FIXED-FIELD ALTERNATING GRADIENT TRANSPORT LINE FOR LASER-PLASMA ACCELERATED ELECTRONS

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Abstract

A transport line for laser-plasma accelerated electrons is proposed based on the Fixed Field Alternating Gradient (FFA) concept. To accommodate the large LPA energy spread and match the beam to a transverse gradient undulator (TGU), the lattice incorporates matching cells with high-temperature superconducting quadrupoles providing gradients of up to 220 T/m, along with a compact FFA dispersion creator. This configuration allows strong focusing and dispersion tailoring within a compact geometry. The study represents an initial step toward demonstrating the feasibility of FFA-based dispersion management for LPA-driven light sources, while outlining the requirements of further optimization.

INTRODUCTION

Accelerators, with dimensions ranging from meters to kilometers, are essential tools for fundamental research as well as for applications in medicine and industry. In recent years, significant effort has been devoted to developing compact accelerator concepts. Laser plasma acceleration (LPA) is one of the most promising approaches, in which an ionized plasma medium sustains electric field gradients up to 100 GeV m^{-1}, far exceeding those of conventional radio-frequency (RF) accelerators.

In LPAs, a high-power terawatt (TW) laser pulse excites a plasma wave, generating a strong longitudinal electric field that accelerates electrons to GeV energies over cm-scale distances, as illustrated in Fig. 1. Moreover, LPAs naturally produce ultra-short electron bunches with micrometer-scale lengths, making them attractive for compact radiation sources and free-electron lasers (FELs) [1].

Figure 1: Schematic sketch of an LPA acceleration process.

Despite these advantages, LPA beams exhibit intrinsic limitations, in particular percent-level energy spread and milliradian divergence, which make beam transport and matching challenging. To efficiently utilize such beams, advanced transport lines are required.

A promising approach combines an optimized transport line with a transverse gradient undulator (TGU). In a TGU, the magnetic field varies transversely, allowing electrons with different energies to emit radiation at the same wavelength by traversing different transverse positions. The resonance condition is given by

$$\lambda = \frac{\lambda_u}{2\gamma^2(x)}\left(1 + \frac{K_u^2(x)}{2}\right), \tag{1}$$

with λ the radiation wavelength, λ_u the undulator period, γ the electron's Lorentz factor, and K_u the undulator parameter, the latter two being matched as a function of transverse position x, which enables narrow-bandwidth radiation even for beams with large energy spread [2, 3]. At KIT, several compact transport lines have been developed for LPA applications at energies between 120 MeV and 700 MeV, using both normal-conducting and high-temperature superconducting (HTS) magnets [4–6]. These studies demonstrate the feasibility of compact transport systems, while highlighting the need for improved solutions capable of handling large energy spread with high acceptance.

Fixed Field Alternating Gradient (FFA) concepts offer a promising approach for high-acceptance beam transport. Originally developed for hadron accelerators [7–9], FFA optics have more recently been extended to transport lines [10, 11]. FFA lattices combine static magnetic fields with alternating-gradient focusing, providing stable transport over a wide momentum range. The magnetic field, in the case of a circular accelerator, is characterized by a geometrical field index K and defined as

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^K F(\theta). \tag{2}$$

where B_0 is the vertical field at reference radius r_0 and $F(\theta)$ is a periodic variation with azimuthal angle θ. In transport lines, in the limit of large reference radius r_0, one can write

$$B_y(x,\theta) \approx B_{y0}\exp\left(mx\right)F(\theta), \tag{3}$$

where $m = K/r_0$ is the normalised field index. FFAs provides controlled orbit separation and large momentum acceptance [10].

In this work, an FFA-based transport line is proposed for LPA electron beams at a reference energy of 300 MeV. By combining a compact FFA dispersion creator with strong focusing provided by HTS quadrupoles, the design aims to generate the required dispersion and match the beam to a TGU within a compact geometry.

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LATTICE CONCEPT

The transport line is designed to capture and manipulate laser-plasma accelerated (LPA) electron beams at a reference energy of 300 MeV and match them to the requirements of a transverse gradient undulator (TGU). The design follows a modular approach, consisting of two matching sections and a central dispersion-generating stage based on a scaling FFA cell. The lattice concept builds upon the FFA principles introduced in the previous section. At the beamline entrance, the LPA beam is characterized by a small transverse size, vanishing dispersion, and large divergence. The initial beam parameters and the target matching conditions at the TGU entrance are summarized in Table 1. Efficient capture of such beams requires strong focusing immediately after the source. The optical design of both matching cells is performed using ELEGANT and OPA [12, 13] while the performance of the dispersion creator is evaluated using PyZgoubi [14], ensuring a smooth transition from the source to the FFA section and from the FFA section to the TGU.

Table 1: Initial LPA Beam Parameters and Target Matching Conditions at the TGU Entrance

Parameter	**LPA Source**	**TGU Matching**
Beam size $\sigma_{x,y}$	4 μm	–
β_x	1.6 mm	0.2 m
β_y	1.6 mm	0.6 m
α_x	0	0
α_y	0	0
Dispersion D	0	20 mm
Dispersion slope D'	0	≈ 0

Matching Cell I

The first matching cell adapts the LPA beam to the acceptance of the dispersion section. It is based on a doublet of high-temperature superconducting (HTS) periodic quadrupoles, providing strong focusing in both transverse planes within a compact footprint. The magnets consist of a sequence of pancake coils with alternating current directions, forming a periodic focusing structure, as illustrated in Fig. 2. The use of ReBCO-based HTS technology enables gradients of up to 220 T/m, allowing efficient capture of the highly divergent LPA beam [15]. The evolution of the beta functions is shown in Fig. 3, demonstrating strong focusing and effective beam confinement.

