OFF-RESONANCE LANDAU CAVITY FIELD PROBE CALIBRATION

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Abstract

Higher harmonic RF cavities are used by synchrotron light sources for lengthening the bunches in order to improve Touschek lifetime and increase incoherent synchrotron tune spread. The MAX IV 1.5 GeV storage ring passive Landau cavities are used for this purpose. These cavities are fitted with probe loops which monitor the amplitude of the fields. In order to have an accurate reading of the field amplitude experienced by the beam, each probe loop requires a calibration factor. This calibration factor is usually found using an on-resonance calibration method, which relies on a model cavity shunt impedance. In this paper, an alternative off-resonance calibration method is presented. This method is less dependent on the accuracy of the model shunt impedance.

THE MAX IV 1.5 GEV STORAGE RING

The MAX IV laboratory houses two electron storage rings, the 1.5 GeV storage ring, and the 3 GeV storage ring. This paper will only treat the 1.5 GeV ring. Relevant parameters for this ring can be found in Tab. 1. The ring has two nominally identical passive Landau cavities, which will be referred to as Cavity A and Cavity B.

Table 1: Cavity Parameters of the MAX IV 1.5 GeV Storage Ring

Main cavity frequency	f_0	100 MHz
Beam energy	E_0	1.5 GeV
Main cavity voltage	V_{MC0}	519 kV
Harmonic number	h	32
Energy loss per turn	U_0	114 kV
Landau cavity shunt impedance	R_s	5.55 MΩ
Landau cavity higher harmonic	N	3

ON-RESONANCE CALIBRATION

The field in a passive cavity operating on resonance is given by [1]

$$V_{HC} = |\tilde{F}| I R_s, \tag{1}$$

where $\tilde{F}$ is the complex bunch form factor [2], I is the beam current, and R_s is the cavity shunt impedance.

The probe loop calibration of the cavity can be found by recording the probe loop voltage at a number of currents and fitting a linear function to the probe loop voltage vs. beam current. In the case of the passive Landau cavities at MAX IV, R_s/Q is found using Superfish [3], from which R_s is obtained by multiplying by a measured Q [4]. However, this procedure may give up to 10% error in R_s, which will propagate linearly to the probe loop calibration factor. Previous measurements of the bunch profiles of the 1.5 GeV storage ring have indicated an error of the probe loop calibration of the order of 7 %. This could be partially explained by a similarly sized error in the model shunt impedance used for the calibration [5].

OFF-RESONANCE CALIBRATION

An alternative to the on-resonance probe loop calibration is the off-resonance calibration. This method relies on approximating the total harmonic cavity field as a small perturbation to the main cavity accelerating voltage and calculating the effect this perturbation has on the synchrotron frequency. This method has the benefit of not being as dependent on the accuracy of the modelled R_s.

Self-Consistent Form Factor

The total energy loss per turn for an electron circulating in a storage ring equipped with n passive harmonic cavities is given by

$$U_{total} = U_0 + U_{HC0} = U_0 + e \sum_n V_{HC0,n} \cos(\psi_{HC,n}), \tag{2}$$

where U_0 is the energy loss per turn for a machine without passive harmonic cavities, and $\psi_{HC,n}$ is the tuning angle of the nth harmonic cavity, given by

$$\psi_{HC,n} = \arccos\left(\frac{V_{HC0,n}}{I R_{s,n} |\tilde{F}|}\right), \tag{3}$$

The total RF voltage of a ring with active main cavities and n passive harmonic cavities is

$$V_{RF}(\varphi) = V_{MC}(\varphi) + \sum_n V_{HC,n}(\varphi). \tag{4}$$

The main cavity voltage is given by

$$V_{MC}(\varphi) = V_{MC0} \sin(\varphi + \varphi_s), \tag{5}$$

where φ_s is the synchronous phase, given by

$$\varphi_s = \pi - \arcsin\left(\frac{U_{total}}{eV_{MC0}}\right). \tag{6}$$

Similarly, the harmonic cavity voltage is given by [2]

$$V_{HC,n}(\varphi) = V_{HC0,n} \sin(N\varphi + \varphi_{HCs,n}), \tag{7}$$

where the synchronous phase of the harmonic cavities, $\varphi_{HCs,n}$, is given by

$$\varphi_{HCs,n} = \psi_{HC,n} - \pi/2 - \varphi_{FF}, \tag{8}$$

where φ_{FF} is the form factor phase [2].

