DEVELOPMENT OF THE SPIN TUNE MODEL FOR EDM INVESTIGATIONS AT STORAGE RINGS

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Abstract

Electric Dipole Moments (EDMs) are sensitive probes of CP violation and could help address some open questions of the Standard Model. Recent advances in storage ring and polarimetry technologies enabled the development of a new experimental method for EDM investigations. One of the main parameters involved in the EDM studies via storage rings is the Spin Coherence Time (SCT), *i.e.*, the time during which the spins of all particles in a stored beam precess coherently. A long SCT increases the experimental sensitivity and minimises the statistical uncertainties. To identify the optimal working conditions, the single-particle spin tune, defined as the number of spin precessions around the vertical axis per revolution, must be tracked with extremely high precision. A lattice-independent model was developed to accurately track the spin tune of single charged particles in a variety of storage rings, both existing and proposed, and was directly applied to SCT optimisation. In this contribution, the model was tested on the hybrid storage ring, a new-generation device conceived for EDM investigations, which uses electrostatic deflectors for confinement and magnetic quadrupoles for focusing.

INTRODUCTION

Despite its remarkable success, the Standard Model (SM) leaves several fundamental open questions. Among them, the observed matter-antimatter imbalance in the Universe, known as baryon asymmetry, is one of the most compelling [1]. The underlying mechanism, called baryogenesis, is an asymmetric process that requires CP violation [2]. The baryon asymmetry is quantified by the baryon-to-photon density ratio η [3]. While the SM predictions are consistent with $\eta \sim 10^{-18}$ [1], the measurements are of the order of 10^{-10} [4]: this means that the CP violation incorporated in the SM is insufficient to account for the observed baryon asymmetry. Therefore, new sources of CP violation beyond the SM are needed. A powerful probe of CP violation is the Electric Dipole Moment (EDM) [5]. It is an intrinsic fundamental property of particles, defined as the permanent asymmetry in the distribution of electric charges within the particle with respect to the spin orientation. In the SM the EDMs are predicted to be extremely small, but many SM extensions predict significantly larger EDMs that could be within or closer to the reach of current and future experiments [6]. Since direct measurements of the EDM are experimentally challenging because of its tiny magnitude, very high precision techniques are required. One of them involves the use of storage rings [7]. By injecting into a storage ring

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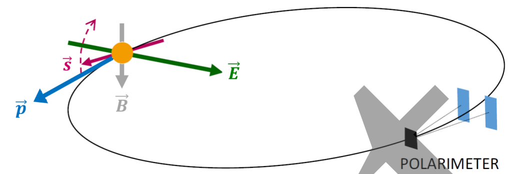

Figure 1: Schematic of the principle of the EDM measurement for a single-particle.

a longitudinally polarized particle beam under the frozen spin condition, *i.e.*, with all spins remaining aligned with their momenta, the existence of a non-vanishing EDM can be revealed as the build-up of vertical polarization, due to the interaction of the particle EDMs with an external radial electric field. This polarization build-up can be measured using a polarimeter, as shown in Fig. 1. The spin tune and the Spin Coherence Time (SCT) are two fundamental quantities in the EDM investigations using storage rings. The single-particle spin tune is the number of spin precessions around the vertical axis per revolution and is defined as [8]:

$$\nu_{s,0} = \frac{f_{\mathrm{spin}}}{f_{\mathrm{rev}}}, \tag{1}$$

where f_{spin} is the spin frequency and f_{rev} is the revolution frequency. When a beam is considered, since particles are distributed in phase-space, each of them experiences slightly different fields and has its own spin tune. As a result, there is a distribution of the spin tunes of the individual particles within the beam, called spin tune spread. For a polarized beam, this spread causes spin decoherence and, consequently, depolarization. The SCT is the time over which the beam polarization decreases to 1/e of its initial value [9]. It must be maximised, within the limits set by the beam lifetime and the machine operation, because it determines how long the EDM signal can accumulate. To obtain a long SCT, it is necessary to minimise the spin tune spread and, thus, to accurately track the single-particle spin tune. A lattice-independent model for accurately tracking the spin tune of single charged particles has been developed in a variety of storage rings, both existing and proposed. The single-particle spin tune can be expressed as a sum of contributions arising from synchrotron and betatron motion, as well as from chromaticities. The novelty of this work lies in its direct application in SCT optimisation: it is possible to determine the lattice optical properties which minimise the spin tune spread and, thus, maximise the SCT. In this contribution, the model is applied to the hybrid storage ring, a new-generation device for EDM investigations that combines electrostatic deflectors for confinement and magnetic quadrupoles for focusing.

