IPAC'26 Preliminary proceedings (edited version): WEP5107

COUPLED-BUNCH INSTABILITIES WITH A DEFOCUSING HIGHER-HARMONIC RF SYSTEM

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Abstract

Longitudinal coupled-bunch instabilities limit the beam intensity in many synchrotrons. They can be mitigated by adding a higher-harmonic radiofrequency (RF) system, which modifies the potential well. Of particular interest is the bunch lengthening mode (BLM), in which the higher-harmonic voltage produces a defocusing effect at the bunch centre. Decreasing the peak line density is beneficial, for example, to reduce space-charge and intrabeam scattering. This study investigates the impact of a higher-harmonic RF system in BLM on the intensity threshold and growth rate of coupled-bunch instabilities. The influence of beam parameters such as the bucket filling factor, narrowband impedance properties, and RF system configuration is highlighted. Regimes in which BLM is advantageous or disadvantageous are compared to the ones for a single-harmonic RF (SRF) system. The results are based on semi-analytical solutions of the Vlasov equation as well as macro-particle simulations using the BLonD simulation code.

INTRODUCTION

Coupled-bunch instabilities (CBI) are driven by long-range wakefields, which are produced by a narrowband (NB) impedance source and link the bunches in a synchrotron. If the CBI intensity threshold is exceeded, the affected bunches become unstable, resulting in coherent oscillations in the longitudinal plane. This leads to beam quality degradation and, in the worst case, to beam loss. To lift the intensity limitations due to CBI, double-harmonic RF systems (DRF) have been implemented in several synchrotrons [1, 2]. Although the impact of DRF systems on CBI is well understood and discussed in the literature [3, 4], the interplay among different beam parameters, impedance, and RF systems has not yet been fully disentangled.

Since the BLM offers many benefits for lepton and hadron rings by reducing peak line density, we will investigate how different parameters affect the CBI threshold in BLM. For this purpose, we will utilise the stability diagram (SD) method introduced in [5]. Since we will only use normalised parameters for an analytical equation, the results can be applied to any synchrotron and reflect general properties of CBI. In addition, macro-particle simulations validate the stability diagram method and shed light on the growth rate behaviour for different RF settings.

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MAIN EQUATIONS AND DEFINITIONS

To simplify the calculation, we move from conjugate coordinates particle energy, ΔE, and RF phase deviation, ϕ, of the conventional equations of motion to energy, $\mathscr{E}$, and phase, ψ, of the synchrotron oscillation defined as

$$\mathscr{E} = \frac{\dot{\phi}^2}{2\omega_{s0}^2} + U_t(\phi) \text{ and} \tag{1}$$

$$\psi = \operatorname{sgn}(\eta \Delta E)\frac{\omega_s(\mathscr{E})}{\sqrt{2}\omega_{s0}} \int_{\phi_{\max}}^{\phi} \frac{1}{\sqrt{\mathscr{E} - U_t(\phi')}} d\phi', \tag{2}$$

where U_t is the total potential, ω_s the angular synchrotron frequency, ω_{s0} the angular frequency of small-amplitude synchrotron oscillations in the SRF system, h the harmonic number, and η the slip factor [6, 7].

We assume stationary bunches with a binomial distribution function, $\mathscr{F}$, independent of ψ:

$$\mathscr{F}(\mathscr{E}) = \frac{1}{2\pi\omega_{s0}A_N}\left(1 - \frac{\mathscr{E}}{\mathscr{E}_{\max}}\right)^{\mu}, \tag{3}$$

where μ is the form factor, and A_N is a normalisation factor.

A small perturbation function $\tilde{\mathscr{F}}$ is added to this stationary distribution, which is periodic in ψ and satisfies the linearised Vlasov equation [6].

The perturbation's phase advance from bunch to bunch is $2\pi l/M$, where M is the number of equidistant bunches, and l is the coupled-bunch mode. The Lebedev equation provides the following set of solutions [8, 9]:

$$\tilde{\lambda}_p^l(\Omega) = \frac{\zeta M}{h} \sum_{k=-\infty}^{\infty} G_{pk}(\Omega) \frac{Z_k(\Omega)}{k} \tilde{\lambda}_k^l(\Omega), \tag{4}$$

with the frequency, Ω, which defines the location in the synchrotron side bands, the intensity parameter

$$\zeta = \frac{-qN_p h^2 \omega_0}{V_0 \cos\phi_{s0}} \tag{5}$$

and the matrix elements G_{pk} coupling the different harmonics (p and k) of the line density perturbation $\tilde{\lambda}^l$:

$$G_{pk}(\Omega) = - i\omega_{s0}^2 \sum_{m=1}^{\infty} \int_0^{\mathscr{E}_{\max}} \frac{d\mathscr{F}(\mathscr{E})}{d\mathscr{E}} \frac{I_{mp}(\mathscr{E}) I_{mk}^*(\mathscr{E}) \omega_s(\mathscr{E})}{\Omega^2/m^2 - \omega_s^2(\mathscr{E})} d\mathscr{E}. \tag{6}$$

where m is the mode number and I_{mp} and I_{mk} are the wave integrals defined in [9].