The complex form factor is given by the Fourier transform of the density distribution of the electron bunch, ρ, at the Nth

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harmonic of the main cavity frequency, ω_{RF}, normalised by the DC component [2]:

$$\tilde{F} = |\tilde{F}|e^{i\varphi_{FF}} = \frac{\mathscr{F}[\rho(\varphi)]_{\omega=N\omega_{RF}}}{|\mathscr{F}[\rho(\varphi)]_{\omega=0}|}. \tag{9}$$

The equilibrium bunch density distribution can be expressed as

$$\rho(\varphi) \propto \exp\left(\frac{U(\varphi)}{E_0 h\alpha\sigma_E^2 2\pi}\right), \tag{10}$$

where E_0 is the beam energy, h is the harmonic number, α is the momentum compaction factor, σ_E is the energy spread of the beam, and $U(\varphi)$ is the longitudinal potential energy given by

$$U(\varphi) = e\int_0^{\varphi} V_{RF}(\varphi')\mathrm{d}\varphi'. \tag{11}$$

Through Eqs. (3), (8), (9), and (10), a complex form factor can be calculated using a complex form factor as input. This sequence of calculations results in the self-consistent equation

$$\tilde{F} = f(\sum_n V_{HC0}, V_{MC0}, U_0, I, R_s, \sigma_E, \alpha, E_0, \tilde{F}), \tag{12}$$

where all arguments of the function f, except $\tilde{F}$, are assumed to be known. For a set of known inputs, the value of $\tilde{F}$ can be found iteratively.

Residual Function

For longitudinal motion with small oscillation amplitudes, the following proportionality of the synchrotron frequency holds [6]

$$f_s^2 \propto \frac{\mathrm{d}V_{RF}}{\mathrm{d}\varphi}\Big|_{\varphi_s}. \tag{13}$$

Assuming one well-known synchrotron frequency, f_{s0}, and corresponding accelerating voltage gradient evaluated around the synchronous phase $\frac{\mathrm{d}V_{RF,0}}{\mathrm{d}\varphi}|_{\varphi_s}$, it is possible to calculate the synchrotron frequency of a perturbed voltage gradient, $\frac{\mathrm{d}V_{RF,1}}{\mathrm{d}\varphi}|_{\varphi_s}$, as

$$f_{s,1} = f_{s,0}\sqrt{\frac{\mathrm{d}V_{RF,1}}{\mathrm{d}\varphi}\Big|_{\varphi_s} \Big/ \frac{\mathrm{d}V_{RF,0}}{\mathrm{d}\varphi}\Big|_{\varphi_s}} \tag{14}$$

as long as the perturbation is small enough that the small oscillation amplitude requirement still holds. In this case, $f_{s,0}$ was chosen to be the zero beam current synchrotron frequency, so that the corresponding unperturbed accelerating voltage becomes $V_{RF,0} = V_{MC}$. The perturbed accelerating voltage is the sum of voltages from the active and passive cavities at a non-zero beam current, as given by Eq. (4).

Equations (12) and (14) can be combined with the measured synchrotron frequency, $f_{s,meas}$, to produce a residual function

$$g(\sum_n V_{HC0}) = \left(f_{s,meas} - f_{s,1}(\sum_n V_{HC0})\right)^2. \tag{15}$$

For a measured synchrotron frequency, the total harmonic cavity voltage can be found as the voltage which minimises Eq. (15).

Measurement

The probe loop calibration factor was found by recording the probe loop amplitude, beam energy spread, and synchrotron frequency at a number of different beam currents. The energy spread was measured using the diagnostic beamline [7], while the synchrotron frequency was measured using the bunch-by-bunch system [8]. The Landau cavities were tuned further from resonance than during normal operation in order to ensure that Eq. (14) holds. As the harmonic cavity fields were lower, the beam was instead kept longitudinally stable using bunch-by-bunch feedback. Due to the limited power of this feedback system, the maximum measurement current was 100 mA, compared to the normal operational current of 500 mA.

The off-resonance calibration method is only able to measure the total effective probe loop calibration of both harmonic cavities in the ring. This results in the effective probe loop calibration given by the weighted sum

$$v_a K_a + v_b K_b = k(v_a + v_b), \tag{16}$$

where v_a and v_b are the probe loop voltages, K_a and K_b are the individual calibration factors, and k is the measured total calibration factor. For a collection of m measurement points Eq. (16) can be written in the matrix form

$$VK = kV\begin{bmatrix}1\\1\end{bmatrix}, \tag{17}$$

where $V = \left[v_{a,1},...,v_{a,m}; v_{b,1},...,v_{b,m}\right]^T$, $K = \left[K_a, K_b\right]^T$, and k is the diagonal matrix $diag(K) = \left[k_1,...,k_m\right]$. K can be found as the least squares solution of Eq. (17).

RESULTS

The measured synchrotron frequency and the corresponding calculated harmonic cavity voltage vs the sum of probe loop voltages can be seen in Fig. 1. This measurement corresponds to an approximately equal signal amplitude from the probe loops of the two harmonic cavities.

In order to find the individual probe loop calibration factor of each cavity, two additional measurements were done. The ratio of Cavity A to Cavity B probe loop read-back in these measurements were 1.65 and 0.60, respectively. These measurements were also done over a larger stored beam current range, in order to increase the measurement accuracy. The results of the two measurements can be seen in Figs. 2 and 3, and the resulting calibration factors in Tab. 2.