HYBRID STORAGE RING

Hybrid [10, 11] means that it combines electric bending elements and magnetic focusing elements. It is a 24-fold symmetric ring with an 800 m circumference, composed of FODO cells, each alternating between straight and bending sections. Each straight section hosts a quadrupole magnet for focusing, while each bending section consists of two electric deflectors for confinement. A sextupole magnet is superimposed on each quadrupole. Two straight sections house the injection points for the clockwise (CW) and counter-clockwise (CCW) proton beams, while a third section contains the radio-frequency (RF) cavity. Frozen spin is achieved at the magic momentum of 700 MeV. The layout of the hybrid ring lattice, built using the software Bmad [12], is shown in Fig. 2. The main advantages of the hybrid ring are the absence of dipole fields, which can mimic the EDM signal, and the simultaneous circulation of two counter-rotating particle beams which help control systematic effects. Moreover, the electric field provided by the confinement system directly contributes to the EDM signal.

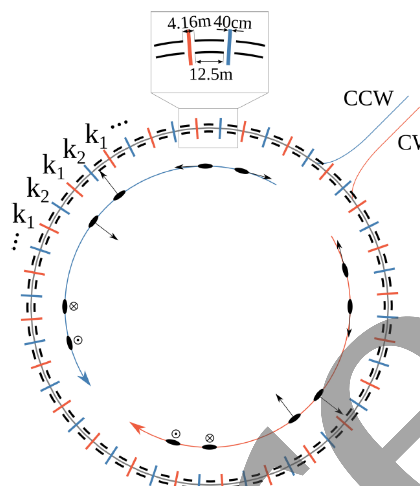

Figure 2: Layout of the hybrid ring lattice.

SPIN TUNE MODEL

The spin tune model, compactly written using indices $i \in \{1,2,3\} \wedge k \in \{1,2\}$ and the Einstein summation convention, is:

$$\langle \Delta\nu_s \rangle = w_i \epsilon^i + W_{ik} \epsilon^i \xi^k. \quad (2)$$

Here, $\Delta\nu_s$ is the relative spin tune and $\langle\rangle$ denotes an average over several synchrotron oscillations. The ϵ^i are the phase-space properties, whose elements are the horizontal emittance (ϵ_x), the vertical emittance (ϵ_y), and the longitudinal emittance (ϵ_z), which scale with the areas of the respective phase-space ellipses. The strengths of each contribution are encoded in the first-order coefficients w_i. This term accounts for the effects of the quadrupole fields due to the particle being offset from the reference orbit. On the other hand, ξ^k accounts for the second-order corrections to these contributions through sextupole fields. These are the lattice optical properties or betatron chromaticities, whose elements are the horizontal chromaticity (ξ_x) and the vertical chromaticity (ξ_y). Only transverse chromaticities are considered, consistent with the two sextupole families available in the hybrid ring lattice. The strengths of each corresponding contribution are encoded in the second-order coefficients W_{ik}. The diagonal elements account for the correction term on the emittance gradient due to the betatron chromaticities in each phase-space, while the off-diagonal elements set the coupling strengths among them. If the phase-spaces are completely decoupled, the off-diagonal terms are expected to be negligibly small.

Derivation

The starting point for the derivation of the spin tune model is Eq. (1). A more general expression can be obtained from the Thomas–BMT equation at first-order [13]:

$$\nu_{s,0} = G\gamma - \frac{r(G+1)}{\gamma(\beta + r)}, \quad (3)$$

where $\beta = v/c$ is the particle velocity normalized to the speed of light, γ is the relativistic factor, G is the gyromagnetic anomaly and $r = E/(Bc)$ is the normalized electric-to-magnetic field ratio. This equation shows that the single-particle spin tune depends on particle velocity through the relativistic factor γ. As a consequence, particles with different energies exhibit different spin tunes and the spin tune spread within the beam arises naturally. However, this expression refers to the reference particle, defined as the particle that exactly follows the design orbit. In a real storage ring, particles are distributed in phase-space and deviate from the reference trajectory. Therefore, it is convenient to introduce the relative spin tune, defined as the deviation of the spin tune of an off-momentum particle with respect to that of the reference particle:

$$\Delta\nu_s = \sigma_0 \delta + \sigma_1 \delta^2 + ..., \quad (4)$$

where $\delta = \Delta p/p$ (p is the particle momentum) is the momentum offset and σ_0 and σ_1 are the first and second-order spin tune factors. Transverse motion introduces an additional dependence through quadrupole fields, leading to a linear contribution to first-order in the transverse emittances:

$$\Delta\nu_s = \sigma_0 \delta + \sigma_1 \delta^2 + h_1 \epsilon_x + h_2 \epsilon_y = \sigma_0 \delta + h_i \epsilon^i, \quad (5)$$

where δ^2 is associated with the longitudinal emittance and absorbed into the phase-space properties. In addition, sextupole fields, introduced to correct chromatic aberrations, induce a further dependence through the transverse chromaticities. This contribution is also linear to first-order, yielding:

$$\Delta\nu_s = \sigma_0 \delta + h_i \epsilon^i + H_{ik} \epsilon^i \xi^k. \quad (6)$$

Up to this point, the relative spin tune has been considered at a fixed momentum offset. However, in the presence of longitudinal focusing provided by the RF cavity, the momentum offset undergoes synchrotron oscillations:

$$\delta = \langle\delta\rangle + \delta_a \sin(\omega_L \phi + \phi_{L,0}), \quad (7)$$

where $\langle\delta\rangle$ is the steady-state momentum offset, δ_a is the oscillation amplitude, ω_L is the synchrotron angular frequency and $\phi_{L,0}$ is the initial synchrotron phase. Substituting Eq. (7) into Eq. (6) and using the constraint imposed by the longitudinal focusing ($\langle\Delta t/t\rangle = 0$), the steady-state momentum offset averages to zero. Furthermore, under this constraint, the coefficients h_i and H_{ik} are recast into effective coefficients w_i and W_{ik}, leading to the steady-state relative spin tune described by Eq. (2).

