PICOSECOND BUNCH-CENTROID STABILIZATION IN SSMB STORAGE RINGS VIA LOW-R/Q HARMONIC CAVITIES AND GILD-BASED FEEDFORWARD COMPENSATION

Peizhi Fang, Xiujie Deng, Yanqing Jia, Zhilong Pan, Shuxin Zheng*, Chuanxiang Tang, Hongjuan Yao
Department of Engineering Physics, Tsinghua University, Beijing, China

Abstract

The Steady-State Micro Bunching (SSMB) scheme holds great promise for generating high-power extreme ultraviolet (EUV) radiation. To achieve a 1 kW EUV output, a storage ring operating with an average current of 1 A and harmonic cavities for bunch lengthening is required to mitigate intra-beam scattering (IBS). However, the combination of high beam current and harmonic cavities induces severe transient beam loading (TBL) and periodic transient beam loading (PTBL), leading to bunch center shifts that severely degrade the laser-electron modulation efficiency. This paper evaluates the stringent tolerance for bunch shift and propose a mitigation strategy that combines low-$(R/Q)_h$ cavity design to suppress PTBL and a newly developed Gap-Induced Linear Decomposition (GILD) analytical model to cancel TBL. Unlike numerical methods, the GILD model provides a closed-form expression for the required compensation voltage. Macroparticle tracking utilizing the `STABLE` code verifies that this approach restricts the bunch center shift to the picosecond level.

INTRODUCTION

The reversible SSMB scheme utilizes laser-electron beam modulation to generate longitudinally coherent synchrotron radiation. To achieve an ambitious 1 kW EUV light source, the storage ring design necessitates an average beam current of 1 A [1]. Furthermore, to reduce the peak current of the microbunch and consequently control the intra-beam scattering (IBS) effect, harmonic cavities must be employed for bunch lengthening.

However, the combination of high beam current and harmonic cavities introduces severe beam loading challenges that directly threaten the modulation process. Specifically, the Transient Beam Loading (TBL) effect leads to different longitudinal shifts for individual bunches along the fill pattern [2]. This misalignment between the electron bunches and the center of the pulsed laser significantly degrades the modulation efficiency. In addition, the Periodic Transient Beam Loading (PTBL) effect, which is inherently a coupled-bunch instability, results in low-frequency periodic variations in both the bunch center position and bunch length [3]. This similarly drives the bunches away from the optimal modulation center, further deteriorating the overall modulation performance.

In this paper, we first analyze the sensitivity of laser-electron modulation to the bunch center shift and quantitatively estimate the resulting degradation of EUV radiation power. To address these stringent requirements, we propose a comprehensive mitigation strategy. The centerpiece of this strategy is the development of a Gap-Induced Linear Decomposition (GILD) analytical model, which provides a deterministic template for feedforward compensation to suppress TBL-induced shifts. Concurrently, we establish a strict low-(R/Q) requirement for the harmonic cavity to fundamentally prevent PTBL instability. Finally, macroparticle tracking simulations using the `STABLE` code [4] are presented to demonstrate the validity of the proposed GILD-based mitigation strategy.

TOLERANCE OF BUNCH SHIFT IN MODULATION

The SSMB scheme for high-power EUV radiation requires precise synchronization between the electron bunch and the laser. We focus on the echo-enabled modulation, where the second-stage modulation demands significantly higher peak power than the first. To optimize laser efficiency, the second-stage laser pulse is designed with an RMS length σ_L comparable to the electron bunch length σ_τ. Consequently, any longitudinal shift $\Delta\tau$ of the bunch center relative to the laser pulse degrades the interaction overlap and the resulting radiation power.

