

THEORETICAL ANALYSIS OF A NOVEL SEEDED FREE-ELECTRON LASER SCHEME WITH THE SAME LAYOUT AS EEHG

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Abstract

Seeded free-electron lasers (FELs) are advanced, accelerator-based light sources characterized by high coherence and stability. In our previous research, we proposed an intelligent optimization framework for studying the micro-bunching process in seeded FELs. Using this framework for exploratory optimization, we identified a novel seeded FEL scheme. This scheme employs an optical layout identical to echo-enabled harmonic generation (EEHG), but by adjusting the parameters of the seed lasers and chicanes, it achieves bunching factors exceeding the theoretical limit of conventional EEHG. In this paper, using a representative set of realistic beam parameters, we demonstrate how this novel scheme induces a high harmonic density modulation in the electron beam. Then, we theoretically analyze this novel seeded FEL, presenting its underlying principles.

INTRODUCTION

In a high-gain single-pass free-electron laser (FEL), the radiation field grows exponentially from spontaneous emission as the electron beam develops density modulation at the radiation wavelength, ultimately reaching saturation within a single undulator pass and yielding gigawatt-level coherent output in the extreme ultraviolet and X-ray regimes [1]. The most commonly used mode, self-amplified spontaneous emission (SASE) [2], typically starts from shot noise, resulting in limited temporal coherence [3]. Seeded FELs introduce stable external seed lasers to interact with the electron beam. After density modulation, the electron beam forms micro-bunching, which generates harmonic radiation that undergoes coherent amplification. Since the quality of the output radiation is similar to that of the seed lasers, seeded FELs offer advantages over the SASE mode, such as narrow bandwidth and a stable central wavelength.

An intelligent optimization framework for studying the micro-bunching process in seeded FELs was developed in our earlier work [4]. By pre-defining feasible components and their parameter ranges, this framework can automatically generate optimized seeded FEL configurations for specific performance objectives. Using it, we have successfully reproduced previously proposed seeded FEL schemes. Furthermore, we have identified a novel seeded FEL scheme with the same layout as that of echo-enabled harmonic generation (EEHG) [5, 6], as shown in Fig. 1. The longitudinal phase space evolution of the novel scheme differs from that of

EEHG, which results in its ability to achieve a larger bunching factor. In this paper, we will present the characteristics of this novel scheme and compare it with EEHG. We will also explain the underlying physical principles governing the micro-bunching process with theoretical analysis.

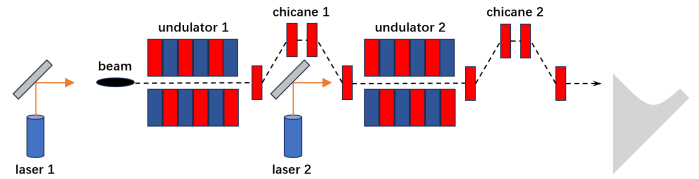


Figure 1: Schematic layout of the novel scheme, with same layout as EEHG. The dashed line represents the path of the beam.

CHARACTERISTICS OF THE NOVEL SCHEME

To intuitively illustrate the characteristics of the novel scheme, we first present a specific example. We adopt standard FEL parameters for the initial electron beam, with a beam energy of 1.2 GeV and an rms energy spread of 150 keV. The seed lasers shown in Fig. 1 have the same wavelength of 260 nm. It is known that in EEHG, the energy modulation induced by the first seed laser is typically significantly stronger than that of the second. In contrast, the novel scheme relies on a strong energy modulation induced by the second seed laser as the key to generating micro-bunching, where undulator 1 provides an amplitude of 532.5 keV and undulator 2 provides an amplitude of 1.29 MeV. The dispersive strengths of the two chicanes are 0.50 mm and 0.05 mm, respectively, which are much smaller than those in EEHG.

The longitudinal phase space of the beam after passing through undulator 1 and chicane 1 is shown in the left plot of Fig. 2, where no separated energy bands are formed, unlike in EEHG. Then, after passing through undulator 2 and chicane 2, shown in the middle and right plots of Fig. 2, the final phase space distribution in the novel scheme approximates a non-uniform multi-peak distribution. The bunching factors for the novel scheme at different harmonic numbers are shown in Fig. 3, together with those for the EEHG scheme optimized at the 15th harmonic for ease of comparison. In EEHG, the bunching factors are typically optimized for some specific harmonic numbers, with most of the other harmonic components being strongly suppressed. However, the novel scheme not only achieves larger bunching factors at the targeted harmonic numbers but also generates significant

