MULTI-BUNCH LONGITUDINAL BEAM DYNAMICS IN THE ELETTRA 2.0 STORAGE RING WITH A DOUBLE-RF SYSTEM

S. Dastan*, S. Di Mitri, E. Karantzoulis, S. Krecic, M. Lonza, K. Manukyan
Elettra-Sincrotrone Trieste S.C.p.A., Trieste, Italy

Abstract

We present a systematic multi-bunch longitudinal beam dynamics study for the Elettra 2.0 storage ring equipped with a double RF system (main RF cavity plus passive third-harmonic cavity). The higher-order modes (HOMs) of the main RF cavity, obtained from 3D eigenmode simulations, are used as input to coupled-bunch instability models. For these HOMs, we compute growth rates using a simple harmonic-oscillator (SHO) model and an extended model including the nonlinear double-RF potential and Landau damping (HHC), and determine the HOM detuning required to bring all growth rates below the damping capability of the longitudinal feedback. The analytical predictions are benchmarked against multi-bunch tracking in Elegant. In parallel, we optimise the double-RF parameters for both uniform and non-uniform fillings using mbtrack2 tracking with the ALBuMS (Algorithm for Longitudinal Multibunch Beam Stability) algorithm. For the operational main-cavity detuning, we identify narrow third-harmonic cavity detuning windows that avoid Robinson-type and periodic transient beam loading (PTBL) instabilities while providing near-flat RF potentials, bunch-lengthening factors up to 4.4 and lifetime improvements up to 3–4. These results define a robust working point for the Elettra 2.0 double-RF system.

INTRODUCTION

In this work, the longitudinal stability problem is studied through two complementary groups of instabilities. The first group consists of HOM-driven coupled-bunch instabilities. These are driven by higher-order modes of the main RF cavity and are analyzed using HOM parameters obtained from electromagnetic simulations. This framework is particularly relevant for bunch-lengthened beams in nonlinear or near-quartic longitudinal potentials, where the instability growth rate and Landau damping must be treated self-consistently. The second group consists of instabilities intrinsic to the double RF system itself. These include Robinson instabilities (dipole/AC, quadrupole, and zero-frequency/DC), fast mode-coupling instability, and periodic transient beam loading (PTBL). In the terminology of the semianalytical ALBuMS [1] framework, the first two correspond to coupled-bunch mode $l = 0$, while PTBL corresponds to $l = 1$. These mechanisms can dominate different regions of parameter space and strongly constrain the accessible harmonic-cavity operating window. The purpose of this paper is therefore to combine these two levels of analysis: first, the study of HOM-driven coupled-bunch growth rates and their reduction via Landau damping in the double-RF potential; second, the global exploration of the harmonic-cavity operating region to identify stable windows that avoid Robinson-type and PTBL instabilities while preserving near-flat-potential conditions, bunch lengthening, and lifetime improvement.

HOM-DRIVEN COUPLED-BUNCH INSTABILITIES

Modes Above Instability Threshold

The higher-order modes (HOMs) of the main RF cavity are obtained from electromagnetic simulations up to 7 GHz [2], which corresponds to the first cutoff frequency for Elettra 2.0 [3]. To identify the modes that can drive longitudinal coupled-bunch instabilities, a rough impedance threshold criterion is applied [4]:

$$Z_{\parallel}^{\mathrm{thresh}} = \frac{1}{N_c} \cdot \frac{1}{f_{\mathrm{HOM}}} \cdot \frac{2E_0 Q_s}{I_0 \tau_s} \tag{1}$$

Here, N_c is the number of cavities, f_{HOM} is the HOM frequency, E_0 is the beam energy, Q_s is the synchrotron tune, I_0 is the total beam current, and τ_s is the longitudinal damping time.

For Elettra 2.0, $Q_s = 2.6 \times 10^{-3}$ without the 3HC and $Q_s \approx 1.0 \times 10^{-3}$ with the 3HC at a detuning of $\sim$ 80 kHz. From this analysis, 78 HOMs are found that exceed the instability threshold in the presence of the third-harmonic cavity (3HC), as shown in Fig. 1. A filtering procedure is applied based on the separation of frequency scales [5], $\omega_s \ll \omega_{\mathrm{HOM}}/(2Q_{HOM}) \ll \omega_0$, where ω_s, ω_{HOM}, ω_0, and Q_{HOM} denote the synchrotron frequency, HOM frequency, revolution frequency, and HOM quality factor, respectively. This ensures that each HOM effectively couples to a single revolution harmonic, with the strongest instability near $f_{\mathrm{HOM}} \approx N f_{\mathrm{rev}}$. After filtering, 69 HOMs are retained.