Equilibrium Electron Distribution

To evaluate the shift tolerance, potential well distortion from the harmonic cavity and synchronous energy loss must be considered. Following [2], the equilibrium longitudinal density $\rho(\tau)$ is:

$$\rho(\tau) = \rho_0 \exp\left(\frac{\omega_0}{2\pi E_0 \alpha_c \sigma_\epsilon^2}\int_0^\tau [e_0 V_c(t) - U_0]dt\right), \quad (1)$$

where α_c is the momentum compaction, σ_ϵ is the energy spread, E_0 is the beam energy, and U_0 is the energy loss per turn. The total dual-cavity voltage is $V_c(t) = V_f\cos(\omega_{\mathrm{rf}}t + \varphi_f) + V_h\cos(h\omega_{\mathrm{rf}}t + \varphi_h)$, where V and φ denote the voltage and phase of the fundamental (f) and h-th harmonic (h) components.

Bunching Factor and Radiation Power

Assuming a Gaussian temporal envelope for the second-stage laser pulse, the normalized bunching factor $R(\tau)$ is determined by the Bessel function J_{n+1}:

$$R(\tau) = \frac{J_{n+1}[a\exp(-\tau^2/2\sigma_L^2)]}{J_{n+1}(a)}, \quad (2)$$

---

[*] zhengsx@tsinghua.edu.cn

where $n = 76$ is the modulation order and a is the modulation amplitude with optimal value $a_{\text{opt}} \approx n + 0.81n^{1/3}$ [5].

The integrated coherent radiation power $W(\Delta\tau)$ as a function of the bunch center shift $\Delta\tau$ is the overlap integral:

$$W(\Delta\tau) \propto \int_{-\infty}^{\infty} [R(\tau) \cdot \rho(\tau + \Delta\tau)]^2 d\tau. \quad (3)$$

Numerical Analysis

As illustrated in Fig. 1(a), the laser-modulated bunching factor reaches its peak at the center and decays rapidly toward the edges. In this configuration, the RMS length of the laser pulse is set to approximately twice that of the electron bunch.

From the bunch profile, it is evident that the microbunching is primarily concentrated within the flat-top region. Consequently, a significant longitudinal shift of the electron bunch leads to a substantial reduction in the effective beam current capable of forming micro-bunches, which severely impacts the output radiation power.

Figure 1(b) depicts the relationship between the bunch center shift and the resulting radiation power. It can be observed that, adopting a 5% power drop as the threshold, the tolerance for the bunch center shift is approximately 20 ps. This remains a challenge for a 1 A average current ring with harmonic cavities., necessitating some mitigation strategy for beam loading effects.

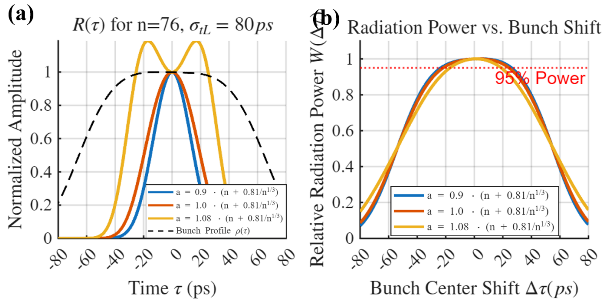

Figure 1: Numerical analysis of radiation characteristics: (a) normalized bunching factor $R(\tau)$ under different modulation strengths a and bunch profile $\rho(\tau)$. (b) relative radiation power versus bunch shift $\Delta\tau$.

BEAM LOADING AND MITIGATION STRATEGY

The bunch shift in SSMB is driven by two distinct mechanisms: Transient Beam Loading (TBL) from the fill gap, and Periodic Transient Beam Loading (PTBL) from mode-1 instabilities. For the baseline parameters in Table 1, both effects can induce jitter exceeding 100 ps, far beyond the 20 ps tolerance.

Our mitigation strategy is based on the principle that a low $(R/Q)_h$ for the harmonic cavity is a fundamental prerequisite. By minimizing $(R/Q)_h$, the beam-cavity interaction is weakened at the source, thereby suppressing the Periodic Transient Beam Loading (PTBL) effect [6–8]. However, relying solely on a passive harmonic cavity with low $(R/Q)_h$ is insufficient to meet requirements, as residual Transient Beam Loading (TBL) still induces approximately 50 ps of phase jitter. Consequently, an active feed-forward compensation scheme for the main RF cavity is mandatory [9]. In this work, the Gap-Induced Linear Decomposition (GILD) analytical model is developed to provide the precise compensation template required to achieve the necessary beam stability.