The general solution is then found from

$$\det\left|\delta_{pk} + \frac{\zeta M}{h} G_{pk}(\Omega) \frac{Z_k(\Omega)}{k}\right| = 0. \quad (7)$$

Most impedance sources in accelerators are well described by the resonator model according to $Z(\omega) = R/(1 + iQ(\omega/\omega_r - \omega_r/\omega))$, where R is the shunt impedance, ω_r the resonant frequency, and Q the quality factor of the resonance.

If exclusively one narrowband impedance with a large Q is present, only the G_{pk} matrix elements with $k = p = k_{\mathrm{nb}} = \lfloor \omega_r/\omega_0 \rceil$ (rounding to the nearest integer) contribute to the determinant in Eq. (7), yielding the simplified equation for the intensity threshold:

$$\zeta_{\mathrm{th}} = \frac{k_{\mathrm{nb}} h}{Z_{k_{\mathrm{nb}}}(\Omega) G_{k_{\mathrm{nb}} k_{\mathrm{nb}}}(\Omega) M}. \quad (8)$$

The instability threshold for such a case can then be calculated from stability diagrams [5]. For the results shown, we will use $\phi_{\max}$ as a variable for the normalised half bunch length ranging from 0 to π (full bucket).

DOUBLE RF-SYSTEMS

The voltage of a DRF system is described by

$$V_{\mathrm{RF}}(\phi) = V_0[\sin(\phi + \phi_{s0}) + r_v \sin(r_h\phi + r_h\phi_{s0} + \Phi_2)], \quad (9)$$

with V_0 denoting the voltage magnitude of the main RF system, ϕ_{s0} the phase of the synchronous particle, and r_v and r_h the voltage magnitude and harmonic number ratios of the two RF systems. For the sake of simplicity, we consider a stationary beam above transition energy, $\phi_{s0} = \pi$. In the case of perfect BLM, the relative phase offset $\Phi_2 = \pi$ leads to a flattened voltage wave in the centre of the bucket. As

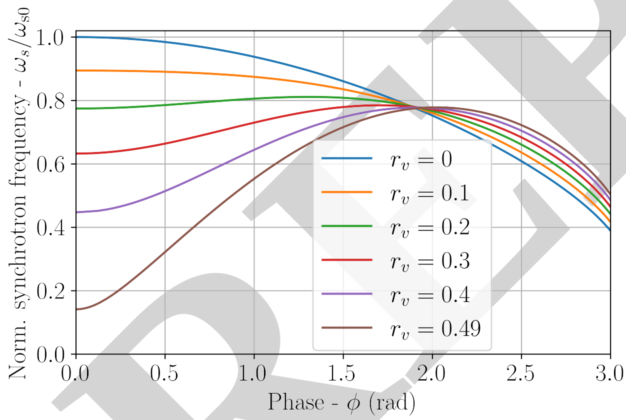

Figure 1: Synchrotron frequency as a function of the half bunch length for a DRF system ($r_h = 2$) in BLM with different voltage ratios, r_v. Bunches are considered to be short for $\phi_{\max} << 1$.

a consequence of the modified RF wave, the synchrotron frequency as a function of the synchrotron oscillation energy, $\omega_s(\mathcal{E})$, changes depending on r_h and r_v. This function is an important contributor to the final stability behaviour of the beam according to Eq. (6). While an overall large synchrotron frequency spread is advantageous for beam stability, the local derivative of $\omega_s(\mathcal{E})$ at the location of the potential instability, $\Omega_{\mathrm{unstable}} = \omega_s(\mathcal{E}_{\mathrm{unstable}})$, has higher relevance. Figure 1 shows that for large enough voltage ratios, the inner part of the bunch profits from a larger $\omega_s'(\mathcal{E})$, while the outer parts have a smaller $\omega_s'(\mathcal{E})$.

