A NEW BEAM DYNAMIC DESIGN PROCEDURE

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Abstract

The beam dynamics design of a Radio Frequency Quadrupole (RFQ) is usually optimized to match the beam parameters at the entrance and exit of the cavity. This design step defines the vane geometry—such as the distance from the vane tips to the beam axis, the vane-tip radius, the modulation type and amplitude, and the modulation length—and also provides the inter-vane voltage for the next stage: the RF design. Based on the inter-vane voltage profile along the cavity, the 2D cross-section is then adjusted to match the local cutoff frequency of the quadrupole mode, typically by modifying the quadrant volume where the magnetic field is dominant. In this paper, we propose a new approach to RFQ beam dynamics design. Instead of considering only the geometry near the beam axis, the full 2D cross-section—including the quadrant region—is integrated directly into the beam dynamics optimization. The resulting inter-vane voltage can then be obtained by preserving the magnetic-field region's area along the cavity, which may simplify the manufacturing process.

INTRODUCTION

At CEA/IRFU, the RFQ design process is divided into several stages [1], including beam dynamics [2], RF design [3], and thermomechanical analysis. Each stage provides inputs to the next, and iterative feedback is often required when the expected specifications are not met, leading to time-consuming optimization loops.

In an RFQ, the transverse geometry of the cavity can be separated into two regions: the vane tips close to the beam axis, which are determined by beam dynamics requirements, and the outer part of the quadrant, which is mainly used to adjust the RF properties of the structure. The RFQ cavity can be modeled as a succession of longitudinally varying waveguide cross-sections, each characterized by a local cutoff frequency. This local cutoff frequency directly governs the intervane voltage distribution along the cavity required for the beam dynamics.

In conventional design approaches, the beam dynamics stage defines only the vane-tip geometry, while the RF design stage adjusts the outer quadrant geometry to achieve the cutoff frequency and voltage profile. This separation introduces a weak coupling between beam dynamics and RF constraints, resulting in iterative design cycles. Additional requirements, such as maintaining a constant cavity volume for mechanical or thermal considerations, further increase the complexity of the optimization.

To address these limitations, this paper proposes a method to derive an analytical approximation of the cutoff frequency as a function of the main geometrical parameters of the RFQ cross-section, within a predefined range of validity. This approach enables the estimation of the local cutoff frequency directly during the beam dynamics design stage, allowing indirect control of the outer geometry and facilitating the enforcement of additional constraints such as constant cavity volume.

In addition, an analytical estimation of the RF power dissipation is introduced, providing a tool for early-stage optimization of the cavity energy efficiency.

The paper first introduces the key geometrical parameters describing the RFQ cross-section, along with the relationship between local cutoff frequency and intervane voltage. Then, the methodology used to construct the analytical approximations for cutoff frequency and power dissipation is presented. Finally, these formulations are integrated into the beam dynamics design process, and a design example satisfying RF and mechanical constraints is obtained.

CELL DESIGN

From a microwave point of view, an RFQ is merely a waveguide-based circuit, consisting of small 3D devices at the end regions, connected by multiple cells of waveguide with constant cross-section. The geometrical parameters for a quarter cell are presented in Fig. 1.

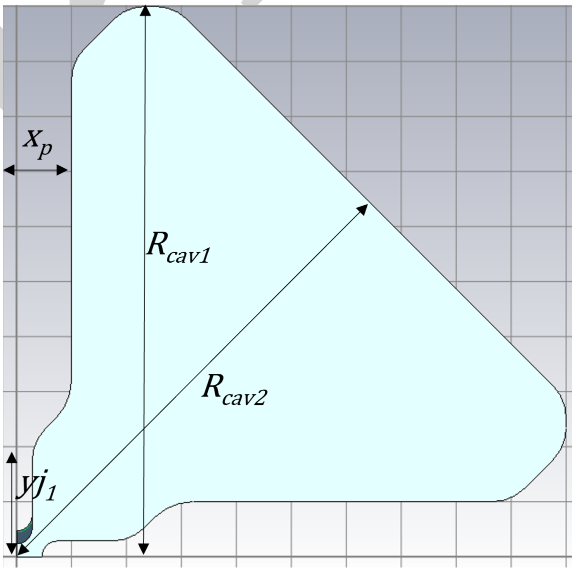

Figure 1: Geometrical parameters of quarter cross section.

Mechanical considerations impose constraints on the vane width x_p: it must be large enough to accommodate cooling channels, while being as small as possible to minimize RF power losses.

The beam dynamics design stage mainly controls the geometrical parameters close to the vane tips. The parameter r_0 represents the average distance from the vane to the beam axis, while ρ is the radius of curvature of the vane tips. The modulation length is denoted by L_{cell}, and its amplitude depends on the modulation parameter m. The vane distance to the beam axis, $r(z)$ is defined in Fig. 2.

