

# MODELLING OF COUNTER-ROTATING MULTI-TURN WAKEFIELDS IN THE MUON COLLIDER RCS CHAIN

L. S. Thiele<sup>\*, 1, 2</sup>, S. Albright<sup>1</sup>, R. Calaga<sup>1</sup>, H. Damerau<sup>1</sup>, A. Grudiev<sup>1</sup>,  
I. Karpov<sup>1</sup>, E. Lamb<sup>1</sup>, S. Lauber<sup>1</sup>, U. van Rienen<sup>2, 3</sup>, S.-A. Udongwo<sup>2</sup>

<sup>1</sup>CERN, Geneva, Switzerland

<sup>2</sup>Institute of General Electrical Engineering, University of Rostock, Rostock, Germany

<sup>3</sup>Department of Life, Light and Matter, University of Rostock, Rostock, Germany

## Abstract

The future multi-TeV muon collider facility will employ a superconducting radiofrequency (RF) system with thousands of cavities in the rapid-cycling synchrotron (RCS) chain. This RF system, shared by both beams, will accelerate two counter-rotating  $\mu^+$  and  $\mu^-$  bunches simultaneously. Due to the high intensity of up to  $2.7 \times 10^{12}$  particles per bunch, the induced voltage in both the fundamental and several higher-order modes is significant. A strong impact on longitudinal beam quality and the luminosity in the collider is expected. Modelling wake potentials in the RF system of the RCS chain is challenging. The long-range wakefields will interact with both the co- and counter-rotating bunches due to the high quality factor and corresponding long decay constants of the fields in the cavities. These two-beam interactions are studied, with particular attention to the specificities caused by the directionality of the induced fields.

## INTRODUCTION

The proposed muon collider facility aims to collide two muon bunches at up to 10 TeV centre-of-mass energy, offering a resource-efficient path to highest energy collisions with leptons. Following cooling and pre-acceleration, the complex consists of a chain of four RCSs that sequentially accelerate the bunches from 63 GeV to 5 TeV. The first two accelerators are planned to be located in the same tunnel, while the last two are to be placed in separate tunnels. Due to the short muon lifetime of  $\tau_\mu \approx \gamma \times 2.2 \mu\text{s}$  in the lab frame, the acceleration needs to be performed rapidly but under the stringent constraint of high beam quality in the collider. The relativistic Lorentz factor,  $\gamma$ , increases with energy. If the longitudinal emittance can not be preserved, the available luminosity will be impaired. The rapid acceleration of the bunches requires a large voltage, which is planned to be provided by thousands of 9-cell 1.3 GHz TESLA cavities [1]. An overview of the RF system parameters is given in Table 1.

The RF system is intended to be common to the  $\mu^+$  and  $\mu^-$  bunches to enable a more cost- and energy-efficient acceleration. Both bunches are expected to have the same intensity of  $2.7 \times 10^{12}$  particles per bunch at the injection of RCS1, resulting in a high instantaneous and low average beam current. In combination with the small aperture of

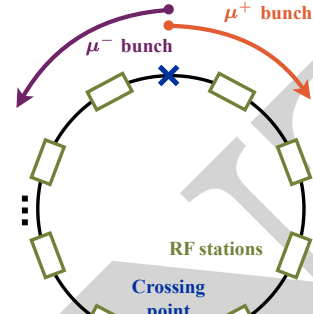


Figure 1: RF system distribution in the muon collider RCS chain. Due to the counter-rotating beams, the time difference between passage of the two bunches will be different in every RF station.

TESLA cavities, this will lead to high induced voltages of several megavolts in both the longitudinal and transverse planes, potentially perturbing the beam and causing emittance growth, instabilities, or beam loss. The high quality factor of the superconducting cavities will cause the induced voltages to persist until the passage of the counter-rotating bunch. The co-rotating beam on the next turn will also be affected. The external quality factor of the fundamental mode (FM) is expected to be around  $10^6$ , while the quality factors of the higher-order modes (HOMs) are at least an order of magnitude lower in the strongly damped TESLA cavity. The high accelerating voltage of several gigavolts per turn, combined with the fast acceleration rate, results in a large phase-advance of the fast synchrotron motion [2]. To reduce the effective synchrotron tune and preserve beam stability,

Table 1: Main RF system parameters from the consolidated parameter report of the MuCol project [2]. The stable phase,  $\phi_s$ , is defined from the rising zero-crossing of the sine-wave.