SHORT BUNCHES

For short bunches, the voltage waves (fundamental and higher harmonic) can be expressed with the Taylor series, yielding contributions of the first and third orders. This, so called, small angle approximation (SAA) simplifies the terms of $I_{mp}(\mathcal{E})$ and $\omega_s(\mathcal{E})$ in Eq. (6). An approximate CBI threshold for SRF in the assumption of a binomial particle distribution has been derived in [10]:

$$\zeta_{\mathrm{th,SRF}} = \frac{h\phi_{\max}^4 \omega_r}{16MR\omega_0} \min_{x\in[0,1]} (1-x)^{1-\mu} \Big/ J_1\left(\frac{x\omega_r\phi_{\max}}{\omega_0 h}\right), \quad (10)$$

where J_1 is the Bessel function of the first kind and order.

In the case of DRF, analytical approximations for CBI [5, 11], as well as for loss of Landau damping in bunch shorting mode [12] have been published.

Following these methods, a scaling for DRF in BLM can be formulated, yielding the relation

$$\zeta_{\mathrm{th,DRF,BLM}} = \left|1 - r_v r_h^3\right| \zeta_{\mathrm{th,SRF}}. \quad (11)$$

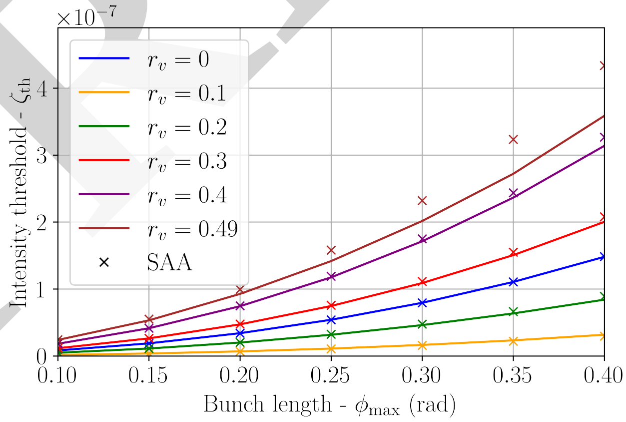

Figure 2: Comparison of bunch length dependency of the intensity threshold ζ_{th} for SRF ($r_v = 0$) and BLM ($r_h = 2$, $\mu = 2$). The solid lines represent results obtained from the stability diagram method without analytical approximations, and the crosses represent results from the small-angle approximation (SAA).

Depending on r_h, a voltage ratio, $r_{v,0}$, is found for which the CBI threshold in BLM drops to zero: $r_{v,0} = 1/r_h^3$. The reason for the decrease of ζ_{th} for $r_v \to r_{v,0}$ is the compensation of the third-order terms of the RF-voltage for BLM, leading consequently to $\omega_s'(\mathcal{E}) = 0$ for $r_v = r_{v,0}$. Beyond the compensation regime, $\omega_s'(\mathcal{E})$ switches sign and for $r_v > 2r_{v,0}$ BLM offers higher intensity thresholds than SRF, independent of the original parameter setting.

The good agreement between exact and approximate solutions can be seen in Figure 2. Only for $r_v \to 1/r_h$, the results deviate more strongly, as the region of validity shrinks for a flat potential well.

LONG BUNCHES

In contrast to short bunches, the location of the instability for long bunches is difficult to predict and might drastically change with the voltage ratio, r_v.

Depending on the resonant frequency of the narrowband impedance source, ω_r, the location of the potential instability might be in an area with decreased or increased $|\omega_s'(\mathscr{E}_{\text{unstable}})|$. In consequence, the combination of certain ω_r, r_v, and ϕ_{max} might lead to strong gains or losses for ζ_{th} in comparison to SRF.

To illustrate this interplay, Figure 3 shows the relative intensity threshold ratio versus ω_r and ϕ_{max}. The results are calculated for a given growth rate to allow a comparison with macro-particle simulations. For the given $r_v = 0.2$, $\omega_s(\mathscr{E})$ has a local maximum around $\phi_{\text{max}} \approx 1.4$, causing significant ζ_{th}-drops for certain ω_r. In the parameter space coloured in blue, the instability has moved to the part of the bunch with low $|\omega_s'(\mathscr{E}_{\text{unstable}})|$, while for the parameter spaces in red, the threshold is still dominated by the core of the beam. The $\omega_s(\mathscr{E})$-spread for small ϕ_{max} becomes comparable to $\Im\{\Omega\} \approx 0.01\omega_{s0}$, resulting in a lower gain in the threshold intensity.