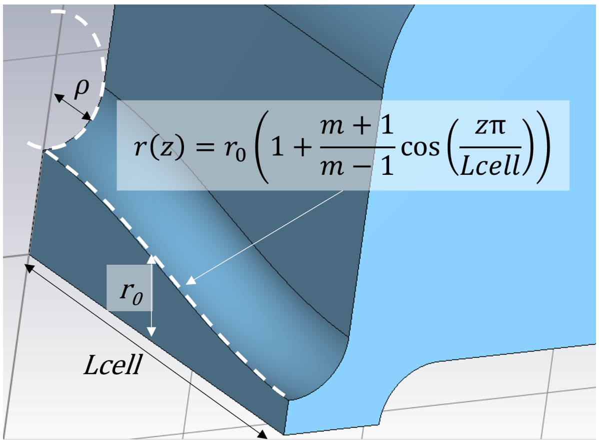

Figure 2: Geometrical parameters close to the beam.

The RF design stage controls the local cutoff frequency f_c. It can be slightly adjusted (typically a few percent) by modifying the equivalent capacitance through the geometrical parameter yj_1. A more effective way to control the cutoff frequency consists of modifying the equivalent inductance by adjusting the cavity volume through the parameters R_{cav1} and R_{cav2}. However, these parameters may be constrained by the slug tuner dimensions, which are used to compensate for manufacturing tolerances.

INTERVANE VOLTAGE AND LOCAL CUTOFF FREQUENCY

According to the transmission line model, the intervane voltage $V(z)$ is related to the local cutoff frequency $f_c(z)$ through:

$$\frac{\mathrm{d}^2V(z)}{\mathrm{d}z^2} = \left(\frac{\omega_c(z)^2}{c^2} - \frac{\omega_0^2}{c^2}\right)V(z)\,, \tag{1}$$

where $\omega = 2\pi f$ for a specified frequency f and f_0 is the cavity resonance.

The intervane voltage profile required by beam dynamics is used as an input in the RF design stage and determines the local cutoff frequency $f_c(z)$:

$$f_c(z) = f_0\sqrt{1 + \frac{1}{V(z)}\frac{\mathrm{d}^2V(z)}{\mathrm{d}z^2}\left(\frac{c}{2\pi f_0}\right)^2}\,. \tag{2}$$

The geometrical parameters of each cell may then be adjusted to satisfy this requirement.

Assuming an analytical formula providing the cutoff frequency based on the geometric parameters is available, these parameters can be directly derived from it at the beam dynamic design stage, avoiding systematic use of RF simulations and allowing more constraints on the geometry.

ANALYTICAL FORMULA FOR CUTOFF FREQUENCY AND LOSS

Data Generation

To estimate an analytical formula for the cutoff frequency, a set of 2400 configurations was generated. These configurations should be close enough to the one finally used by the beam dynamics. The range for each geometric parameter is presented in Table 1. The last line is a criterion to select the cells generated randomly, which implies that the largest amplitude of the modulation has to correspond to longer cells.

Table 1: Geometric Parameters Ranges

Parameter	**Range**	**Unit**
r_0	[2.8;3.6]	mm
m	[1;2.5]	None
ρ/r_0	[0.5;1.1]	None
r_0/L_{cell}	[1/15;1.2]	None
R_{cav1}	[80;100]	mm
R_{cav2}	[80;100]	mm
$m^{2.5}r_0/L_{cell}$	[0.5;0.9]	None

The parameter yj_1 is fixed to 20 mm and x_p to 10 mm. For each configuration, the cutoff frequency f_c and the loss per meter $Loss$ are computed using CST Microwave Studio (Dassault Systèmes). The total simulation time is approximately 2 days. The results for configuration i are gathered into vectors x_i and y_i. Assuming a linear relationship between these vectors, a constant term is included in x_i to account for an offset:

$$x_i = \begin{pmatrix} r_0/L_{cell}\sqrt{m-1} \\ \rho/r_0 \\ m \\ r_0 \\ R_{cav1} \\ R_{cav2} \\ 1 \end{pmatrix} y_i = \begin{pmatrix} f_c \\ Loss \end{pmatrix}\,. \tag{3}$$

Analytical Formula Approximation

The 2400 configurations are gathered into the matrix X (respectively Y), whose columns correspond to the vectors x_i (respectively y_i). The relationship is assumed to be linear:

$$Y = AX \tag{4}$$

The analytical approximation for the cutoff frequency and loss per meter is obtained using a least-squares approach:

$$\widetilde{A} = YX^T(XX^T)^{-1} \tag{5}$$

The comparison between the results from the 3D simulation on CST $(f_c, Loss)$, and the approximated values via the analytical formula $(\tilde{f_c}, \tilde{Loss})$ are presented in Figs. 3 and 4.

