OPTIMIZING A LOW-GAIN FEL OSCILLATOR FOR THE SAPS STORAGE RING: A PARAMETER STUDY*

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Abstract

Achieving higher spectral brightness in fourth-generation light sources like the Southern Advanced Photon Source (SAPS) through integrated free-electron laser (FEL) oscillators presents significant challenges. In response, we present a systematic optimization of the parameters for a low-gain FEL oscillator on the SAPS storage ring. Our work quantitatively evaluates how a transverse gradient undulator (TGU) modifies the ring's equilibrium beam parameters and analyzes the effects of harmonic operation and undulator length on FEL gain. These findings offer essential design guidance for implementing such FEL oscillators in storage rings.

INTRODUCTION

Fourth-generation storage-ring light sources provide high-brightness synchrotron radiation with excellent stability and high repetition rate. However, further improving the spectral brightness, especially in the hard x-ray range, remains an important goal for future light-source development. One possible approach is to integrate a low-gain x-ray FEL oscillator into a storage ring, where coherent radiation can be built up in an optical cavity while preserving the high-repetition-rate nature of the ring-based source [1]. For storage-ring-based FEL oscillators, the relatively large equilibrium energy spread and transverse beam size can strongly limit the single-pass FEL gain. A transverse gradient undulator (TGU), combined with a local dispersion, provides a possible way to improve the transverse overlap between electrons with different energies and the radiation field, thereby enhancing the gain. This concept is particularly attractive for storage rings, but it also introduces additional constraints. When the TGU is installed directly in a storage-ring straight section, the local dispersion and the TGU field can modify the equilibrium beam parameters, especially the vertical emittance. Therefore, the FEL gain optimization must be performed together with an evaluation of the resulting storage-ring equilibrium parameters. In this work, we study a low-gain FEL oscillator based on the SAPS storage ring parameters. Because the SAPS beam energy is 3.5 GeV, harmonic radiation is considered to reach the hard x-ray photon-energy range compatible with high-reflectivity Bragg crystal cavities. We first evaluate the effect of a vertically oriented TGU on the storage-ring equilibrium parameters. We then optimize the main FEL oscillator parameters, including the vertical dispersion, TGU gradient, Rayleigh length, and undulator length. The study identifies the parameter range required to obtain a practical single-pass gain and clarifies the main physics and technical challenges for implementing such an FEL oscillator in SAPS.

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IMPACT OF THE TGU ON THE STORAGE-RING EQUILIBRIUM PARAMETERS

We consider installing the TGU directly in a storage-ring straight section, avoiding the frequent extraction and reinjection required in a bypass configuration. Although operationally simpler, this layout can modify the ring equilibrium parameters, especially the emittance, through the combined effect of the TGU and local dispersion. An analytical treatment, to be reported elsewhere, shows that a vertically oriented TGU in a section with vertical dispersion D modifies the vertical dispersion as

$$\varepsilon_y = C_q\gamma^2 \frac{I_{5,\mathrm{TGU}}}{j_{y,\mathrm{TGU}}\left(I_{2,0}+I_{2,\mathrm{TGU}}\right)}. \tag{1}$$

Here $j_{y,\mathrm{TGU}}$ is given by

$$j_{y,\mathrm{TGU}} = 1 - \frac{I_{4,\mathrm{TGU}}}{I_{2,0}+I_{2,\mathrm{TGU}}}. \tag{2}$$

The TGU contributions to the radiation integrals are

$$I_{2,\mathrm{TGU}} = \frac{B_0^2 L_u}{2(B\rho)^2}, \tag{3}$$

$$I_{4,\mathrm{TGU}} = \frac{B_0^2 L_u}{(B\rho)^2}\left[\alpha\,(D+\eta_0) - \frac{3}{8}\frac{B_0\eta_0}{B\rho}\right], \tag{4}$$

and

$$I_{5,\mathrm{TGU}} = \frac{B_0^3 L_u}{(B\rho)^3}\left[\frac{4}{3\pi}\frac{(D+\eta_0)^2}{\beta_{y0}} - \frac{3}{4}\frac{\eta_0\,(D+\eta_0)}{\beta_{y0}} + \frac{4}{15\pi}\left(\frac{4}{\beta_{y0}}+\beta_{y0}k_u^2\right)\eta_0^2\right]. \tag{5}$$

