SPACE CHARGE DYNAMICS IN RING-BASED ELECTRON COOLER

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Abstract

The Ring Electron Cooler (REC) is designed to provide cooling to 275 GeV protons at Electron Ion Collider (EIC). The REC is a non-magnetized RF-based electron cooler employing a 150 MeV electron storage ring. Electron bunches of 21 nC are needed to achieve cooling rates required by EIC. The tune shift in the electron storage ring induced by e-bunch space charge (SC) is ≈0.14 in both planes. This paper investigates the SC-driven beam dynamics in the REC, the resulting emittance growth, and potential beam loss.

INTRODUCTION

The Ring Electron Cooler (REC), is under design as a possible solution to the cooling need for the Electron Ion Collider's (EIC) Hadron Storage Ring (HSR) at its top energy of 275GeV. In the REC, cooling of the protons is accomplished via electron cooling [1] in a 170m long cooling section where the cooling ring and HSR overlap. The electron current in the REC is maintained by top-up injection of a high charge 21nC bunches. This high charge per bunch, means the space charge will be an important addition to the dynamics, shifting the tune and increasing emittance. As damping the growth from beam-beam scattering and intrabeam scattering is accomplished by a section that contains 18 2.4T wigglers each 4.2m long [2], image currents in this section also have the potential to shift the tune.

Previous estimates of the space charge tune shift indicated a substantial shift of ≈0.14 in both planes. This tune shift gives indication that space charge may be important to beam stability and emittance growth. In this paper, we determine the tune shift achieved in the full lattice through tracking using a Bassetti-Erskine approach and determine the severity of space charge on bunch stability and emittance increase.

TRACKING WITH SPACE CHARGE

Initial estimates of the space charge tune shift from direct space charge, used the first term of the formula [3, 4]

$$\Delta Q_{x,y} = -\frac{N r_0}{4\pi\beta^2\gamma}\left(\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}\left(\sigma_x+\sigma_y\right)}\right\rangle\frac{\varepsilon_0^{x,y}}{\gamma^2 B_f} + \left\langle\beta_{x,y}\right\rangle\frac{\varepsilon_1^{x,y}}{h^2 B_f} + \left\langle\beta_{x,y}\right\rangle\beta^2\frac{\varepsilon_2^{x,y}}{g^2}\right) \quad (1)$$

where N is the number of particles per bunch, β and γ are the relativistic factors, the ε's are the Laslett coefficients, B_f is a bunching factor, h is the height of the vacuum chamber, and g is the pole gap. Initial estimates used a smooth β approximation for a round beam where $\left\langle\beta_{x,y}\right\rangle = L/\left(2\pi Q_{x,y}\right)$, simplifying the equation to be independent of lattice specific optics. Under this approximation, the tune shift estimate was ≈0.14 in both planes. As an optimized lattice now exists, these estimates can be compared against tracking with space charge.

In order to track the number of turns necessary to collect meaningful data of space charge dynamics, a suitable model must first be chosen. The damping time of the ring is 50,000 turns, meaning that effects from space charge on this order could be important. This time scale makes 3D self-consistent models too time intensive to yield results in a reasonable amount of time, so in this study an approximate model is used that neglects longitudinal kicks and gives a transverse kick that is given by the Bassetti-Erskine formula [5] with properly chosen coefficients

$$K_y + iK_x = \frac{r_e N}{\gamma^3\sigma_z}\exp\left[\frac{-z^2}{2\sigma_z^2}\right]\sqrt{\frac{\sigma_x+\sigma_y}{\sigma_x-\sigma_y}}\left\{\mathrm{erf}\left[\frac{x+iy}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right] - \exp\left[-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right]\mathrm{erf}\left[\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2\left(\sigma_x^2-\sigma_y^2\right)}}\right]\right\} \quad (2)$$

In order to measure the space charge tune shift in the bunch, a 5000 particle bunch was tracked over 2000 turns using Bmad [6], with the tune of the particles determined by a NAFF algorithm of the final 1000 turns. The space charge model was the before mentioned Bassetti-Esrkine with the assumption of the constant emittance. The result of this tracking was the tune footprint in Fig 1, where it can be seen that the highest shifted particles appear near the estimated tune shift. It can additionally be seen that tune shift sits just past a strong sextupole resonance, with many particles sitting along the resonance line.

While using the assumption of a constant emittance is appropriate for determining the initial tune shift, in order to study long term effects, any change in the size of the beam should be accounted for. This was done by updating the emittance used in the Bassetti-Erskine model every 1000 turns, which was small when compared to multiple damping periods. This changes the space charge kick overtime, and, for a beam with presumably increasing emittance, this will lead to a weakening of the space charge tune shift. This can be seen in Fig. 2 where the core tune shift is plotted over the run.

The effect this has on the full bunch is shown in Fig. 3, where it can be seen that the tune footprint begins to shrink, with particles beginning to leave the region of the sextupole resonance. Despite crossing resonances, increased particle losses in tracking are not observed.