RCS	1	2	3	4
$E_{inj}$ [GeV]	63	313	750	1500
$E_{ej}$ [GeV]	313	750	1500	5000
$\phi_s$ [°]	148	153	134	118
$V_{RF}$ [GV]	27.6	17.5	15.7	72.7
$n_{turns}$	18	55	66	55
$C$ [km]	5.99	5.99	10.7	35.0
$\alpha_p$ [ $10^{-4}$ ]	10.4	9.0	3.2	2.1

\* leonard.thiele@uni-rostock.de

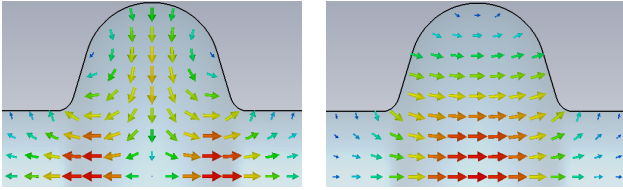


Figure 2: Two eigenmodes of a single-cell TESLA-like cavity, simulated in CST Studio Suite<sup>®</sup> [3]. From a symmetric eigenmode (left), the bunch will receive an equivalent energy kick, regardless of the propagation direction, while an asymmetric eigenmode (right) will cause opposite energy kicks for the two propagation directions.

the RF voltage must be distributed across multiple RF stations around the circumference, as depicted in Fig. 1.

## R/Q CALCULATION FOR COUNTER-ROTATING BEAMS

The longitudinal geometric shunt impedance,  $R/Q$ , can be calculated from the electric field distribution obtained in an electromagnetic solver. For this study, the eigenmodes were computed with CST Studio Suite<sup>®</sup> [3]. The  $R/Q$  of mode  $m$  can be calculated using [4]

$$\begin{aligned} \frac{R}{Q_m} &= \frac{|V_{\text{eff}, m}|^2}{\omega_m U} \\ &= \frac{1}{\omega_m U} \cdot \left| \int_{-l/2}^{l/2} E_{z, m}(z) e^{i\omega_m z/c} dz \right|^2, \end{aligned} \quad (1)$$

with the length of the object,  $l$ , the energy stored in the field,  $U$ , the complex electric field in  $z$ -direction on the beam axis,  $E_{z, m}$ , the effective voltage per mode,  $V_{\text{eff}, m}$ , and the angular resonance frequency of the mode,  $\omega_m$ . In most accelerators, this value is used to calculate the induced voltage of each eigenmode using the resonator model. Figure 2 shows the electric field vectors for two different eigenmodes in a 3-cell TESLA-shaped cavity. A particle entering the cavity from either side will encounter equivalent field vectors (Fig. 2 left), or opposite field vectors (right). This effect is not a feature of multi-cell cavities but rather a property of the mode pattern in general. The induced voltage calculation from the  $R/Q$  needs to be extended by a directionality constant that accounts for the phase shift. More complex mode patterns can be analysed by computing

$$\overline{V_{\text{eff}, m}} = \int_{-l/2}^{l/2} E_{z, m}(z) e^{-i\omega_m z/c} dz, \quad (2)$$

and comparing it to  $V_{\text{eff}, m}$ . The application of this value in the calculation of the induced voltage is discussed below.

## INDUCED VOLTAGE CALCULATION

To study the effects of induced voltages, the macroparticle tracking code Beam Longitudinal Dynamics (BLonD) [5] is used. In the code, a time profile of the bunch distribution is generated by projection of a particle distribution. These profiles are convolved with the wake potential of a point

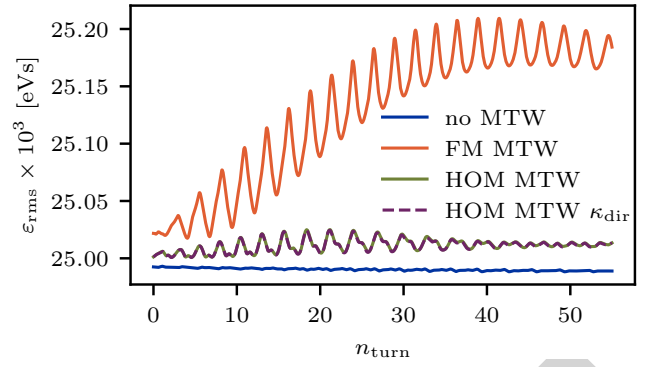


Figure 3: RMS emittance evolution during the acceleration in RCS2 for different modelling of the intensity effects. The RF system is distributed over 32 stations.

charge,  $w_m$ , to calculate the induced voltage in the resonant object. Since the HOM decay constants are long with respect to the time distance between passages, the induced voltages of previously passed bunches must be considered in addition to the current ones. The time differences between the bunch passages vary across RF stations, requiring the induced voltage to be calculated individually for each station.

