IMPEDANCE ANALYSIS OF LONG VACUUM CHAMBERS VIA LONGITUDINALLY REDUCED GEOMETRIES

I. C. de Almeida, A. P. B. Lima, F. H. de Sá
Brazilian Synchrotron Light Laboratory, Campinas, Brazil

Abstract

Beamlines of modern synchrotron light sources demand the installation of long undulators with small gaps in the storage ring. The vacuum chamber of these devices requires time-consuming wakefield simulations for impedance estimation. Furthermore, these structures can exhibit trapped modes, demanding extensive eigenmode simulations for impedance characterization. This paper presents a method to estimate a conservative impedance envelope for trapped modes from a longitudinally reduced geometry, reducing both computational time and numerical artifacts inherent to long simulations. The proposed methodology is validated through the impedance analysis of the APPLE-2 undulator for the SIRIUS IPE beamline.

INTRODUCTION

Modern light sources demand long insertion devices with small gaps to achieve stronger magnetic fields and improved photon beam properties. These chambers might present a large horizontal aperture and a small vertical aperture compared to the standard storage ring vacuum chamber. Such geometries can support beam excited trapped modes below the beam pipe cutoff frequency, requiring extensive wakefield and eigenmode simulations for accurate impedance characterization. For long devices, the large number of trapped modes and the computational burden associated with large meshes become a major challenge. This paper therefore presents a method to estimate a conservative impedance envelope for trapped modes using eigenmode simulations of a longitudinally reduced geometry.

To demonstrate this approach, the impedance analysis of the APPLE-2 undulator for the SIRIUS IPE beamline is used as a case study. As shown in Fig. 1, the 3.774 m aluminum straight section of the APPLE-2 undulator previously used at the Swiss Light Source (SLS) has a horizontal aperture of 57 mm and a vertical aperture of 8.3 mm [1]. The copper transitions from the undulator elliptical chamber to the standard 24 mm inner diameter round vacuum chamber of SIRIUS were developed in-house.

The wakefield simulations of the complete geometry of the undulator with the tapered transitions were performed using ECHO3D [2] with a bunch length of 2.5 mm, a mesh step size of 0.5 mm in all three dimensions and a wakelength of 25 m. Figure 2 shows the real part of the vertical dipolar impedance, where numerous vertical dipolar modes were identified to be trapped below the beam pipe TE_{11} mode cutoff of 7.32 GHz. The characterization of these modes through a reduced geometry is discussed in the following sections.

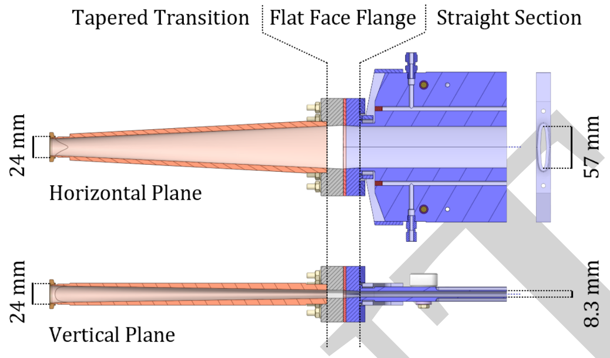

Figure 1: Horizontal and vertical views of a part of the aluminum straight section of the APPLE-2 undulator and one of the copper transitions to the standard 24 mm round vacuum chamber of SIRIUS. The elliptical cross section of the straight section is also shown.