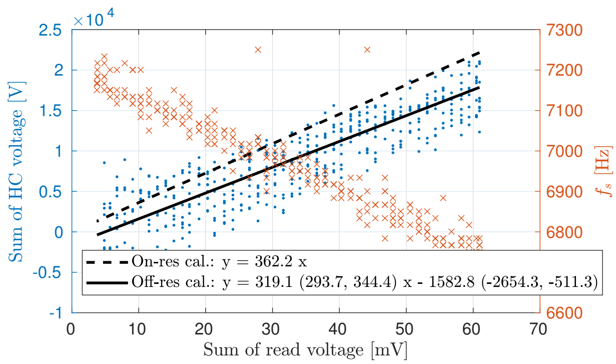

Figure 1: The measured f_s and calculated $\sum_n V_{HC,n}$ vs the sum of harmonic cavity probe loop voltage. The fields in Cavity A and B were approximately equal.

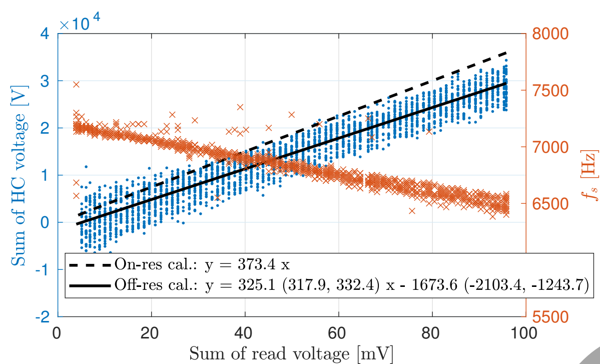

Figure 2: The measured f_s and calculated $\sum_n V_{HC,n}$ vs the sum of harmonic cavity probe loop voltage. The field in Cavity A was ~ 40 % lower than the field in Cavity B.

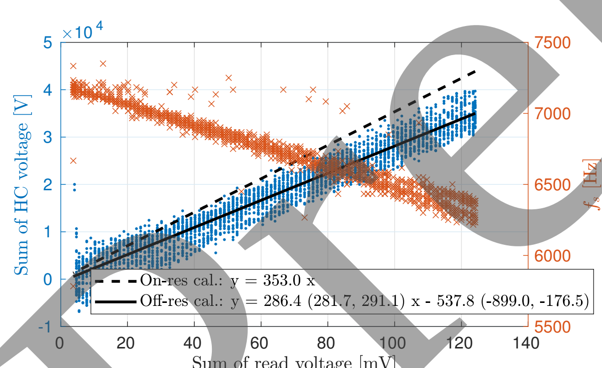

Figure 3: The measured f_s and calculated $\sum_n V_{HC,n}$ vs the sum of harmonic cavity probe loop voltage. The field in Cavity A was ~ 65 % higher than the field in Cavity B.

Table 2: Calibration Factor (CF) and Confidence Interval (CI) of the Individual Probe Loop Calibration Measurements

	CF [V/mV]	95 % CI
K_a	227.2	[211.8, 242.6]
K_b	389.4	[370.6, 408.2]
$K_{a,on-res}$	321.5	
$K_{b,on-res}$	406.5	

Sensitivity Analysis

The accuracy of the off-resonance probe loop calibration depends on the accuracy of the know quantities used in the calculation. In order to estimate how an error in a variable affects the resulting harmonic cavity voltage, a sensitivity analysis was performed. The results of this analysis can be seen in Fig. 4. The analysis was performed by individually changing each input parameter and fitting a first-order polynomial to the results.

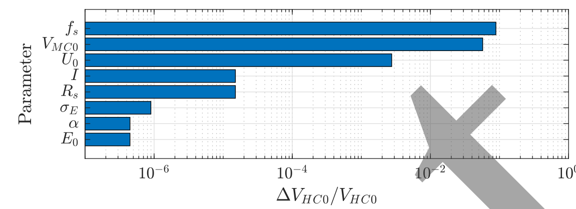

Figure 4: Sensitivity analysis of each of the input parameters for the harmonic cavity voltage calculation. For each parameter, the relative change of V_{HC0} corresponds to a relative parameter change of 1 %.

DISCUSSION AND CONCLUSION

From the measurement presented in Fig. 1, the total probe loop calibration constant was found to be 319.1 V/mV, CI 95 % [293.7, 344.4], which is a reduction of 11.9 % compared to the on-resonance calibration, 362.2 V/mV. This reduction is in the same direction and order of magnitude as the one expected from previous measurements of the bunch profile [5]. The individual calibration factors, calculated from the measurements presented in Figs. 2 and 3 corresponded to a reduction of 29 % and 4 %, for cavities A and B respectively, compared to the factors measured on-resonance. From Fig. 4, it is clear that any error to the harmonic cavity shunt impedance, R_s, does not propagate linearly to the harmonic cavity voltage, unlike the on-resonance calibration (see Eq. (1)).

The off-resonance harmonic probe loop calibration is a promising complement to the on-resonance calibration. It is significantly less dependent on an accurate value of R_s, making it useful in detecting potential errors in a modelled R_s. The method is reliant on an accurate measurement of f_s, which can generally be achieved to sufficient accuracy (order of 0.1 %) by increasing the averaging. Its ability to find the calibration factor of an individual cavity depends on how wide a range of uneven field distributions are measured.

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