To incorporate the counter-rotation into the calculation of the long-range induced voltage, the directionality constant,

$$\kappa_{\text{dir}, m} = \begin{cases} 1 & \text{if } V_{\text{eff}, m} = \overline{V_{\text{eff}, m}} \\ -1 & \text{if } V_{\text{eff}, m} = -\overline{V_{\text{eff}, m}} \end{cases} \quad (3)$$

is introduced. This approach is only valid for symmetric structures.

Especially in the first two RCSs, the long-range induced voltages will interact with both the beam propagating in the same direction on the next turn and the counter-rotating beam. A distinction must be made between the source of the induced voltage and the propagation direction of the witness beam to determine if  $\kappa_{\text{dir}, m}$  needs to be applied in the induced voltage calculation. For the calculation of the induced voltage, we define

$$\kappa_{\text{dir}, m, \text{eff}} = \begin{cases} \kappa_{\text{dir}, m} & \text{if } k_n \oplus k_0 \\ 1 & \text{else,} \end{cases} \quad (4)$$

with the direction of the current bunch,  $k_0$ , and the propagation direction of the previous  $n$ -th bunch,  $k_n$ .

## IMPLEMENTATION OF INDUCED VOLTAGE CALCULATION

The new major revision of BLonD, BLonD V3, features an execution pipeline in which each element can be freely placed [6]. This pipeline can be reversed to determine the element order for the counter-rotating beam. With this architecture, tracking the induced-voltage contributions from both beams becomes feasible.

In principle, the induced voltage could even be calculated continuously throughout the turn. However, the required time resolution at the bunch passage, combined with the

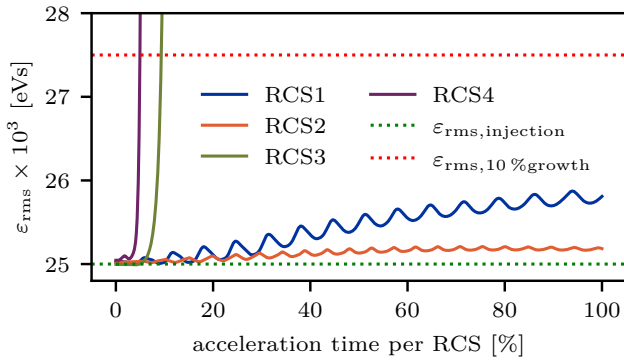


Figure 4: RMS emittance evolution during the acceleration with the fundamental mode multi-turn induced voltage in different RCSs with the RF system being distributed over 32 stations. 10% emittance growth represents the overall budget throughout the acceleration chain.

high harmonic number of the accelerators, makes it numerically impractical. Instead, the induced voltage is computed only at the relevant times of the beam passages.

The high quality factor of some HOMs requires keeping track of past bunch passages, their propagation directions, and their cavity crossing times. The total induced voltage for the current turn can be calculated using

$$W(t) = \sum_{n=0}^{n_p} \left\{ \lambda_n(t) * \left[ \sum_{m=0}^{n_m} \kappa_{\text{dir}, m, \text{eff}} \times w_m(t - t_n) \right] \right\}, \quad (5)$$

with the number of past profiles,  $n_p$ , the number of modes,  $n_m$ , and the passage time of the profiles,  $t_n$ .

## RESULTS

The parameters of the cavity eigenmodes were calculated from simulation data of a 9-cell TESLA cavity with two HOM dampers and a fundamental power coupler. The HOM dampers were optimised to reduce the quality factors of the most prominent HOMs.

To analyse the impact of the different intensity effects, a simulation of RCS2 was performed that included individual effects. Figure 3 shows that the multi-turn wake (MTW) on the FM has the largest impact on the longitudinal emittance growth during acceleration. The contribution of the HOMs is smaller, and the definition of  $\kappa_{\text{dir}}$  does not affect the bunch emittance. This is expected because the induced voltage of the FM always constructively interferes, as the bunch is synchronous with the FM frequency. Since the HOM frequencies are not integer multiples of the revolution frequency, the induced voltages will interfere constructively or destructively, resulting in lower amplitudes. Additionally, the combined  $R/Q$  of the studied HOMs is lower than the  $R/Q$  of the FM. A realistic simulation of the beam dynamics would include the HOM MTWs with  $\kappa_{\text{dir}}$  and the FM MTW. Figure 4 presents the evolution of the longitudinal rms emittance during acceleration in the different synchrotrons, assuming a matched initial distribution and only considering the FM MTW. While the bunches in RCS1 and RCS2 experience minor emittance growths, the huge induced voltage in