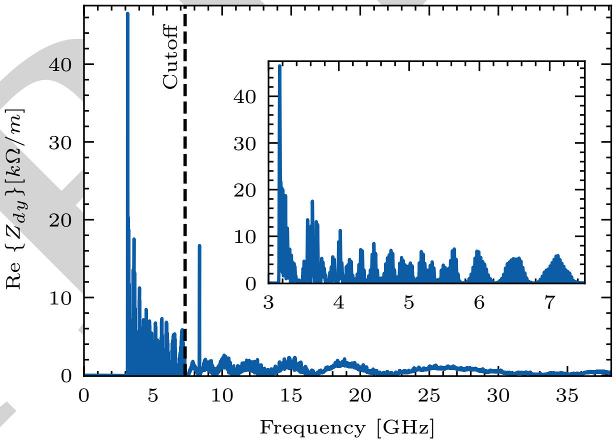

Figure 2: Real part of vertical dipolar impedance obtained from wakefield simulations of the complete undulator chamber. The detail shows the frequency range below the beam pipe cutoff.

EIGENMODE SIMULATIONS

To estimate the vertical dipolar shunt impedance of the trapped modes, the mode frequency, the longitudinal electric field, and the surface integral of the squared magnitude of the tangential magnetic field can be obtained from eigenmode simulations using ANSYS High-Frequency Structure Simulator (HFSS) [3]. Since the transverse electric and magnetic fields are related to the longitudinal electric field through the Panofsky-Wenzel theorem, the vertical dipolar shunt impedance is given by Eq. (1).

$$R_y = \frac{\left|\int_{-\infty}^{\infty} E_z(0, y_0, z, t = z/c) e^{jkz} dz\right|^2}{2Pky_0^2} = \frac{\left|V_\parallel(0, y_0)\right|^2}{2Pky_0^2}, \tag{1}$$

where $E_z(0, y_0, z, t)$ is the longitudinal electric field displaced vertically from the chamber center by y_0, $V_\parallel(0, y_0)$ is the corresponding voltage, $k = \omega/c$ is the wavenumber of the mode, where ω is the mode frequency and c is the speed of light, and P is the total power dissipated on the surface given by Eq. (2).

$$P = \frac{R_s(\rho_{Cu})}{2} \int_{T1+T2} |H_t|^2 dS + \frac{R_s(\rho_{Al})}{2} \int_S |H_t|^2 dS. \tag{2}$$

The first term of Eq. (2) is the power dissipated in the copper transitions (T_1 and T_2) and the second term is the power dissipated in the aluminum straight section (S). The integrals are given by the eigenmode simulations with perfect electric conductor (PEC) boundary conditions. The real part of the material dependent surface impedance R_s is given by Eq. (3).

$$R_s = \sqrt{\frac{\omega \mu_0 \rho}{2}}, \tag{3}$$

being μ_0 the vacuum magnetic permeability and ρ the electrical resistivity of the material at room temperature. Since the shunt impedance is independent of the field amplitudes, the simulations can be performed with a field normalization so that the stored energy is 1 J for each mode.

REDUCED GEOMETRY METHOD

To understand how the trapped mode spectrum scales with the straight section length and to motivate the reduced geometry approach, a simplified rectangular cavity model can be considered. For a rectangular cavity with horizontal aperture a, vertical aperture b and longitudinal length d, with $d > a > b$, the resonant frequency of the TE_{mnp} and TM_{mnp} modes are given by Eq. (4).

$$f_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}, \tag{4}$$

where μ and ϵ are, respectively, the magnetic permeability and electric permittivity of the material filling the cavity. The overall dominant mode is the TE_{101} and the dominant TM mode is the TM_{110}. For cavities with the same transverse dimensions, changing the length will not change the mode profiles. The only effect will be to increase their frequencies as the geometry is shortened. However, if $d \gg a$, the frequency of the modes TE_{mn1} of cavities with different lengths will be approximately equal for the same m and n.

As an approximation, a rectangular cavity with vertical and horizontal apertures equal to the ones of the APPLE-2 undulator ($a = 57$ mm and $b = 8.3$ mm) will have the mode frequencies given in Table 1 for different straight section lengths. Even for the complete straight section length, the frequencies are slightly lower than those of the actual undulator chamber since its cross section is smaller than the one of the rectangular approximation and the effective length is longer than the straight section length due to the transitions.

