DESIGN AND IMPLEMENTATION OF A CAVITY VOLTAGE FEEDBACK TRACKING ALGORITHM FOR DOUBLE RF SYSTEMS

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Abstract

In fourth-generation storage rings, double RF systems are widely employed to lengthen bunches, thereby enhancing the Touschek lifetime and suppressing IBS effects. However, the harmonic cavity (HC) in such systems can drive various types of longitudinal instabilities, which severely impede effective bunch lengthening. In theory, a low-level feedback (LLRF) system can be used to selectively modify the cavity impedance to suppress these longitudinal instabilities. In this paper, we design a physical model for such a LLRF feedback system and implement its functionality in tracking code. This implementation is subsequently integrated as a module into an existing longitudinal tracking simulation program. Tracking results from the new code show excellent agreement with theoretical impedance analysis, providing strong support for the design of feedback parameters.

INTRODUCTION

The use of a harmonic cavity (HC) to stretch bunches for alleviating intra-beam scattering and improving the Touschek lifetime has become a common practice in the design of fourth-generation light sources. Nevertheless, an HC may also drive severe instabilities, among which the longitudinal coupled-bunch instability (LCBI) driven by the HC fundamental mode impedance is particularly prominent. LCBI causes deviations of the bunch centroid and bunch length from the stretched equilibrium, seriously degrading the performance of a double RF system.

Based on experience with single RF systems, low-level feedback (LLFB) techniques can suppress coupled-bunch instabilities driven by the fundamental mode of the main cavity. For instance, a combination of direct feedback (DFB) and one-turn feedback (OTFB) can significantly reduce the fundamental mode impedance seen by the beam overall, and can also selectively lower the impedance of the RF cavity at revolution frequency harmonics. Both PEP-II [1, 2] and CERN [3] have systematically studied RF system models that incorporate both DFB and OTFB.

The concept of employing the DFB system to suppress HC-driven instabilities in double RF systems has recently been proposed [4]. However, systematic studies on the selection of feedback parameters and the analysis of suppression effectiveness are still lacking. To address this gap, we designed a physical model of the DFB+OTFB feedback system for the HC and implemented it in the STABLE code—a GPU-accelerated multi-particle, multi-bunch tracking method. Taking the Hefei Advanced Light Facility (HALF) as an example, we investigated the suppression of the periodic transient beam loading (PTBL) effect, a specific LCBI, using closed-loop cavity impedance analysis and the upgraded STABLE code.

DIRECT FEEDBACK AND ONE-TURN FEEDBACK

Direct feedback (DFB) can suppress LCBI by introducing a high-gain, low-latency loop in the RF cavity amplifier chain, thereby substantially reducing the fundamental mode impedance seen by the beam. Recently, ALBA implemented a digital DFB system and experimentally verified its effectiveness in reducing the fundamental mode impedance [5]. ALBA also plans to use an active third-harmonic cavity with DFB for bunch lengthening in its upgraded fourth-generation light source, ALBA-II [6]. Furthermore, SuperKEKB employed characteristic equation analysis and time-domain simulations to quantitatively study how DFB, together with other control loops, improves stability, and provided a recommended set of feedback parameters [7].

Unlike DFB, the one-turn feedback (OTFB) is a special type of comb-filter feedback. It selectively enhances the gain or applies a phase shift at the synchrotron frequency sidebands via a “one-turn delay plus a designed filter”, thereby specifically reducing the HC fundamental mode impedance around the revolution frequency harmonics. OTFB was initially proposed for the CERN Super Proton Synchrotron (SPS), where the distance between the generator and the cavity precluded fast RF feedback [8]. It was later used in PEP-II, in series with a fast RF feedback similar to that employed in the Large Hadron Collider (LHC) [9].

Combining the two feedback methods and following the studies at PEP-II and CERN, we present the DFB+OTFB feedback model implemented in this work, as shown in Fig. 1.

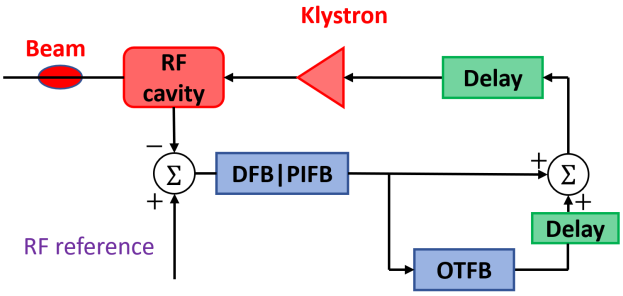

Figure 1: Block diagram of the combined DFB+OTFB feedback model.