To evaluate these effects quantitatively, we employ the updated baseline parameters of the SSMB storage ring, as summarized in Table 1.

Table 1: Main Parameters of the SSMB Storage Ring

Parameter	**Value**	**Unit**
Beam energy E_0	600	MeV
Circumference C_0	~300	m
Average current I_0	1	A
Fundamental RF f_f	500	MHz
Harmonic RF f_h	1500	MHz
Main cavity voltage V_f	1.0	MV
Energy loss per turn U_0	300	keV
Momentum compaction α_c	1×10^{-3}	-

Analysis and Suppression of PTBL

Periodic Transient Beam Loading (PTBL) is a longitudinal coupled-bunch instability that occurs in passive harmonic cavities even under uniform filling patterns. We usually limit $(R/Q)_h$ threshold to suppress PTBL.

First, an analytical $(R/Q)_h$ threshold formula provided by He *et al.* [6] is employed. By substituting the SSMB storage ring parameters—including the 1 A average current, beam energy, and synchronous phase—into the analytical limit, we establish the basic requirement for the harmonic cavity $(R/Q)_h < 40\ \Omega$.

To further refine this limit, semi-analytical calculations are performed using the `pycolleff` code. [8] We fix the harmonic cavity's geometric shunt impedance $(R/Q)_h$ and scan the beam current to identify the instability threshold. As shown in the calculation, with $(R/Q)_h$ set to 40 Ω, the PTBL threshold current exceeds the design goal of 1 A. Therefore, maintaining the harmonic cavity $(R/Q)_h \le 40\ \Omega$ is established as a critical design requirement to ensure longitudinal stability during high-current operation.

Analytical Model and Suppression of TBL

To quantify the impact of the bunch gap on transient beam loading and provide a physical basis for compensation, we first derive an analytical framework for the beam-induced voltage. Consider a storage ring containing bunch trains, where each train consists of n_t filled buckets with charge q_b, followed by a gap of n_g empty buckets. The bucket spacing is t_b, yielding a total period $T = (n_t + n_g)t_b$. Under steady-state conditions, the cavity voltage $\tilde{V}_b(\tau)$ is periodic with T.

The voltage evolves through immediate beam kicks and subsequent decay. For high-Q cavities, where the dimensionless decay factor satisfies $|\alpha| \ll 1$, we apply periodic boundary conditions to the discrete recurrence relations. By performing a first-order Taylor expansion in the continuous-time limit $\tau \in [0, T)$, we derive the Gap-Induced Linear Decomposition (GILD) model. This framework rigorously decomposes the beam-induced voltage into a constant steady-state term and a time-dependent transient term:

$$\tilde{V}_b(\tau) = \tilde{V}_{b,\mathrm{st}} + \tilde{V}_{b,\mathrm{tr}}(\tau), \tag{4}$$

where the steady-state component $\tilde{V}_{b,\mathrm{st}}$ represents the macroscopic average beam loading:

$$\tilde{V}_{b,\mathrm{st}} = -2I_0 \frac{n_t}{n_t + n_g} \frac{R_L}{1 - i \tan\psi}. \tag{5}$$

Here $I_0 = q_b/t_b$ is the average bunch current, R_L is the loaded shunt impedance, and ψ is the detuning angle. The transient component $\tilde{V}_{b,\mathrm{tr}}(\tau)$ captures the intra-train variations:

$$\tilde{V}_{b,\mathrm{tr}}(\tau) = \begin{cases} -(R/Q)_{\mathrm{tot}} I_0 \frac{n_g}{n_t+n_g} \omega_{\mathrm{rf}}(\tau - \tau_0), & 0 < \tau \le n_t t_b \\ +(R/Q)_{\mathrm{tot}} I_0 \frac{n_t}{n_t+n_g} \omega_{\mathrm{rf}}(\tau - \tau_0), & n_t t_b < \tau \le T \end{cases} \tag{6}$$

where $\tau_0 = n_t t_b$ denotes the end of the bunch train ,and $(R/Q)_{\mathrm{tot}} = (R/Q)_f + h(R/Q)_h$ represents the total effective geometric impedance contributing to the transient.