Growth Rate Analysis

The growth rates of the filtered HOMs are evaluated using two approaches.

SHO model: The simple harmonic oscillator (SHO) model assumes a rigid bunch without Landau damping. In this case, 28 modes exhibit growth rates exceeding the longitudinal feedback capability of Elettra 2.0, which can damp instabilities with growth rates below 1000 s^{-1}, as shown in Fig. 2. To reduce all growth rates below this threshold, a maximum HOMs detuning of ±69 kHz is required. The SHO model shows symmetric behavior around resonance.

HHC model: The higher-harmonic cavity (HHC) [5] model includes Landau damping arising from the nonlinear

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* sara.dastan@elettra.eu

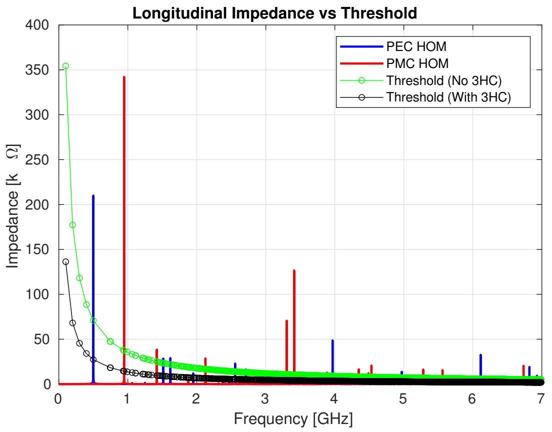

Figure 1: Longitudinal impedance of HOMs compared to the instability threshold. PEC (perfect electric conductor) and PMC (perfect magnetic conductor) denote the boundary conditions used in the electromagnetic simulations.

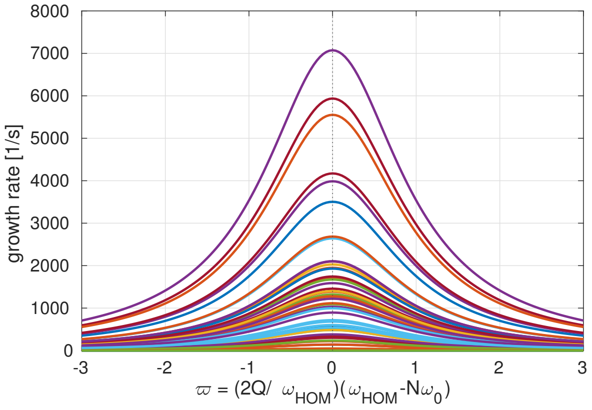

Figure 2: Growth rates of HOMs as a function of normalized detuning $\bar{\omega}$ using the SHO model. Each curve corresponds to a different HOM.

double-RF potential. In this case, only 8 modes remain unstable above 1000 s^{-1}, as shown in Fig. 3. The maximum required HOMs detuning to suppress these modes is significantly reduced to about −46 kHz (or ~ 30 kHz depending on the detuning direction). Unlike the SHO model, the response is asymmetric around resonance.

Tracking Validation

The analytical predictions are benchmarked against multi-bunch tracking simulations performed with Elegant [6]. The simulations are carried out assuming uniform filling with 432 bunches, each represented by 10^5 macroparticles. The tracking is performed for a representative mode (HOM7), with frequency $f_{\mathrm{HOM}} = 2.5553$ GHz, quality factor $Q = 45900$, and shunt impedance $R_s = 72800\ \Omega$. The HOMs are treated as effectively on resonance ($f_{\mathrm{HOM}} = Nf_{\mathrm{rev}}$), since only modes with bandwidths much larger than ω_s are considered, corresponding to the most critical excitation condition. The simulation includes synchrotron radiation, radiation damping, and nonlinear momentum compaction up to second order. The third-harmonic cavity is modeled as an active RF system. The beam is tracked for 80 000 turns, with a gradual ramp-up of the impedance over the first 5 000 turns to avoid artificial transients.