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IMPLEMENTATION IN STABLE

STABLE is a GPU-accelerated multi-bunch, multi-particle tracking code designed for investigating longitudinal beam dynamics in double RF systems, offering high computational efficiency [10]. While the code originally included a PI feedback module, we have implemented additional DFB and OTFB modules. The difference equation corresponding to the combined DFB and OTFB system is:

$$\Delta I_{\text{out}}(n) = k_p G \Delta I_{\text{in}}(n - N_1) e^{j\phi} + K \Delta I_{\text{out}}(n - N_2), \quad (1)$$

where ΔI_{in} and ΔI_{out} represent the input and output generator current deviations, respectively, and N_1, N_2 denote the number of bunches corresponding to delay times τ and $T_0 - T_G$. The complete feedback system implementation pseudocode is shown in Algorithm 1.

In Algorithm 1, i represents the bunch index, cavity voltage is averaged over groups of m bunches, and k denotes the k-th bunch in the current group; ω_r is the cavity resonant frequency, R_L is the loaded impedance, Q_L is the loaded quality factor, T_0 is the revolution period, n is the cavity harmonic order, and h is the harmonic number; $V_{0,i-1}$ is the cavity voltage induced by the previous bunch during its passage (zero if no bunch is present); V_{g0} is the corrective generator voltage calculated by the feedback loop; V_{ideal} is the target cavity voltage; k_p, k_i, ϕ are DFB or PI feedback parameters (with k_i primarily for eliminating steady-state error and having minimal impact on impedance); G, K are OTFB parameters; d_1, d_2 represent the normalized delays of the two feedback loops, normalized by the bunch spacing; and I_{g0} is the fixed feed-forward generator current.