To mitigate the phase transients, an active feedforward system is employed to synthesize a generator voltage $\tilde{V}_{g,\mathrm{ff}}(\tau)$ that cancels the transient component $\tilde{V}_{b,\mathrm{tr}}(\tau)$ identified in Eq. (6). Ideally, the total beam-induced voltage plus the generator-driven transient should remain constant, requiring $\tilde{V}_{g,\mathrm{ff}}(\tau) = -\tilde{V}_{b,\mathrm{tr}}(\tau)$.

Based on the GILD model, the required generator voltage to achieve perfect compensation is a triangular-like waveform. During the beam passage ($0 < \tau \le n_t t_b$), the compensation voltage $\tilde{V}_g(\tau)$ is:

$$\tilde{V}_g(\tau) = (R/Q)_{\mathrm{tot}} I_0 \frac{n_g}{n_t + n_g} \omega_{\mathrm{rf}}(\tau - \tau_0), \tag{7}$$

and during the gap passage ($n_t t_b < \tau \le T$):

$$\tilde{V}_g(\tau) = -(R/Q)_{\mathrm{tot}} I_0 \frac{n_t}{n_t + n_g} \omega_{\mathrm{rf}}(\tau - \tau_0). \tag{8}$$

Macroparticle Tracking Verification

To validate the analytical GILD model and the effectiveness of the transient beam loading (TBL) suppression, numerical simulations were conducted using the `STABLE` macroparticle tracking code [4].

In Fig. 2(a), where the cavity $(R/Q)_h$ is below the PTBL threshold, the application of feedforward effectively suppresses the bunch centroid shift. The bunch centroids $\langle\tau\rangle$ rapidly converge to near zero. Consequently, a slight increase in the average bunch length σ_τ is observed, confirming that the bunch distribution is successfully maintained in the desired stretched state.

Conversely, Fig. 2(b) shows the regime where $(R/Q)_h$ is above the PTBL threshold. Despite the feedforward intervention, the bunch centroids remain significantly separated and fail to reach a stable equilibrium. This result demonstrates that when the periodic transient beam loading is triggered, feedforward compensation becomes insufficient.

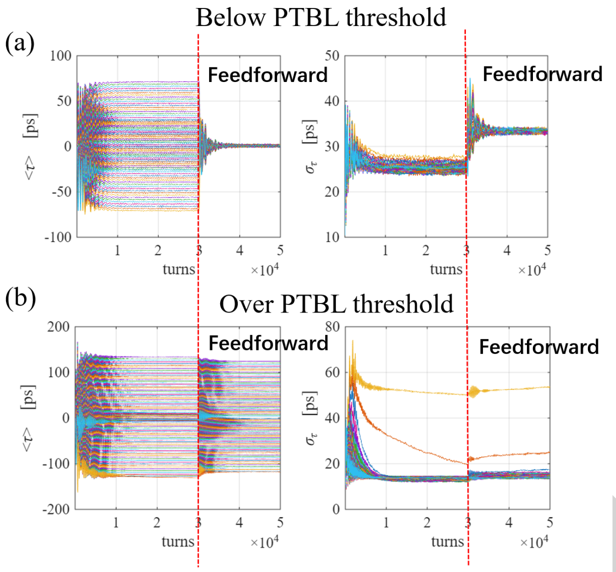

Figure 2: Evolution of bunch centroids $\langle\tau\rangle$ (left) and bunch lengths σ_τ (right) vs. turns. The red dashed line at 3×10^4 turns marks the activation of the TBL feedforward. (a) $(R/Q)_h$ is below the PTBL threshold; (b) $(R/Q)_h$ is above the PTBL threshold.