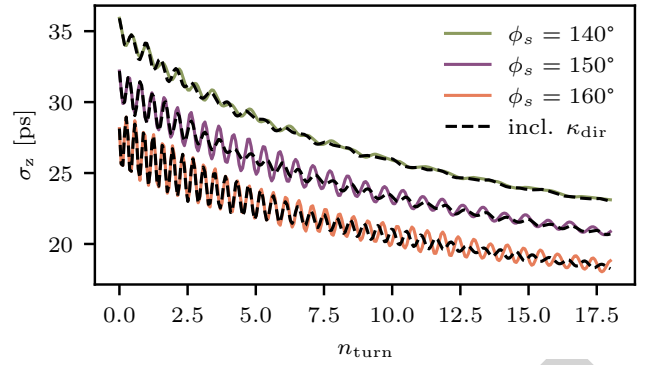


Figure 5: Bunch length evolution during the acceleration in RCS1 with the RF system distributed across 16 stations.

RCS3 and RCS4 causes beam loss within a few turns. The high quality factor of the FM leads to an induced voltage build-up over tens of turns, resulting in a significant phase shift in the cavity voltage. The acceleration in the last two RCSs is performed closer to the crest of the RF waveform, leading to a smaller stable region in phase space that is more prone to perturbations from induced voltages. The effects of the FM MTW could be mitigated with a feedback or feed-forward system.

Even though Fig. 3 indicates that the impact of  $\kappa_{\text{dir}}$  on the emittance is small in RCS2, it should be noted that the effect depends on the accelerator parameters as well as the choice of the stable phase,  $\phi_s$ . Figure 5 shows the evolution of the bunch length during the acceleration in RCS1 for different  $\phi_s$ . The influence of  $\kappa_{\text{dir}}$  (dashed black) on the bunch length oscillation can be observed. The coefficients provide an additional phase mixing term between the co- and counter-rotating induced voltages. For multiple-beam passages,  $\kappa_{\text{dir}}$  can change if modes interfere constructively or destructively, with the exact interaction depending on the cavity position. As a result of the counter-rotating beams, the time between two passages differs from cavity to cavity, adjusting the phasing of the HOMs and changing the total amplitude in the cavities. Due to the strongly damped cavity studied, the effect  $\kappa_{\text{dir}}$  on phase mixing between turns is significantly reduced. The bunch length oscillation could be attributed either to the high synchrotron tune or to a remaining mismatch in the initial distribution.

## CONCLUSION

In this contribution, the interactions between induced voltages from two counter-rotating beams in the same cavity objects were studied for the RCS chain of the muon collider. The MTW of the FM can lead to emittance growth in RCS1 and RCS2. In RCS3 and RCS4, the induced voltage causes beam loss. The MTW of the HOMs had less impact on beam dynamics, resulting in only a slight emittance growth. This difference can be partly attributed to the phase mixing across the different passages, the lower combined  $R/Q$  of the HOMs and the strong damping by the HOM couplers. Future work will focus on investigating possible feedback or feedforward systems to mitigate the effects of the FM MTW.

## ACKNOWLEDGMENTS

This work has been sponsored by the Wolfgang Gentner Programme of the German Federal Ministry of Education and Research (grant no. 13E18CHA). Endorsed by the IMCC. Funded by the European Union (EU). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the EU or European Research Executive Agency (REA). Neither the EU nor the REA can be held responsible for them.

## REFERENCES

- [1] B. Aune *et al.*, “Superconducting TESLA cavities”, *Phys. Rev. Accel. Beams*, vol. 22, p. 092001, 2000, [doi:10.1103/PhysRevSTAB.3.092001](https://doi.org/10.1103/PhysRevSTAB.3.092001).
- [2] R. Taylor *et al.*, “MuCol Milestone Report No. 7: Consolidated Parameters”, Zenodo, 2025, [doi:10.5281/ZENODO.17476875](https://doi.org/10.5281/ZENODO.17476875).
- [3] CST Studio Suite®, Dassault Systèmes, Darmstadt, Germany, 2024.
- [4] H. Padamsee, J. Knobloch, and T. Hays, *RF Superconductivity for Accelerators: 2nd Edition*, Wiley, New York, USA, 2008.
- [5] H. Timko *et al.*, “Beam longitudinal dynamics simulation studies”, *Phys. Rev. Accel. Beams*, vol. 26, p. 114602, 2023, [doi:10.1103/PhysRevAccelBeams.26.114602](https://doi.org/10.1103/PhysRevAccelBeams.26.114602).
- [6] S. Lauber *et al.*, “Recent developments of BLonD for new large scale accelerator facilities”, presented at the IPAC'26, Deauville, France, May 2026, paper THP5005, this conference.

PREPRINT