IMPROVED SPACE-CHARGE SIMULATION STUDIES FOR THE MUON COLLIDER PROTON COMPRESSOR*

M. Sörqvist†,1,2, S. Johannesson[1], N. Milas[1]
[1]European Spallation Source, Lund, Sweden
[2]Lund University, Lund, Sweden

Abstract

In the current design of the International Muon Collider Collaboration proton complex, a compressor ring rotates the proton beam in longitudinal phase space to produce the short pulses required for muon production. Previous simulation studies of this compressor have relied on 2.5D Particle-in-Cell (PIC) space charge in Xsuite, which employs a static mesh (i.e., it does not adapt to an evolving beam size). To validate these results, an adaptive mesh has been implemented and 3D space charge has been tested, followed by a convergence study and a tune sweep. A resonance phenomenon originally believed to be space-charge driven has been investigated, and the selection of an optimal working point has been further explored. The results show that both the adaptive mesh and 3D space charge introduce significant differences relative to the previous configuration, although compression to 2 ns should still be feasible provided the severity of micro-bunching is not significant. This work presents an overview of these studies and discusses their implications for optimization of the compressor.

INTRODUCTION

The proton complex of the Muon Collider comprises production, accumulation and lastly compression of proton bunches [1]. These are then to hit a target for production of pions, which decay to muons. The baseline compression setup for the Muon Collider is an accumulator lattice with 120 ns pulses and a highly dispersive, above transition, compressor ring further reducing the bunch length to 2 ns. An option suggested is to operate at 5 GeV, which this study explores. Lattice functions of the compressor is shown in Fig. 1. The compressor is inspired by the design in [2].

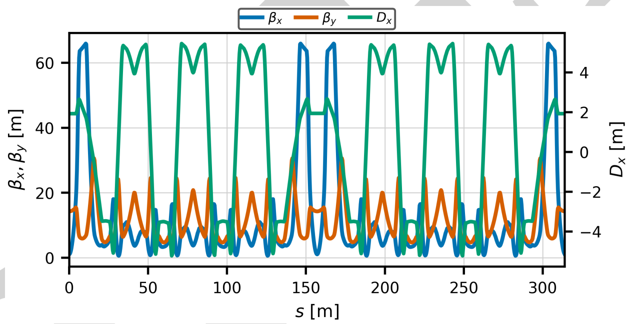


Figure 1: Lattice functions (β_x, β_y, D_x) for the compressor.

Previous simulation studies of compression have used 2.5D Particle-in-Cell (PIC) space-charge simulations in Xsuite [3, 4], which employs a static mesh. Previous simulations indicated no major show-stoppers to achieve a sub 2 ns compression for a two-bunch scheme, however with a fourth-order structure in transverse phase space. In Xsuite, 2.5D space-charge implies solving transverse open boundary Poisson equations over several longitudinal slices. 3D space-charge solves an open boundary 3D Poisson equation.

As the compressor operates in a space-charge dominated regime, the question has been posed as to whether 2.5D would sufficiently model the space-charge dynamics. Furthermore, a static mesh would require significant computational resources to model both initial and final turns with 3D space-charge. Thus an adaptive mesh has been developed for Xsuite which evolves with the beam size over turns.

IMPROVED SIMULATIONS

New simulations have been performed with 3D space-charge and an adaptive mesh (32×32×32 resolution with $1e6$ macro-particles, called "baseline configuration") in Xsuite to validate previous results. The lattice tune ("original tune") was $Q_x = 10.913$ and $Q_y = 5.891$. A comparison of 2.5D vs 3D is shown in Fig. 2 and another comparison of static vs adaptive mesh is shown in Fig. 3. Maximum compression was reached earlier for 3D space charge, and the transverse blow-up is not as significant for 3D. Possibly this is caused by stronger longitudinal-transverse coupling. Significant filamentation is seen for 3D but not 2.5D, perhaps caused by the longitudinal space charge affecting the non-linear longitudinal dynamics. An adaptive mesh yields a larger transverse emittance, possibly due to higher compression levels.

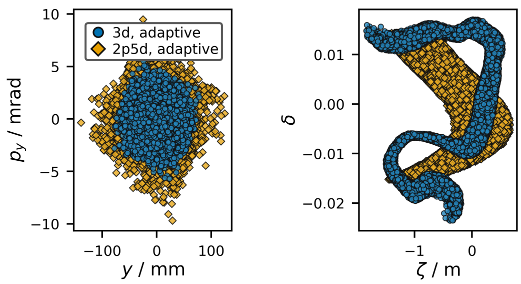


Figure 2: Comparison between 2.5D and 3D space charge with adaptive mesh. Each result is shown at turn of smallest σ_z (turn 41 for 2.5D, turn 38 for 3D).