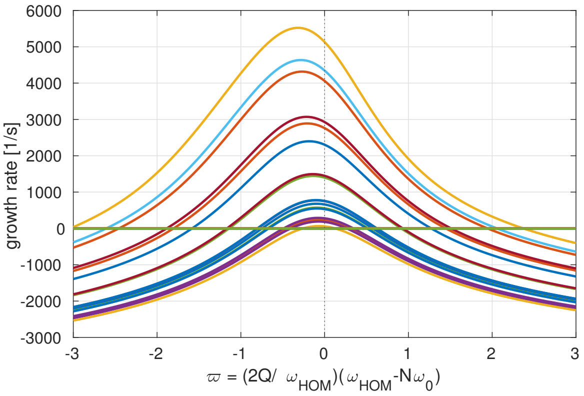

Figure 3: Growth rates of HOMs as a function of normalized detuning $\bar{\omega}$ including Landau damping (HHC model). Each curve corresponds to a different HOM.

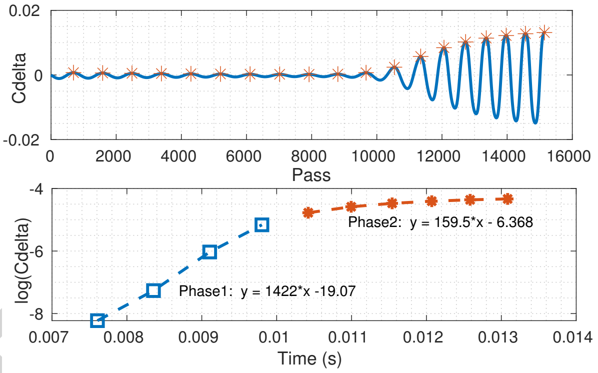

Figure 4: Evolution of the instability growth rate from tracking simulations for HOM7 compared with analytical predictions. The top plot shows the bunch centroid energy deviation (*Cdelta* in Elegant) as a function of turn number, illustrating the growth of longitudinal oscillations. The bottom plot shows log(*Cdelta*) versus time, i.e. the oscillation amplitude in logarithmic scale, from which the exponential growth rate is extracted via linear fitting.

For this HOM, the SHO model predicts a growth rate of 1710 s^{-1}, while the HHC model predicts a reduced growth rate of 228.93 s^{-1}. The tracking results exhibit two distinct regimes. At early times, the instability grows exponentially with a growth rate of 1422 s^{-1}, in agreement with the SHO prediction. As the oscillation amplitude increases, nonlinear effects associated with the double-RF potential lead to phase mixing and Landau damping, reducing the effective growth rate to 159.5 s^{-1}, consistent with the HHC model. These results, shown in Fig. 4, confirm that the instability initially follows the SHO behavior and then transitions to the HHC regime, showing good agreement between analytical models and tracking simulations.

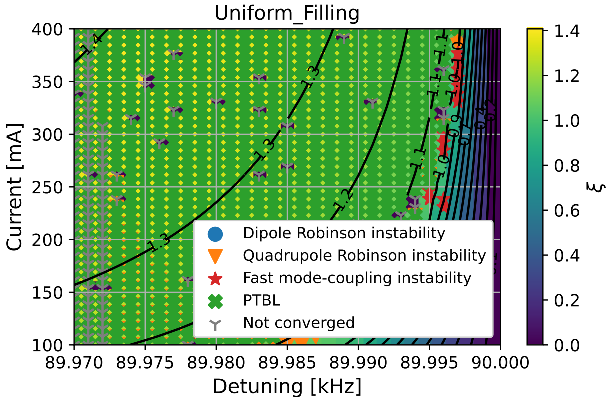

Figure 5: Stability map for uniform filling showing the flat potential parameter $\zeta \equiv -\nu V_2 \sin\theta_2 / V_1 \sin\theta_1$. Values close to 1 correspond to near-flat potential conditions.