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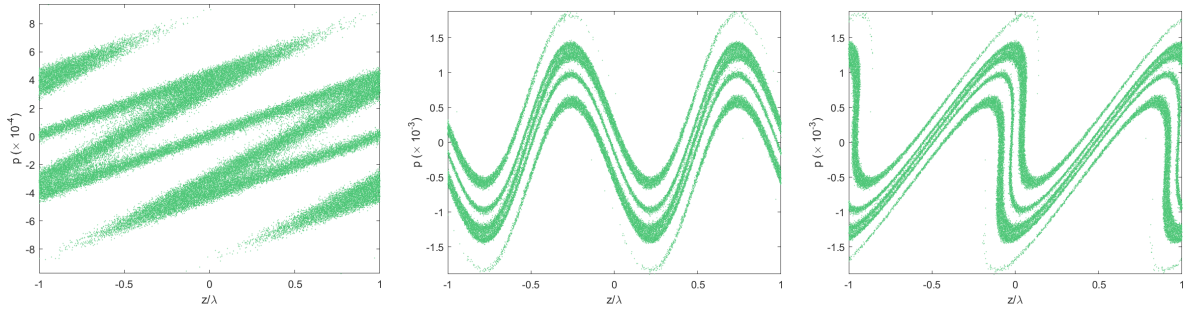


Figure 2: Longitudinal phase space evolution at the exits of chicane 1 (left), undulator 2 (middle), and chicane 2 (right) in the novel scheme.

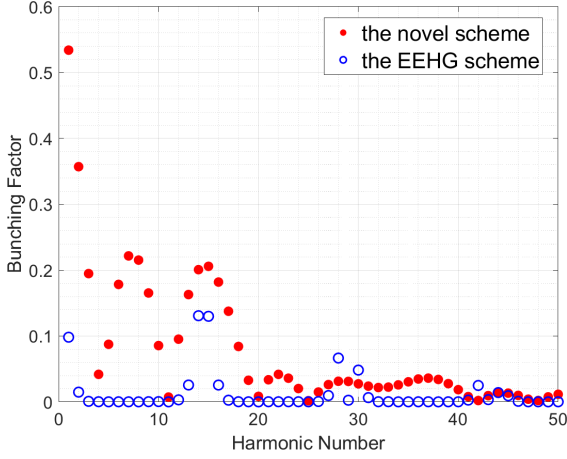


Figure 3: Comparison of the bunching factors of the novel scheme (red dots) and EEHG (blue circles), which are optimized at the 15th harmonic.

components at many other harmonics, thereby providing enhanced tunability.

THEORETICAL ANALYSIS

Using the notation of Ref. [5, 6], we consider an initial Gaussian beam energy distribution with an average energy E_0 and the rms energy spread σ_E , and define the dimensionless energy deviation $p = (E - E_0) / \sigma_E$. Assuming a uniform longitudinal distribution, the initial longitudinal phase-space distribution is $f(z, p) = \frac{N_0}{\sqrt{2\pi}} \exp\left(-\frac{p^2}{2}\right)$, where N_0 is the number of electrons per unit length of the beam.

Since the novel scheme shares the same layout as EEHG, we omit the derivation and directly present the expression of the final distribution function:

$$f_f(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left\{ p - A_2 \sin(K\zeta - KB_2 p + \phi) - A_1 \sin[\zeta - (B_1 + B_2)p + A_2 B_1 \sin(K\zeta - KB_2 p + \phi)] \right\}^2 \right], \quad (1)$$

where all symbols have the same meaning as in Ref. [6]. In the subsequent derivation, we assume the two lasers have the same wavelength, i.e., $K = 1$.

The complex bunching factor at the M -th harmonic can be defined as

$$b_M = \lim_{L \rightarrow +\infty} \frac{1}{2N_0 L} \int_{-L}^{+L} \exp(-iM\zeta) \left[\int_{-\infty}^{+\infty} f_f(\zeta, p) dp \right] d\zeta. \quad (2)$$

Since $\frac{1}{2N_0 L} \exp(-iM\zeta) f_f(\zeta, p)$ is absolutely integrable, the order of the double integration can be interchanged, thus we have

$$b_M = \lim_{L \rightarrow +\infty} \frac{1}{2N_0 L} \int_{-\infty}^{+\infty} f(z, p) \left[\int_{-L}^{+L} \exp(-iM\zeta) d\zeta \right] dp. \quad (3)$$

After lengthy algebraic manipulations, we obtain the final expression for the bunching factor b_M :

$$b_M = \sum_{m=-\infty}^{+\infty} J_m(-A_2 B_2 M) \times J_{M-m}(mA_1 B_1 - A_1(B_1 + B_2)M) \times \exp\left(im\phi - \frac{1}{2}(mB_1 - M(B_1 + B_2))^2\right), \quad (4)$$

where J_m is the Bessel function of the first kind of order m .

When the dispersion strength of chicane 1 is large, meaning that the value of B_1 is large, the exponential term in Eq. (4) decays rapidly as m deviates from the value that minimizes the exponent. The exponent is minimized when $mB_1 - M(B_1 + B_2) = 0$. Since the value of B_2 is non-zero and the maximum value of J_{M-m} decreases as $|M - m|$ increases, the term with $m = M + 1$ can be made dominant by choosing appropriate parameters, thereby maximizing the bunching factor. In this case, the modulus of the bunching factor $|b_M|$ can be simplified to

$$|b_M| = |J_{M+1}(A_2 B_2 M) J_1(A_1 B_1 - A_1 B_2 M)| \times \exp\left(-\frac{1}{2}(B_1 - MB_2)^2\right), \quad (5)$$

which has the same form as the bunching factor formula for EEHG given in Ref. [6]. However, limited by the product of the two Bessel functions, the theoretical maximum bunching factor is given by $0.39/(M + 1)^{1/3}$ [6].