Table 1: Frequencies for some of the modes of a rectangular cavity approximation with transverse dimensions equal to the undulator apertures for different straight section lengths.

Mode	**Frequency [GHz]**		
	d = 3.774 m	**d = 1 m**	**d = 10 cm**
TE_{101}	2.63	2.63	3.03
TE_{201}	5.26	5.26	5.47
TE_{301}	7.89	7.89	8.03
TE_{011}	18.06	18.06	18.12
TM_{110}	18.25	18.25	18.25

Since the beam pipe TE_{11} cutoff frequency is 7.32 GHz, eigenmode simulations of the undulator chamber only presents the TE_{10p}-like and TE_{20p}-like modes. The TE_{20p}-like family has a negligible transverse electric field at the center of the chamber and, therefore, does not contribute to the beam coupling impedance.

The reduced geometry method is based on the field profiles shown in Fig. 3, where it is observed that the longitudinal electric fields of the trapped modes are negligible along the straight section, with the voltage contribution arising primarily from the transition regions[1]. Besides, since the field profiles at the transitions only change slightly with the straight section length, the maximum voltage in Eq. (1) occurs when the electric fields at both transitions are in phase as the beam traverses the structure. On the other hand, the dissipated power in the straight section increases approximately linearly with the straight section length, since the transverse field profiles that define the integral over S in Eq. (2) do not change significantly for different lengths and TE_{10p}-like modes.

Using these properties, it is possible to solve the eigenmodes for a reduced structure, calculate the worst-case voltage and the dissipated power, and then re-scale the power for the complete geometry. While this method does not yield the exact shunt impedances of the trapped modes in the complete device, it provides a conservative impedance envelope as a function of frequency.

Due to the longer straight section, the complete undulator geometry supports a significantly larger number of modes compared to the reduced model since there will be many more TE_{10p}-like modes below the cutoff frequency of the beam pipe. To validate the proposed scaling approach, eigenmode simulations were performed for a 1 m straight section and for a complete geometry with 3.774 m straight section.

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[1] Although the Panofsky–Wenzel theorem formally allows the transverse impedance to be calculated from the longitudinal voltage, its physical interpretation may be misleading. In practice, the beam experiences transverse kicks from the transverse fields distributed mainly along the straight section, S.

Figure 4 shows the shunt impedance envelope obtained from scaling the values calculated from the reduced version and the shunt impedances calculated from the complete geometry. The values obtained from the complete geometry remain below the envelope predicted by the reduced model, demonstrating that this method provides a reliable conservative estimate of the shunt impedance envelope.

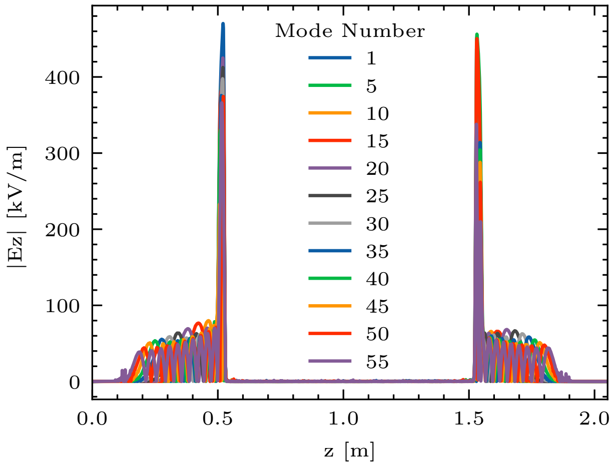

Figure 3: Magnitude of the longitudinal electric field for a 1 m straight section undulator geometry at a line vertically displaced by 1 mm from the chamber center for some of the trapped vertical dipolar modes.