It is thus evident that to effectively control the bunch centroid shift in an SSMB storage ring, suppressing both TBL and PTBL effects is indispensable. This goal can only be achieved through the dual implementation of a low-(R/Q) cavity design and feedforward compensation; neither strategy can be dispensed with, as they address different aspects of the beam stability.

CONCLUSION

To achieve the 1 kW EUV output goal in the SSMB scheme, stringent control of bunch center stability is essential. Our analysis demonstrates that the radiation power tolerance requires the bunch shift to be suppressed below 20 ps, a challenging requirement for a 1 A storage ring with harmonic cavities.

We propose a comprehensive mitigation strategy combining low $(R/Q)_h$ harmonic cavity design and active feedforward compensation. The low $(R/Q)_h$ requirement ($\le 40\ \Omega$) fundamentally suppresses the PTBL instability. Crucially, the newly developed GILD analytical model provides the precise deterministic template for the feedforward scheme, effectively canceling TBL-induced transient phase shifts. Macroparticle tracking simulations with `STABLE` confirm that this dual approach successfully restricts the bunch center shift to the picosecond level, satisfying the modulation efficiency requirements.

REFERENCES

[1] X. J. Deng *et al.*, “Reversible steady-state microbunching in an electron storage ring”, submitted to *Phys. Rev. Accel. Beams*, 2025.

[2] J. M. Byrd *et al.*, “Transient beam loading effects in harmonic rf systems for light sources”, *Phys. Rev. Spec. Top. Accel. Beams*, vol. 5, no. 9, p. 092001, 2002. doi:10.1103/PhysRevSTAB.5.092001

[3] T. He *et al.*, “Periodic transient beam loading effect with passive harmonic cavities in electron storage rings”, *Phys. Rev. Accel. Beams*, vol. 25, no. 2, p. 024401, 2022. doi:10.1103/PhysRevAccelBeams.25.024401

[4] T. He and Z. Bai, “Graphics-processing-unit-accelerated simulation for longitudinal beam dynamics of arbitrary bunch trains in electron storage rings”, *Phys. Rev. Accel. Beams*, vol. 24, no. 10, p. 104401, 2021. doi:10.1103/PhysRevAccelBeams.24.104401

[5] G. Stupakov, “Using the Beam-Echo Effect for Generation of Short-Wavelength Radiation”, *Phys. Rev. Lett.*, vol. 102, p. 074801, 2009. doi:10.1103/PhysRevLett.102.074801

[6] T. He, “Novel perturbation method for judging the stability of the equilibrium solution in the presence of passive harmonic cavities”, *Phys. Rev. Accel. Beams*, vol. 25, no. 9, p. 094402, 2022. doi:10.1103/PhysRevAccelBeams.25.094402

[7] M. Venturini, “Passive higher-harmonic rf cavities with general settings and multibunch instabilities in electron storage rings”, *Phys. Rev. Accel. Beams*, vol. 21, no. 11, p. 114404, 2018. doi:10.1103/PhysRevAccelBeams.21.114404

[8] M. B. Alves, “Theoretical Models for Longitudinal Coupled-Bunch Instabilities Driven by Harmonic Cavities in Electron Storage Rings”, *Phys. Rev. Accel. Beams*, vol. 28, no. 3, p . 034401, 2025. doi:10.1103/PhysRevAccelBeams.28.034401

[9] N. Yamamoto, T. Takahashi, and S. Sakanaka, “Reduction and compensation of the transient beam loading effect in a double rf system of synchrotron light sources”, *Phys. Rev. Accel. Beams*, vol. 21, no. 1, p. 012001, 2018. doi:10.1103/PhysRevAccelBeams.21.012001