To overcome this limitation, the required dispersion strength can be reduced to slow down the decay rate of the exponential term, thereby allowing multiple terms in the

series to superpose. For simplicity, we consider only the contributions of the three most dominant terms for $m = M, M+1$ and $M+2$, i.e.,

$$J_M(-A_2B_2M)J_0(-A_1B_2M) \exp\left(iM\phi - \frac{1}{2}(MB_2)^2\right), \quad (6a)$$

$$J_{M+1}(-A_2B_2M)J_{-1}(A_1B_1 - A_1B_2M) \times \exp\left(i(M+1)\phi - \frac{1}{2}(B_1 - MB_2)^2\right), \quad (6b)$$

and

$$J_{M+2}(-A_2B_2M)J_{-2}(2A_1B_1 - A_1B_2M) \times \exp\left(i(M+2)\phi - \frac{1}{2}(2B_1 - MB_2)^2\right). \quad (6c)$$

Numerical analysis shows that, to maximize the absolute values of $J_0(-A_1B_2M)$, $J_{-1}(A_1B_1 - A_1B_2M)$ and $J_{-2}(2A_1B_1 - A_1B_2M)$, the arguments should be chosen near the local extremum points near zero. This choice ensures that the three function values have the same sign. When M is large, the maximum values of J_M , J_{M+1} and J_{M+2} are approximately equal, and the corresponding arguments, taken to be positive, are also approximately the same. Because of the parity of Bessel functions, the three functions $J_M(-A_2B_2M)$, $J_{M+1}(-A_2B_2M)$ and $J_{M+2}(-A_2B_2M)$ approximately attain their extremal magnitudes simultaneously and exhibit alternating signs. Therefore, the phase ϕ should be chosen as π , so that the three dominant terms add constructively.

Based on the analysis above, the modulus of the bunching factor $|b_M|$ can be simplified to

$$|b_M| \approx |J_{M+1}(A_2B_2M)| \cdot \left| J_0(A_1B_2M) \exp\left(-\frac{1}{2}M^2B_2^2\right) + J_1(A_1B_1 - A_1B_2M) \exp\left(-\frac{1}{2}(B_1 - MB_2)^2\right) + J_2(2A_1B_1 - A_1B_2M) \exp\left(-\frac{1}{2}(2B_1 - MB_2)^2\right) \right|. \quad (7)$$

However, this form is still relatively complex, making it difficult to obtain an algebraic expression for its maximum. Therefore, we seek the maximum using numerical methods. Let $F(B_1, B_2)$ denote the absolute value of the inner sum, so that $|b_M| \approx |J_{M+1}(A_2B_2M)| \cdot F(B_1, B_2)$. We calculate the maximum value of F for different values of A_1 , and the results are shown in Fig. 4. For small A_1 , the maximum value of F increases linearly with A_1 , and can be approximated as $F_{max} \approx 1 + 0.303A_1$. As A_1 increases, F_{max} converges to a limiting value of approximately 2.06. Therefore, for large M , the theoretical maximum bunching factor at the M -th harmonic for the novel scheme is approximately given by $1.38/(M+1)^{1/3}$. It is worth noting that this formula is derived under the condition that all parameters can vary freely. In practice, due to limitations in seed laser power and beam slice energy spread, the achievable bunching factor is usually lower than this theoretical maximum.

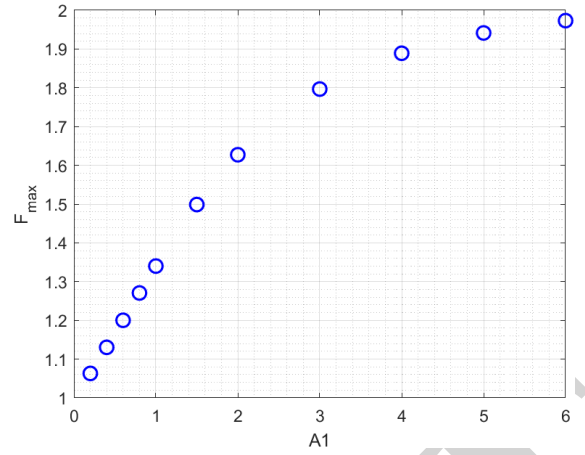


Figure 4: Maximum value of F over B_1 and B_2 as a function of A_1 .

CONCLUSION

In this paper, we reported a novel seeded FEL scheme identified through the intelligent optimization framework in our earlier work. Its layout is the same as that of EEHG, but the component parameters are chosen differently, leading to distinct longitudinal phase space distributions. This scheme can achieve significantly larger bunching factors at the cost of higher final energy spread. We also explained the physical mechanism behind the novel scheme through analytical derivation, and used numerical methods to estimate the theoretical maximum bunching factor. Future work includes refining the theory of the novel scheme, and analyzing its performance and variations under practical parameter constraints.

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