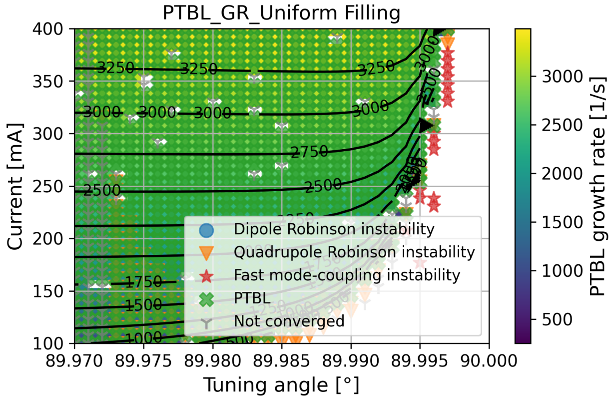

Figure 6: Growth rate of PTBL instability.

DOUBLE RF STABILITY ANALYSIS

Main Cavity Detuning

To identify a safe operating point for the main RF system, a preliminary scan of the main cavity (MC) detuning is performed using the semi-analytical ALBuMS framework. In this study, the third-harmonic cavity (3HC) is modeled as a resonator with $R_s = 1\ \Omega$, such that its effect is negligible. The cavity detuning is expressed in terms of the tuning angle ψ, defined from the cavity voltage and beam loading conditions [1]:

$$\tan\psi = Q_L\left(\frac{\omega_r}{\nu\omega_\mathrm{rf}} - \frac{\nu\omega_\mathrm{rf}}{\omega_r}\right) \tag{2}$$

where ν is the harmonic number of the cavity. The results show that the dominant instability in this configuration is the dipole Robinson instability. By selecting a detuning of -1 kHz for the main cavity, this instability can be avoided, and this value is used as the operational working point in the following analysis.

Uniform Filling (432/432)

The double RF system is then studied assuming uniform filling and using the following cavity parameters:

Table 1: RF Cavity Parameters Used in the Analysis

Cavity	m	R_s (Ω)	Q	Q_L	N_cav
MC	1	3.2×10^6	4.0×10^4	1.3×10^4	4
3HC	3	1.8×10^{10}	2.0×10^8	2.0×10^8	1

Keeping the main cavity detuning fixed at -1 kHz, the third-harmonic cavity detuning is scanned over the range 89.97°–89.999°, corresponding to 7–195 kHz. The results show that for detuning values between 7 and 71 kHz (89.97°–89.997°), the beam is strongly affected by periodic transient beam loading (PTBL) instability, as shown in Fig. 5.

As shown in Fig. 6, the growth rate of the PTBL instability exceeds the longitudinal feedback capability of Elettra 2.0. Focusing on the safe detuning interval, an optimal operating region is identified. In particular, for detuning values between 83 and 89 kHz, the results show that: $\zeta = 0.90$–0.96 (near flat potential), Bunch length: 21–27 ps and Touschek lifetime improvement: ×3.2–4.4. (1000 s^{-1}). Therefore, this detuning region must be avoided.

Non-Uniform Filling (400/432)

For the non-uniform filling pattern of Elettra 2.0 (400/432), the ALBuMS algorithm does not converge reliably over the full detuning range. However, restricting the analysis to the safe interval (72–107 kHz), a stable region can still be identified. In this case, the optimal detuning range is found between 80 and 85 kHz, where: $\zeta = 0.85$–0.90, Bunch length: 15.2–16 ps and Touschek lifetime improvement: ×2.4–2.5.

Tracking Benchmark with mbtrack2

The results are benchmarked using multi-bunch tracking simulations with `mbtrack2` [7, 8]. For uniform filling, at ~ 80 kHz, the bunch length reaches 32.6 ps. For non-uniform filling at the same detuning, the bunch length reaches 25.9 ps. These results confirm that the harmonic cavity is more effective in the uniform filling case, particularly around 80 kHz detuning, where larger bunch lengthening is achieved, and the system operates closer to the flat potential condition.

CONCLUSION

A multi-bunch longitudinal beam dynamics study of the Elettra 2.0 double-RF system has been presented. The inclusion of Landau damping significantly reduces both the number of critical HOMs and the required detuning, from about ±69 kHz in the SHO model to about -46 kHz (or ~ 30 kHz) in the HHC model. Tracking simulations with Elegant confirm the transition from SHO-like growth to a Landau-damped regime. The RF parameter scan identifies operational limits from Robinson and PTBL instabilities, with a stable operating window at higher third-harmonic cavity detuning (83–89 kHz for uniform filling and 80–85 kHz for non-uniform filling). These results define a robust working point for the Elettra 2.0 double-RF system.

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