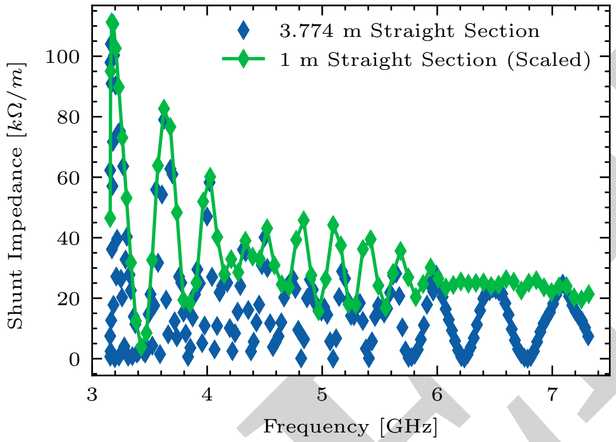

Figure 4: Envelope for the vertical dipolar shunt impedances obtained from the eigenmode simulations of the longitudinally reduced undulator geometry and the shunt impedances obtained from the complete geometry.

All modes with significant vertical dipolar shunt impedance obtained from the eigenmode simulations of the complete geometry correspond to peaks in the vertical dipolar impedance from the wakefield simulations results shown in Fig. 2, confirming that the identified modes correspond to beam-excited trapped modes.

The reduced model simulations took about 12 h, while the simulations for all the modes of the complete geometry required about 96 h. This demonstrates the effectiveness of the proposed method in decreasing the computational time.

However, there is a limit in reducing the straight section length since at some point there will be a mixing of the transitions fields. In addition, Table 1 also shows that when d is not sufficiently large compared to a, the TE_{101} mode frequency becomes larger than the corresponding mode frequency of the complete geometry. In this case, the envelope would not capture the first modes.

Currently, SIRIUS operates with a beam current of 200 mA and a bunch length of about 3.5 mm, with studies being performed to consider the feasibility of reaching 250 mA [4]. After the installation of the third harmonic cavity, nominal operation will be with a beam current of 350 mA and a bunch length larger than 9.0 mm [5,6], being the vacuum chambers designed to withstand up to 500 mA. Figure 5 shows the transverse coupled-bunch instability thresholds for the most demanding operating conditions without and with the third harmonic cavity. The largest trapped mode impedance obtained from the longitudinally reduced geometry simulations is also shown for comparison. These results indicate that no coupled-bunch instability is expected to be driven by the undulator vacuum chamber structure.

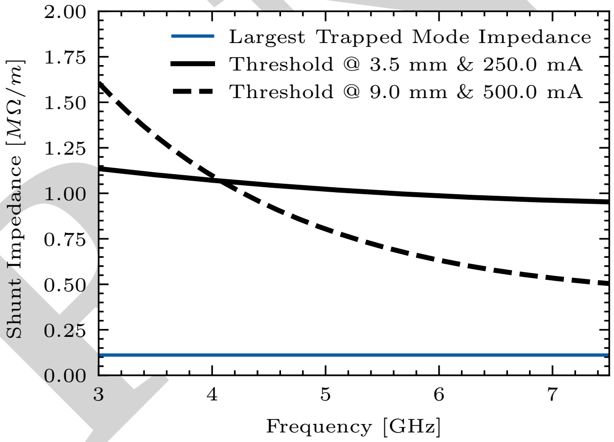

Figure 5: Comparison of the largest vertical dipolar trapped mode impedance calculated from the reduced geometry with the coupled-bunch instability thresholds for the most demanding operating conditions foreseen.

CONCLUSION

A method to characterize trapped modes through eigenmode simulations of longitudinally reduced geometries was presented. The APPLE-2 undulator chamber for the SIRIUS IPE beamline supports trapped vertical dipolar modes below the beam pipe cutoff frequency and this approach allowed the determination of a conservative envelope for the vertical dipolar shunt impedances while reducing the computational cost. The results obtained with the reduced model were validated by comparison with eigenmode simulations of the complete geometry. In all considered operating conditions, the estimated shunt impedances remained below the transverse coupled-bunch instability threshold. The undulator has been successfully installed and is currently operating without beam instabilities attributed to its vacuum chamber.

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