Convergence Study

A particle-per-cell convergence study with 3D adaptive space-charge was conducted. Macro-particle amounts ranging between $5 \cdot 10^3$ and $5 \cdot 10^6$ as well as mesh sizes ranging

* Work endorsed by IMCC
† markus_sorqvist@hotmail.se

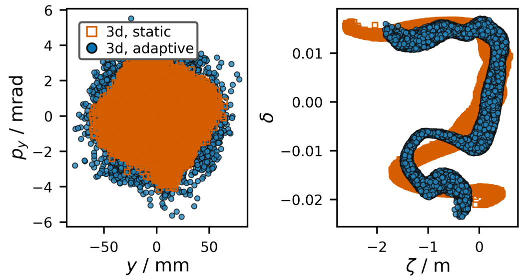


Figure 3: Comparison between adaptive and static mesh with 3D space charge, for turn 38.

between 32 to 160 were used. The result for turn 41 is shown in Fig. 4, indicating convergence at 30 macro-particles per cell. This criterion is fulfilled for the baseline configuration.

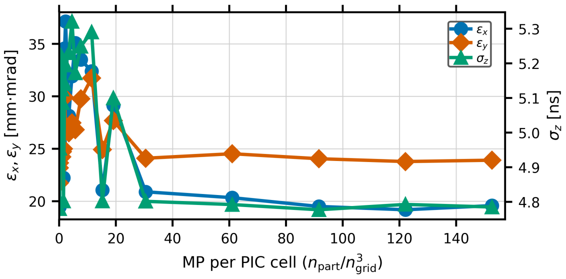


Figure 4: Macro-particle per cell convergence for turn 41.

FOURFOLD STRUCTURE INVESTIGATIONS

It was initially believed that a transverse fourth-order structure was a space-charge driven resonance, but Fig. 5, showing the emittance development per turn in the compressor suggest otherwise.

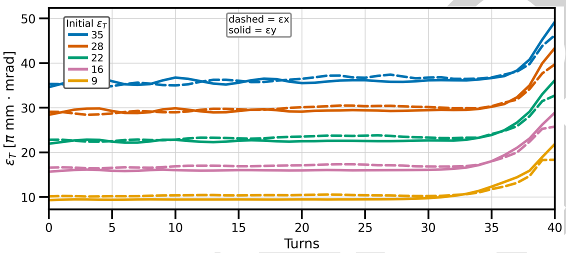


Figure 5: Transverse emittance development for different initial emittances.

As can be seen, the absolute transverse blow-up observed for later turns does not decrease with an increased initial emittance. In a simulation without space-charge (see Fig. 6), the structure disappears in y, while it remains in x. It was determined that while space-charge may amplify the structure and introduce $x-y$ coupling, it is not space-charge driven. This initiated an investigation into the root cause of the structure.

Longitudinal-Transverse Coupling

Through dispersion there will be significant longitudinal-transverse coupling in the lattice even without space charge.

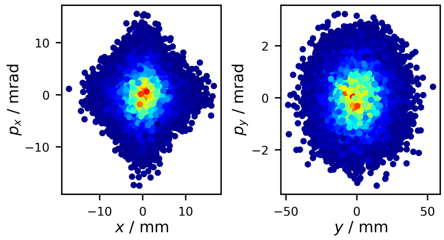


Figure 6: Transverse emittance for a no-space charge simulation at turn 41. The structure persists in x.

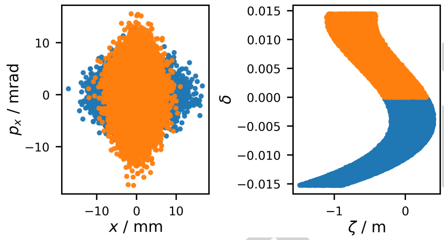


Figure 7: Positive (negative) momentum deviation particles in orange (blue) with no space-charge.

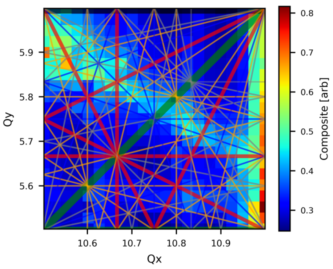
Thus, several no space-charge tests were conducted to evaluate whether the structure could be caused by synchro-betatron coupling. This included moving the RF-cavity from a highly dispersive region to a low-dispersion region, and similarly was done for the observation point. In both cases a structure remained present. Simulating for 164 turns revealed the structure only at turns of high energy spread. The RF-cavity was thereafter removed, and the initial bunch was injected in its ideal, rotated state. The structure was still visible, and it was determined that while longitudinal-transverse coupling was important, synchro-betatron coupling was not the cause.

Non-linear Chromatic Effects

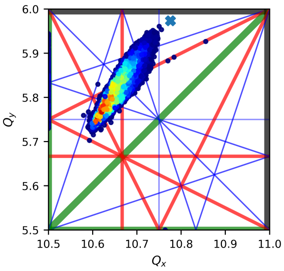
The lattice was originally matched for a chromaticity of $\xi = 0$. Initially, it was thought that the sextupoles could drive a resonance at second order. However, attempts to adjust the chromaticity either did not suppress the structure or caused emittance blow-up. Studies conducted by S. Machida have determined that the structure without space-charge is not a resonance, but a structure possibly driven by non-linear chromaticity and the large momentum spread [5]. This effect is demonstrated in Fig. 7, where positive and negative momentum deviation particles have undergone chromatic beta-beating. Superposing their phase spaces creates a fourfold structure. The addition of more sextupole families may suppress it. With space-charge active, a true resonance may be excited, thus amplifying the structure. With a superperiodicity of 2, this can happen through systematic resonances $4Q_x = 42$ and $4Q_y = 22$.