Algorithm 1: DFB with OTFB Control Algorithm

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Data: Bunch index $i$, control parameters $m$, $q_L$, $\omega_r$,
      $T_0$, $h$, $R_L$, $Q_L$, $k_p$, $k_i$, $\phi$, $G$, $K$, $d1$, $d2$, $n$, $I_{g0}$,
      $V_{\text{ideal}}$
Result: Cavity voltage evolution
for each bunch i do
    Calculate beam loading voltage $V_{b,i}$ according to:
        $V_{b,i} = V_{b,i-1} \exp\left\{\left(j - \frac{1}{2q_L}\right)\frac{\omega_r T_0}{h}\right\} + V_{0,i-1}$
    Calculate generator cavity voltage $V_{g,i}$ according
      to:
        $V_{g,i} = V_{g,i-1} \exp\left\{\left(j - \frac{1}{2q_L}\right)\frac{\omega_r T_0}{h}\right\} + V_{g,0}$
    The sum of the two gives the total cavity voltage:
        $V_{t,i} = V_{b,i} + V_{g,i}$
    if $i \% m = 0$ then
        $k = i/m$;
        Calculate the average cavity voltage $V_{T,k}$ of
          $m$ total cavity voltage;
        Calculate the error current:
            $\Delta I_k = (V_{\text{ideal}} - V_{T,k})/R_L$
        Calculate the generator current deviation
          $\Delta I_{g1,k}$ after DFB:
            $\Delta I_{g1,k} = \left(k_p \Delta I_k + \sum_{p=-\infty}^{k} k_i \Delta I_p\right)\exp(j\phi)$
        Calculate the generator current deviation
          $\Delta I_{g2,k}$ after OTFB:
            $\Delta I_{g2,k} = G\Delta I_{g1,k-d1/m} + K\Delta I_{g2,k-h/m}$
        Calculate new generator cavity voltage $V_{g0}$:
            $V_{g0} = \frac{2\pi n R_L}{Q_L}\left(\Delta I_{g1,k-d1/m} + \Delta I_{g2,k-d2/m} + I_{g0}\right)$
    end
end
```

VERIFICATION OF PTBL SUPPRESSION

The periodic transient beam loading (PTBL) effect, also known as a special type of mode-1 instability with a very low oscillation frequency, can seriously degrade the bunch lengthening performance of a double RF system. This instability is believed to be driven by the imaginary part of the HC fundamental mode impedance, and its severity grows with the product of the HC R/Q and the beam current I_0. In this section, we take the superconducting active double RF system of HALF as an example, with detailed parameters available in Ref. [11], and employ the extended STABLE code to study the influence of the DFB+OTFB feedback on the closed-loop cavity impedance and the PTBL threshold current. It should be noted that, to exacerbate the instability, we increased the R/Q of the HC to 90 Ω.

Closed-Loop Cavity Impedance

Under the narrow-band resonator approximations, the fundamental mode impedance of the harmonic cavity can be expressed as:

$$Z(\omega) = \frac{R_L}{1 + jQ_L\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}, \quad (2)$$

where R_L, Q_L, and ω_r are the loaded impedance, loaded quality factor, and resonant angular frequency of the HC, respectively. Throughout this paper, the $e^{j\omega t}$ convention is adopted for all time-harmonic quantities.

Based on the configuration shown in Fig. 1, the overall closed-loop cavity impedance incorporating both feedback paths can be written as:

$$Z_T(\omega) = \frac{Z(\omega)}{1 + e^{-j\omega\tau}\frac{k_p}{R_L}Z(\omega)e^{j\phi}[1 + \frac{G}{1-Ke^{-j\omega T_0}}e^{-j\omega(T_0-T_G)}]}. \quad (3)$$

where τ is the DFB delay, k_p is the DFB gain, and ϕ is the DFB phase shift, G is the OTFB gain, K is a constant between 0 and 1, and T_G is the group delay of the comb filter.

Considering the practical digital DFB system, we fixed the delay time at 1000 ns. Using the HALF parameters and comparing the imaginary part of the closed-loop cavity impedance with and without the DFB+OTFB feedback, we selected a set of feedback parameters:

$$k_p = 8.2, \phi = 0.06\pi, G = 0.34, K = 31/32, T_d = -3/8\,T_0.$$

The corresponding comparison of the imaginary part of the closed-loop impedance under these parameters is plotted in Fig. 2.

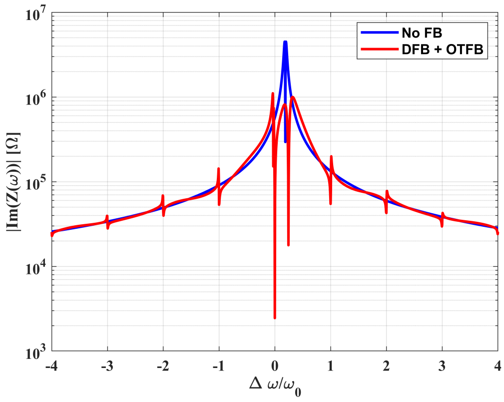

Figure 2: Comparison of the imaginary part of the closed-loop cavity impedance under only PI feedback and DFB + OTFB conditions.

Figure 2 clearly shows that the imaginFigure ary part of the closed-loop cavity impedance is significantly reduced at 0 and $\pm\omega_0$, and that the impedance at higher revolution frequency harmonics is also reduced to some extent. Theoretically, this parameter can effectively suppress PTBL effect.

Tracking Simulation Verification

Using the extended STABLE code, we carried out a detailed comparison of the longitudinal bunch motion under optimum lengthening conditions at the nominal current of 350 mA, between the case with PI-only feedback and that with the DFB+OTFB feedback. In the tracking simulations presented here, we focus on uniform filling patterns. Each bunch contains 20,000 macro-particles, and the tracking spans 200,000 turns. The results are shown in Fig. 3.

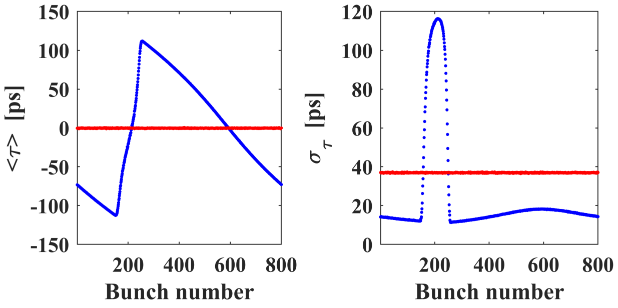

Figure 3: The bunch center (left) and bunch length (right) distribution after 200,000 turns. Blue points represents the data with only PI feedback, and red points represent the data with DFB+OTFB.

Figure 3 indicates that, with the PI-only feedback, strong PTBL effects develop in the bunches and the overall bunch lengthening performance is significantly degraded. In contrast, when the DFB+OTFB feedback is active with the parameters given in Section 4.1, the bunches remain stable and the optimum bunch lengthening is preserved.

CONCLUSION

In this paper, the DFB+OTFB system has been implemented in the particle tracking code STABLE. Combined with theoretical analysis of the closed-loop cavity impedance, its effectiveness in suppressing PTBL has been verified using HALF parameters. The results show that, with properly chosen feedback parameters, the DFB+OTFB system can effectively increase the PTBL threshold current. This study indicates that the upgraded STABLE code can serve as a powerful tool to assist LLFB-based suppression of instabilities in double RF systems.

As the suppression performance strongly depends on the feedback parameters, a detailed investigation of this dependence will be pursued in future work. Additionally, research on instability suppression for triple RF systems will be carried out in parallel [12].

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