Application

To achieve a long SCT, the spin tune spread within the beam must be minimised. Since the spread originates from the dependence of the single-particle spin tune on the phase-space variables, this corresponds to minimising the dependence of the steady-state relative spin tune. From the spin tune model in Eq. (2), the steady-state relative spin tune can be written as:

$$\langle\Delta\nu_s\rangle = \epsilon^i\left(w_i + W_{ik}\xi^k\right). \tag{8}$$

The spin tune spread is therefore governed by the dependence on the phase-space variables encoded in the term in parentheses. To minimise the spread, the steady-state relative spin tune must become independent of the phase-space variables to first-order. This condition corresponds to imposing:

$$w_i + W_{ik}\xi^k = 0. \tag{9}$$

Solving this system yields the optimised chromaticities:

$$\xi^k_{\text{opt}} = -(W^+)^{ik}w_i, \tag{10}$$

where W^+ denotes the pseudo-inverse of the matrix W_{ik}. Under this condition, the spin tune spread is minimised, leading to a maximisation of the SCT. Since Eq. (9) corresponds to an overdetermined system, a unique solution for the chromaticities is not guaranteed in general. In this case, Eq. (10) represents the condition for a partial SCT optimisation. A complete SCT optimisation can in principle be achieved in lattices providing a third sextupole family.

Simulations

The spin tune model was first used to determine the model coefficients. Each coefficient was extracted by isolating the corresponding term in Eq. (2) and setting the relevant phase-space variable, while keeping all other degrees of freedom fixed to zero. Owing to the linear dependence of the steady-state relative spin tune on the phase-space variables and chromaticities, the coefficients were obtained from a linear fit to the simulated data. Then the model was benchmarked using approximately 10^4 independent single-particle tracking simulations, each characterized by randomly sampled values of momentum offset, transverse emittances, and chromaticities. The steady-state relative spin tune values were obtained in two ways: simulated values, extracted from the simulated data, and calculated values, obtained using Eq. (2). Finally, the residuals, defined as the difference between the simulated and calculated values, were evaluated to assess the reliability of the model. SCT optimisation condition was validated by tracking a beam of approximately 10^3 particles over 10^6 turns in the optimised lattice configuration, with independently sampled momentum offsets and transverse emittances. The evaluation of the SCT was obtained from the time evolution of the polarization vector magnitude by fitting the curve with a second-order polynomial. The software Bmad was used to perform single-particle beam dynamics and spin tracking simulations.

RESULTS

Figure 3 shows the histogram of the residuals: the distribution is sharply peaked around zero, indicating that the model reproduces the simulated values with negligible systematic bias. Figure 4 shows the polarization vector magnitude as a function of time, yielding $\tau = (90.6 \pm 1.4)$ s.

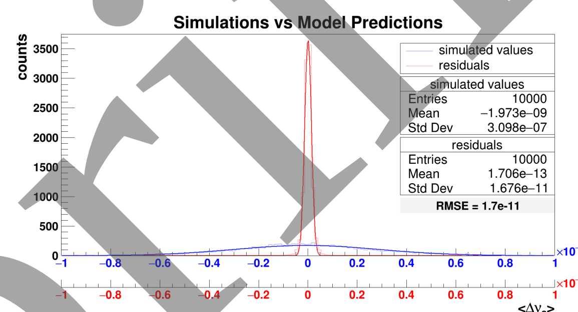

Figure 3: Histogram of the residuals to compare simulated and model-predicted steady-state relative spin tune values.

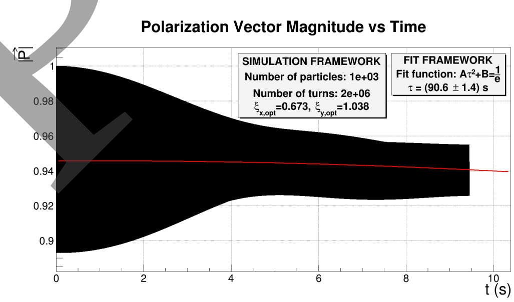

Figure 4: Plot of the polarization vector magnitude as a function of time to evaluate the SCT.

CONCLUSIONS

A model for accurately tracking the spin tune of single charged particles was developed and tested on the hybrid storage ring. It enables the estimation of the single-particle spin tune when direct measurements are not feasible and finds direct application in SCT optimisation. Future perspectives include broadening the applicability of the model to both existing and proposed storage rings and achieving full SCT optimisation considering storage rings with three sextupole families. Looking ahead, the spin tune model represents a significant advancement in EDM studies via storage rings.

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