The parameter η_0 is defined as

$$\eta_0 = \frac{eB_0(\alpha D - 1)}{mc\gamma^2 k_u^2}. \tag{6}$$

This expression allows us to evaluate the contribution of the TGU to the equilibrium emittance. Using the SAPS storage-ring parameters, we then scan the vertical dispersion D and the TGU gradient α to quantify their impact on the beam emittance. Figure 1 shows the resulting dependence of the vertical emittance on D and α.

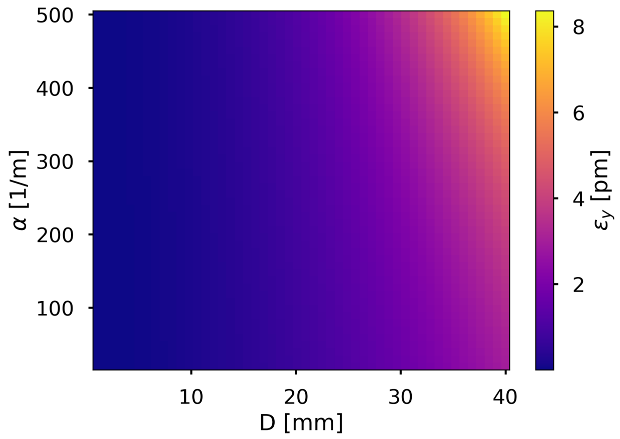

Figure 1: Vertical emittance as a function of the vertical dispersion D and the TGU gradient α for the SAPS storage ring.

PARAMETER OPTIMIZATION OF THE LOW-GAIN FEL OSCILLATOR

Considering the beam energy of the SAPS storage ring, $E_0 = 3.5$ GeV, it is challenging for the fundamental radiation of an FEL oscillator to reach the hard x-ray range with practical undulator parameters. On the other hand, high-reflectivity Bragg crystal cavities are typically designed for photon energies in the several-keV to few-tens-of-keV range. For example, diamond-based x-ray FEL oscillator studies have considered high-reflectivity crystal cavities in the hard x-ray range, with representative photon energies around 10 keV and a feasible range extending up to several tens of keV. Therefore, harmonic radiation is required in the present design. To optimize the oscillator parameters, we use the theory of a low-gain harmonic FEL oscillator [2]. In this framework, the small-signal gain can be expressed as

$$G = -c_h \frac{\int d\eta\, d\mathbf{x}\, d\mathbf{p}\, \bar{F}(\mathbf{x},\mathbf{p},\eta;0)\frac{\partial}{\partial\eta}\left|\mathscr{A}_\nu(\eta,\mathbf{x},\mathbf{p})\right|^2}{\mathscr{N}_\nu}, \tag{7}$$

where

$$\mathscr{A}_\nu(\eta,\mathbf{x},\mathbf{p}) = \int d\phi\, A_\nu^{(0)}(\phi,0)U_\nu^*(\eta,\phi,\mathbf{p},\mathbf{x}), \tag{8}$$

and

$$\mathscr{N}_\nu = \int \left|A_\nu^{(0)}(\phi)\right|^2 d\phi. \tag{9}$$

Here $\bar{F}$ denotes the averaged smooth background distribution of the electron beam. The vertical emittance enters this distribution and is evaluated using Eq. (1). In this study, the undulator period is chosen to be $\lambda_u = 2.26$ cm, and the third harmonic radiation is used. For the SAPS beam energy of 3.5 GeV, this corresponds to an undulator parameter of $K = 1.29$, yielding a radiation wavelength of 0.146 nm, or a photon energy of approximately 8.5 keV.