TUNE SCAN

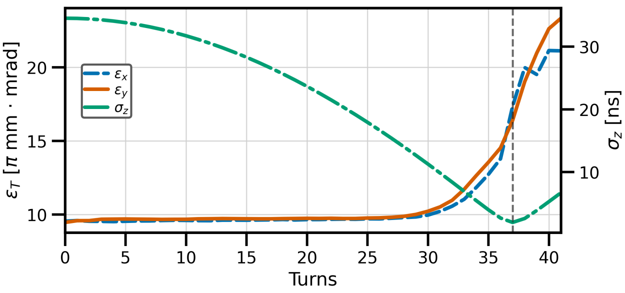
To evaluate the new space-charge configuration, and the possibility of a new working point where the fourfold structure would be weaker, a tune scan was performed. Transverse emittance, bunch length and particle loss for each simulations' optimal σ_z turn was evaluated, with a composite result shown in Fig. 8a. One feasible candidate avoiding the systematic resonances is [10.776, 5.974], with shortest σ_z at turn 37, shown in Fig. 8 (b-c). The space-charge will shift the particles away from the vertical integer line, although studies with modeled imperfections will need to validate the choice. A bunch length of 1.95 ns was achieved for this and many other candidates.



(a) Composite tune scan results.



(b) Candidate tune spread.



(c) Candidate emittance and bunch length.

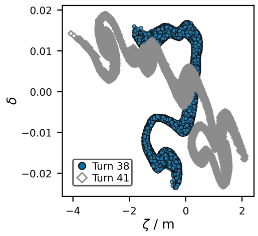
Figure 8: Tune scan as well as results for candidate [10.776, 5.974] (turns 0-37 in (b), (0-41 in (c)).

MICRO-BUNCHING

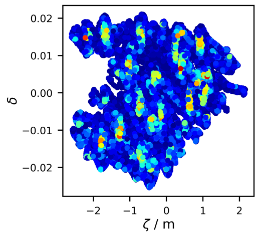
In Fig. 9a, 3D adaptive mesh (baseline configuration, original tune) is compared longitudinally at turns 38 and 41. At turn 41, significant micro-bunching has emerged. Another test increased only the longitudinal mesh resolution from 32 to 160, and macro-particle count was increased from $1e6$ to $8e6$ for particle-per-cell margin (original tune). The result is shown in Fig. 9b.

It is discussed in [6] how a coarse mesh may alias short-wavelength (high-k) forces, causing force modulation and artificial instabilities. In Fig. 9a, the rapid mesh re-expansion may cause previously resolved high-k fluctuations to alias on the coarser mesh, driving the observed microbunching.

Also [6] notes that PIC lacks "displacement invariance": the force between two particles depends not only on their separation but also on their positions relative to the mesh. Piecewise constant interpolation further introduces force jumps when particles cross cell boundaries. Although Xsuite uses piecewise continuous interpolation [4, 7], re-meshing can reintroduce such effects because particle positions relative to the mesh change discontinuously. The resulting force fluctuations, better resolved on a fine mesh and amplified by z–δ coupling and the negative mass instability, may contribute



(a) Baseline configuration at turns 38 versus 41, microbunching for the latter.



(b) Transverse mesh of 32, a z-mesh of 160, and 8 million macro-particles.

Figure 9: Two types of micro-bunching shown in longitudinal phase space: After mesh re-expands for baseline configuration (a), before mesh re-expands (turn 38) with increased longitudinal mesh and macro-particle amount (b).

to the microbunching in Fig. 9b, where the mesh has not yet re-expanded.

Alternatively, the negative mass instability itself may dominate, since the ring operates above transition with initially small energy spread. In that case, a lattice re-design may be required, as the shortest rms bunch length in Fig. 9b remains more than 1 ns longer than for the same tune (original) in the baseline configuration.

FUTURE WORK

Future work should include simulations with a high-resolution static mesh and cross-checks with other codes, preferably those implementing boundary conditions. Empirical data is further needed for extensive validation.

CONCLUSION

Results indicate that both employment of 3D over 2.5D space charge, as well as an adaptive mesh over a static one result in significant differences. The feasibility of compression to 2 ns with the improved configuration is shown. A fourfold structure is likely driven by sextupoles and amplified by space-charge. Microbunching has been observed, although the cause has not yet been determined if it is physical, numerical, or both. A re-design with more sextupole families may be needed to curb a four-fold structure, and further considerations must be taken if the microbunching is physical. In this case, general performance may need to be re-evaluated.

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