There is a very good agreement for both analytical formulas. The accuracy of the estimated frequency is less than

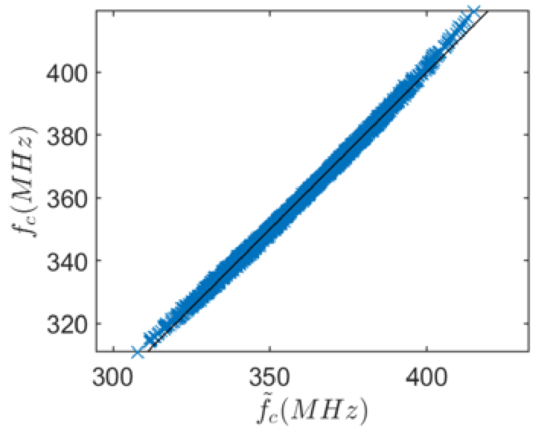

Figure 3: Analytic formula $\tilde{f}_c$ vs. 3D simulation f_c.

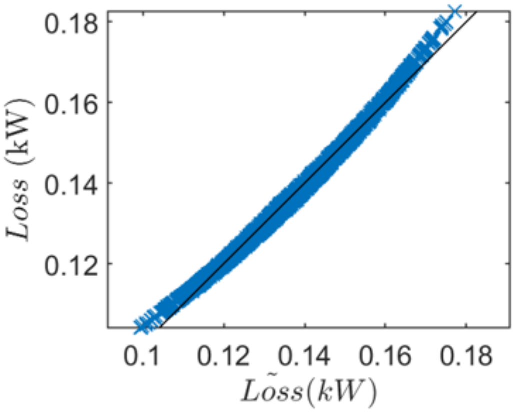

Figure 4: Analytic formula $\tilde{Loss}$ vs. 3D simulation $Loss$.

1%, and for the loss, about 5%. This error can decrease to 2% if we only consider the cell whose frequency is close to the cavity frequency.

IMPLEMENTATION INTO THE BEAM DYNAMIC DESIGN

The beam dynamics have been optimized for a 6 m-long RFQ cavity operating at 352 MHz with a beam current of 55 mA. Several configurations were investigated.

For each configuration, the intervane voltage defines the cutoff frequency profile $f_c(z)$. Assuming constant values for R_{cav1} and R_{cav2}, the optimized parameters $r_0(z)$ and $m(z)$ are used to determine $\rho(z)$. The new constraint does not degrade the optimized beam dynamics performance, as the transmission remains around 99%.

These geometrical parameters are then used in the RF design stage. Each cell is simulated on CST, and the parameter $yj_1(z)$ is slightly adjusted, around 20 mm, to match the target cutoff frequency and compensate for the approximation error of the analytical model. The final adjusted $yj_1(z)$ parameter along the cavity is illustrated in Fig. 5.

The required correction remains within ±1 mm, demonstrating the accuracy of the analytical approximation for the cutoff frequency.

Assuming continuous wave (CW) operation and neglecting 3D effects, the total RF power loss is estimated using the analytical model to be 463 kW, while the CST simulation predicts 461 kW, further validating the accuracy of the proposed method.

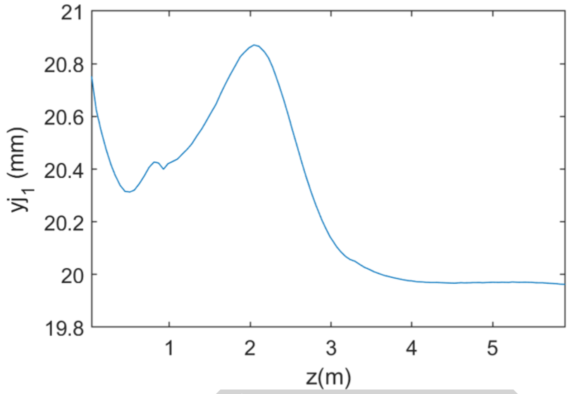

Figure 5: yj_1 adjustment along the RFQ cavity.

CONCLUSION

In this work, a method has been proposed to incorporate RF design constraints directly into the beam dynamics design stage of an RFQ. A direct relationship between the intervane voltage and the geometrical parameters has been established.

An analytical approximation of the cutoff frequency, as well as the RF power dissipation, has been derived from a set of numerical simulations. Although this approximation is valid within a predefined range of geometrical parameters, it provides sufficient accuracy for design purposes, with errors below 1% for the frequency and a few percent for the losses.

The proposed approach enables the estimation of RF parameters without relying on systematic 3D electromagnetic simulations, significantly reducing the number of iterations between beam dynamics and RF design stages. This allows additional constraints to be taken into account early in the design process.

The method has been successfully applied to the design of a 6 m long RFQ operating at 352 MHz, showing excellent agreement between analytical predictions and full-wave CST simulations.

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