig.4).

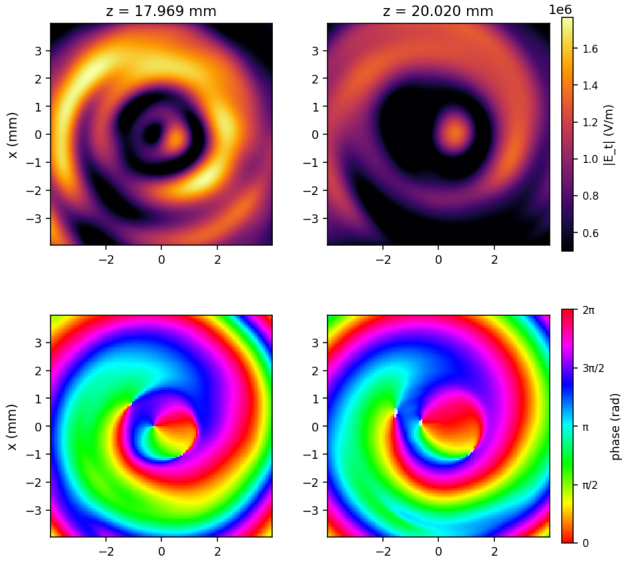

Figure 4: Simulated transverse field downstream of the SPP structure at $z = 17.969$ mm and $z = 20.020$ mm, respectively. Top: $|E_t|$ showing the characteristic donut profile. Bottom: phase showing a 0–2π azimuthal winding, consistent with an LG_{01} mode ($\ell = +1$).

Helically Slotted Structure The second geometry is the helically slotted waveguide, with the same parameters as the spiral phase plate. However, the dielectric is coated within the simulation by a simulated perfect electrical conductor (PEC) layer. A single helical slot of width $w_s = 250$ μm etched out of the PEC outer layer. In this configuration, the drive beam is injected on a helical trajectory with orbital radius $\Delta r = 200$ μm and helical pitch matched to the slot periodicity L_p.

CONCLUSION AND FUTURE WORK

We have presented two approaches for generating OAM THz radiation via DWA, both of which are planned to be tested at the UCLA MITHRA facility. The first uses a spiral phase plate to imprint a helical phase profile onto the TM_{01} wakefield of a standard cylindrical dielectric waveguide. Particle-in-cell simulations of this structure demonstrate successful generation of an OAM THz field. The resulting field with a characteristic doughnut intensity profile and phase profile azimuthally varying from 0–2π matching an LG_{01} mode (Fig. 4). The second novel approach is a structure to generate an LG_{01} like waveguide mode by employing a helically slotted dielectric waveguide structure. This is then paired with a helical drive beam to excite a single chirality eigenstate from an HEM_{11}. This method offers the highest potential advantage as intrinsic OAM generation within the waveguide itself allows for longer waveguide structures and improved BBU, without requiring an external phase shaping.

Both approaches target a 0.273 THz frequency set by the MITHRA beam parameters and dielectric structure geometry. Within the waveguide, the field profile of the LG_{01} beam could enable transverse focusing and longitudinal acceleration of witness beams which offers an opportunity to mitigate beam breakup instabilities that limit the efficiency of DWA.

Near-term experimental plans for this project include the final fabrication and installation of the spiral phase plate and helical slotted structure at MITHRA for initial OAM generation experiments. This OAM will then be characterised by a Shack-Hartmann wavefront sensor to measure the helical phase front and confirm the topological charge of the generated beam to match the expected from the structure [13, 14]. Further simulation studies will explore optimization of the helically slotted geometry to optimize transverse focusing and acceleration by shifting the beam orbital radius, and interaction length to maximize OAM mode. Looking forward, a successful demonstration of controlled OAM THz generation at MITHRA would open applications in structured wakefield-driven acceleration as well as stable Dielectric Wakefield Acceleration, vastly reducing beam breakup and improving beam quality.

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