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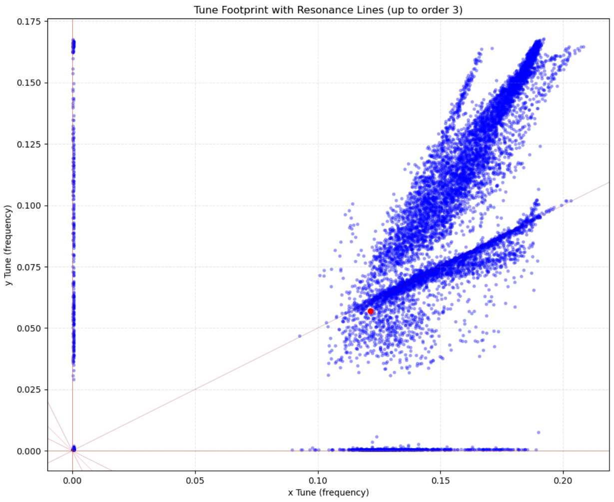


Figure 1: Tune footprint with space charge tune shift. Estimated tune shift of the bunch core shown in red.

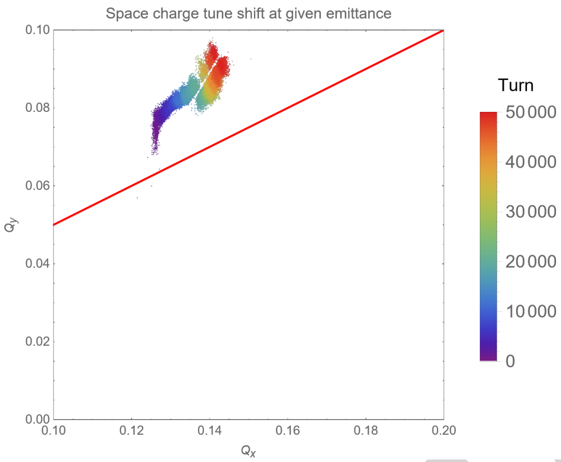


Figure 2: Location of the space charge tune shift as the emittance of the bunch increases over 50000 turns.

EMITTANCE GROWTH

One potential challenge with space charge is the increase in emittance that the beam can see over time. As mentioned previously, this change results in a decrease of the space charge tune shift so we expect this growth to decrease over time. An approximation for the scaling can be found by assuming emittance growth of tune diffusion with a scaling of ΔQ^2. It is then seen from eq. 1 that the growth rate would be

$$R_{SCx,y} = \frac{A_{SCx,y}}{\epsilon_{x,y}\left(\sqrt{\epsilon_x} + \sqrt{\epsilon_y}\right)} \tag{3}$$

where $A_{SCx,y}$ is a scaling factor which will be taken from tracking. This tracking was done for 10^5 turns with the same conditions as before, leading to Figs. 4a and 4b. both of these saw increase over the period on the order of 50%. This tracking neglects the effects of other sources of emittance change like IBS, BBS, and radiation, so to find a new es-

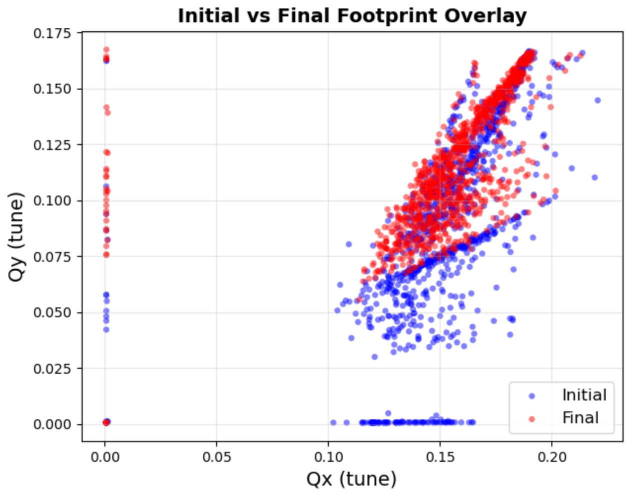
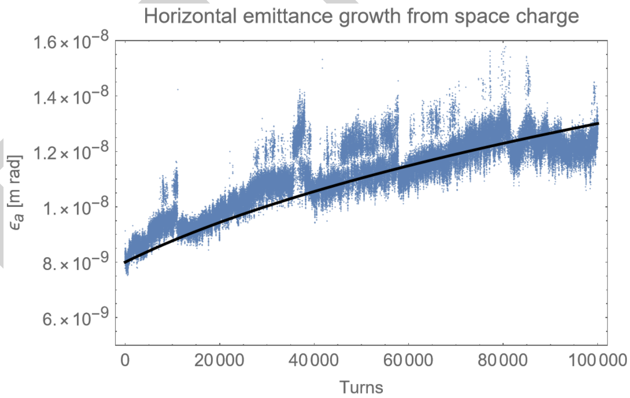