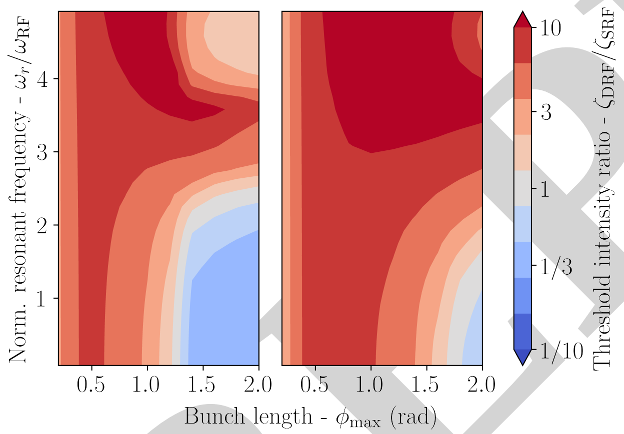

Figure 3: Relative intensity parameter ζ_{th} change from SRF to DRF in BLM with $r_v = 0.2$, $r_h = 4$, $\mu = 2$ (left) and $\mu = 5$ (right) for different bunch length and normalized ω_r. The red areas mark parameter spaces where the intensity threshold gains, while the blue areas mark regions with lowered thresholds.

Increasing the distribution parameter, μ, will push the instability more towards the centre; thus, the blue areas of relative ζ_{th}-loss shrink or disappear, while the red gain regions remain unchanged. The opposite effect holds for a decreased μ.

GROWTH RATE BEHAVIOUR AND MACRO-PARTICLE SIMULATIONS

Different values can be inserted as the imaginary part of Ω into Eq. (6), corresponding to the expected growth rate. The curve obtained that way can be compared with macro-particle tracking. BLonD simulations [13] with different intensities are carried out, and the growth rates, $\Im\{\Omega\}$, of the excited modes are determined.

Figure 4 illustrates the remarkable agreement between both methods. For $r_v \to r_{v,0}$, a drop of the intensity threshold is observable, due to the decrease of $|\omega_s'(\mathscr{E})|$. Despite the longer bunch length ($\phi_{\text{max}} = 1$), the small angle approximation remains a good estimate of the intensity threshold behaviour at $\Im\{\Omega\} \approx 0$. Furthermore, the intensity threshold has a linear growth rate-dependency, if ϕ_{max} or r_v are sufficiently small. Linearity is not maintained when the stability diagram is highly deformed, usually by a change of sign in $\omega_s'(\mathscr{E})$. In these cases, the instability location (Ω_{unstable}) changes significantly with $\Im\{\Omega\}$.

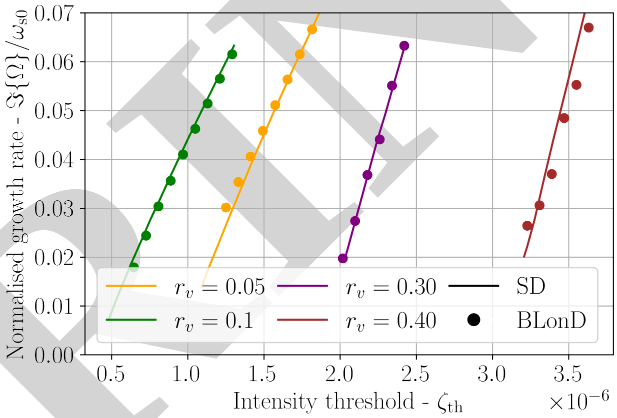

Figure 4: Growth rate evolution for an increasing intensity threshold, ζ_{th}, for different voltage ratios, r_v. Solid lines are obtained from the stability diagram method, and circles are results from the macro-particle simulation software using BLonD. Parameters: $\mu = 2$, $r_h = 2$, $\phi_{\text{max}} = 1$.

CONCLUSION

In this study, we investigated the effect of a DRF system in BLM on the intensity thresholds for CBI. The approximated scaling factor of $|1 - r_v r_h^3|$ for DRF-systems in BLM was compared to the stability diagram method to assess its validity range. Moreover, we presented the condition $r_v \geq 2/r_h^3$ to ensure an increased intensity threshold, independent of μ and ω_r. For longer bunches, we shed light on the interplay of r_v, ϕ_{max}, and ω_r, demonstrating that, against prior believes, intensity threshold gains can still be realised in BLM, even if a local extremum in $\omega_s(\mathscr{E})$ is present.

Finally, macro-particle simulations were used to validate the semi-analytical approach and better understand the ζ_{th}-$\Im\{\Omega\}$-relation.

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