At this photon energy, high-reflectivity Bragg crystals can be considered for a low-gain x-ray FEL oscillator. In particular, diamond C(422) provides a near-backscattering Bragg reflection around 8.5 keV, making it a promising candidate for constructing a low-loss x-ray optical cavity. A detailed dynamical-diffraction calculation will be required to optimize the reflectivity, bandwidth, angular acceptance, thermal load, and output coupling.

Assuming a single-crystal Bragg reflectivity of $R_c = 0.99$, the round-trip reflectivity of a four-crystal cavity is estimated to be $R_c^4 \simeq 0.96$. The corresponding threshold single-pass FEL power gain is therefore

$$G_{\text{th}} = \frac{1}{R_c^4} - 1 \simeq 4.1\%. \tag{10}$$

When output coupling and other cavity losses are included, a practical design should target a small-signal single-pass gain of at least 8%–12%, preferably around 10% or higher.

The above estimate indicates that the oscillator requires a single-pass gain of roughly 10% or higher to provide a reasonable margin against cavity losses. We therefore examine whether such a gain can be achieved within the available straight-section length and under the equilibrium-emittance constraint imposed by the TGU. The length of the straight section is taken to be 6 m, and the peak current of the electron beam is about 63 A. By scanning different values of the TGU gradient α and the vertical dispersion D, we obtain the small-signal gain shown in Fig. 2. For each data point, the gain is evaluated at the optimized detuning parameter.

As shown in Fig. 2, a relatively large gain is obtained near the curve

$$\alpha = \frac{2+K^2}{K^2 D}. \tag{11}$$

which corresponds to the TGU matching condition. However, the maximum gain is only about $g \simeq 9.5\%$. This value is close to the estimated threshold and may not provide a sufficient margin when output coupling, crystal imperfections, and other optical losses are included.

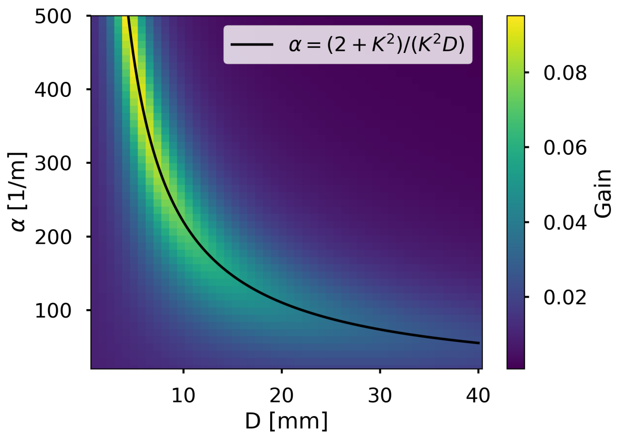

Figure 2: Small-signal gain versus D and α, with μ optimized at each point. The solid black curve shows the TGU matching condition.

To further increase the gain, we optimize other relevant parameters of the oscillator. The optical mode size is an important factor because it affects the transverse overlap between the radiation field and the electron beam. It can be characterized by the Rayleigh length Z_r, which should be chosen consistently with the undulator length L.

For a given L, we scan Z_r and calculate the corresponding small-signal gain. The left panel of Fig. 3 shows the resulting gain map in the (L, Z_r) plane for $D = 0.02$ m, with the TGU gradient α determined from the approximate matching condition discussed above. The results indicate that an optimum Z_r exists for each L, and the optimum value increases with the undulator length.

The optimum Z_r also depends on the vertical dispersion. As shown in the right panel of Fig. 3, a larger D generally requires a larger optimum Z_r. This is expected because a larger dispersion increases the effective vertical beam size, so a larger optical mode size is needed to maintain good transverse overlap between the electron beam and the radiation field.

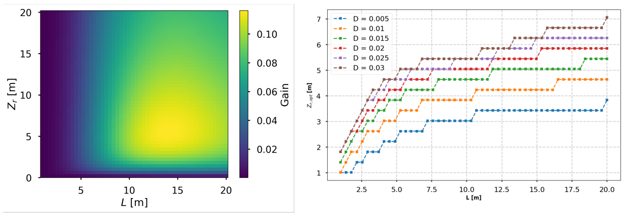

Figure 3: Rayleigh-length optimization. Left: gain map versus L and Z_r for $D = 0.02$ m. Right: optimum Z_r versus L for different D.