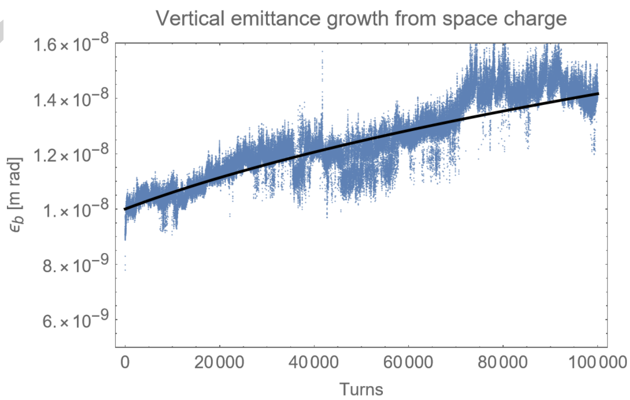
Figure 3: Tune Footprint decreasing in size as the space charge tune shift decreases from emittance change.

timate of the equilibrium emittance, these effects must be balanced.

As the emittance scaling of all of these effects is known, it is possible to arrive at an approximation of the equilibrium by finding the value such that the sum of the rates is zero



(a) Horizontal



(b) Vertical

Figure 4: Emittance increase from space charge with recalculation every 1000 turns.

$$R_{SCx,y} + R_{IBS} + R_{BBS} + R_{RADx,y} + R_0 = 0 \tag{4}$$

where R_0 is the stochastic part of the radiation and each other terms are

$$R_{IBS} = \frac{A_{IBS}}{\epsilon_x \epsilon_y} \tag{5}$$

$$R_{BBS} = \frac{A_{BBS}}{\epsilon_x \epsilon_y} \tag{6}$$

$$R_{RADx,y} = A_{RADx,y} \epsilon_{x,y} \tag{7}$$

where the A's are scaling factors taken from tracking. This gives an equilibrium emittance of 8.16 and 7.88 rad m where less than 1% is contributed from space charge, showing that this emittance growth is substantially less than from IBS and BBS.

IMAGE CURRENT

In addition to the direct space charge of the previous two sections, indirect effects such as image charge or image currents also must be studied. Tune shifts from image charge can largely be reduced through the use of circular beam pipes, however image currents induced in the wiggler pole faces are not as easily reduced and account for almost a fifth of the length of the ring. The image current tune shift is the third term in eq. 1, and, with the optics in the wigglers as shown in fig. 5 it can be seen that the tune shift from image currents is expected to be significantly larger in the vertical direction.

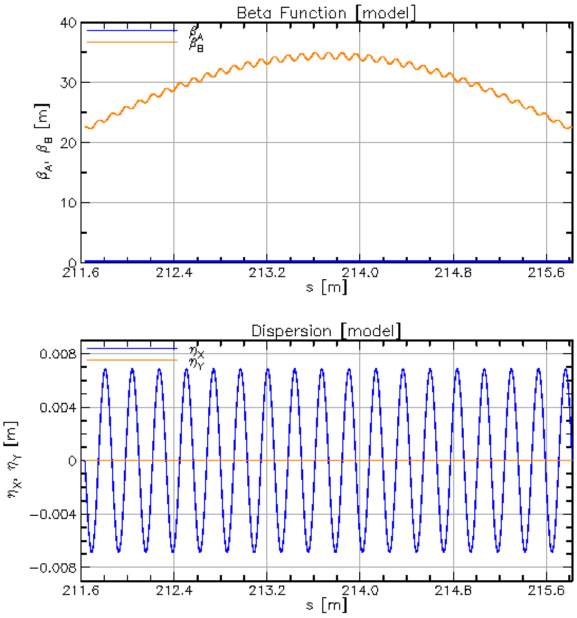


Figure 5: Optics across one wiggler.

When the β-function of the wiggler section is used with a pole gap of 1cm, the resulting tune shift is 1.7×10^{-4} in the horizontal and -1.8×10^{-2} in the vertical direction. This tune shift is largely negligible in the horizontal direction, but the shift in the vertical direction could cause noticeable but minor changes in dynamics when compared to the direct space charge tune shift. As this tune shift is about half of the vertical phase advance in the wigglers, wiggler focusing can be adjusted if needed.

CONCLUSION

At the energy of 150MeV the REC is designed for, space charge effects can produce a notable tune shift, both from direct and indirect sources. In this work, tracking using the Bassetti-Erskine space charge approximation was used to verify the space charge tune shift estimates used in the ring design. Emittance growth effects were also studied. Space charge strength reduction from beam size growth does cause some particles in the bunch to cross resonances, but an increase in particle losses were not seen. Based on this emittance growth, the estimate of new equilibrium emittance does not significantly change, confirming the previous emittance estimates. These results that previous estimates for the space charge tune shift and equilibrium emittance are in line with expected values from tracking, and that while space charge will remain an important area of study for the REC, initial tracking studies are promising and show these effects to be manageable within the REC design.

ACKNOWLEDGMENTS

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