With the above optimization, the optimum TGU gradient is determined by D, and the optimum Rayleigh length is determined by L and D. The gain can therefore be treated as an effective two-parameter function, $G(L, D)$, instead of $G(L, D, \alpha, Z_r)$.

A two-dimensional scan over L and D is then performed. At each scan point, α is calculated from Eq. (2), while Z_r is obtained by interpolating the optimized numerical relation $Z_r = f(L, D)$ obtained from Fig. 3. The results are summarized in Fig. 4. Without imposing any constraint on the vertical emittance, the maximum gain reaches about 23%, corresponding to $L = 12$ m. However, this optimum occurs at $D = 5$ mm, for which the corresponding vertical emittance is only $\varepsilon_y \sim 0.12$ pm, a value that is very difficult to realize in practice. If the undulator length is further restricted to about 6 m, the maximum gain is reduced to $\sim$ 10.8%. Although this value is in principle sufficient for oscillator operation, it corresponds to an even smaller vertical emittance, $\varepsilon_y \sim 0.055$ pm, which is essentially unrealistic. To obtain a more practical operating point, we impose a lower limit of 1 pm on the vertical emittance and repeat the scan over D and L. The corresponding results are also shown in Fig. 4, where data points with $G < 0.1$ are excluded for clarity. Under this constraint, the straight-section (or undulator) length needs to be about 10 m or longer, and the preferred vertical dispersion is around 10 mm. The optimum gain in this case is about 15%, obtained at $D = 13.7$ mm and $L = 15$ m. The lower panels of Fig. 4 provide additional one-dimensional views of the optimized solution space. For $D = 13.7$ mm, the gain as a function of L shows that a sufficiently long interaction length is required to reach the 15% level. By contrast, when the undulator length is fixed at $L = 6$ m, the gain remains below about 4.6% for all values of D, indicating that a 6 m straight section is insufficient for this scheme. For the optimum case with $G \simeq 15\%$, the required TGU gradient is about $\alpha \sim 161\ \mathrm{m}^{-1}$. This is a relatively large value and may require superconducting technology for practical implementation.

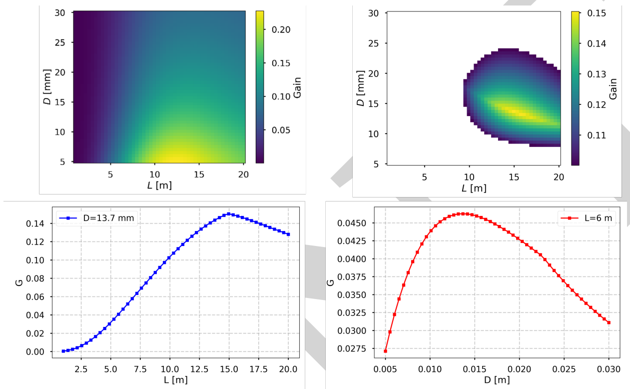

Figure 4: Optimized gain versus L and D. Upper panels: scans without and with the constraint $\varepsilon_y \geq 1$ pm, where only points with $G \geq 0.1$ are shown in the constrained case. Lower panels: gain versus L at $D = 13.7$ mm, and gain versus D at $L = 6$ m.

CONCLUSION

We have optimized a low-gain FEL oscillator for the SAPS storage ring. The results indicate that substantial gain requires a straight section longer than 10 m, with little improvement beyond $L \sim 15$ m. A vertical dispersion of order 10 mm is preferred, implying a large TGU gradient that may require superconducting technology. These requirements highlight both physics and technical challenges for implementation in SAPS, including a long low-emittance straight section, controlled local dispersion, a suitable x-ray cavity, high-reflectivity Bragg crystals, and a high-gradient TGU.

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