FFA Dispersion Creator

The central element of the lattice is a compact four-cell dispersion creator based on a scaling FFA triplet cell in an F–D–F configuration. The design follows the scaling FFA concept introduced earlier, where the vertical magnetic field obeys a power-law dependence on radius, ensuring stable optics over a wide momentum range.

Each cell consists of three combined-function magnets with magnetic lengths of approximately 5 cm, separated by 3 cm drift spaces, forming a quasi-straight transport section.

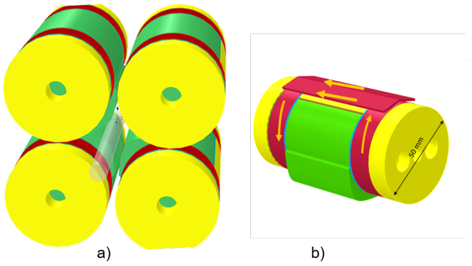

Figure 2: Schematic of the HTS periodic quadrupole: (a) full magnet; (b) one quarter showing alternating current directions in the pancake coils.

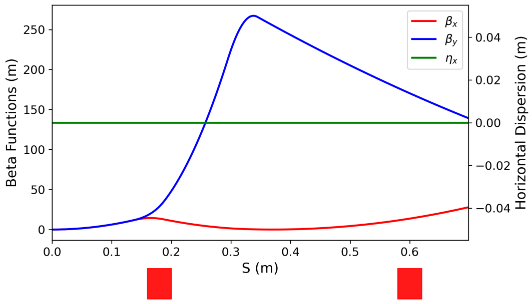

Figure 3: Horizontal and vertical beta functions (β_x, β_y) along Matching Cell I and the horizontal dispersion η_x.The red rectangles indicate quadrupole magnet.

A large reference radius of $R = 10^5$ m is employed to approximate a straight geometry while preserving the scaling properties of the FFA field. The reference magnetic field strengths are approximately +0.9 T in the focusing magnets and −1.4 T in the defocusing magnet, with a field index of $K = 50$. Due to the scaling law $B(r) \propto r^K$, the effective magnetic field experienced along the orbit varies with particle momentum. Consequently, the local field amplitude along the trajectory reaches values of up to ~ 2 T along the orbit. The magnetic field variation along the beam trajectory for different momentum offsets is shown in Fig. 4. The figure illustrates the characteristic F–D–F field structure and the dependence of the field amplitude on the orbit displacement due to the scaling FFA profile. In contrast to conventional dipole-based dispersion sections, the FFA approach provides continuous focusing and large momentum acceptance. Particles with different energies follow distinct but stable trajectories, naturally generating an energy-dependent orbit separation. This enables efficient dispersion generation within a compact footprint.

A configuration with a total length of approximately 1 m is obtained while maintaining sufficient orbit separation for the considered energy spread. The resulting orbits for different momentum deviations are shown in Fig. 5, while the corresponding optical functions and dispersion evolution are presented in Fig. 6.

The dispersion creator provides a horizontal dispersion of approximately 20 mm at the exit, suitable for matching to the downstream TGU. The dispersion was evaluated from the

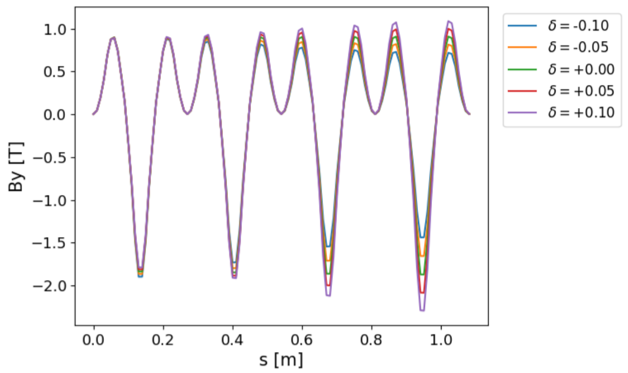

Figure 4: Vertical magnetic field B_y along the longitudinal coordinate s in the FFA-based dispersion creator for different momentum deviations δ.

momentum-dependent orbit separation obtained by multi-momentum particle tracking. Over a momentum range of $\Delta p/p \approx \pm 10\%$, stable beam transport and smooth orbit separation are maintained, while small variations of the optical functions remain consistent with the expected behavior of scaling FFA dispersion creator/suppressor sections [10].

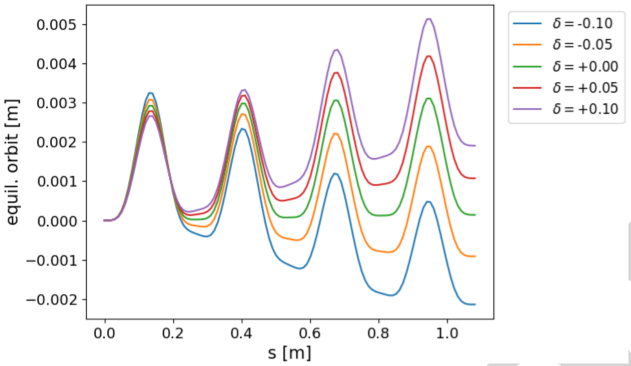

Figure 5: Particle trajectories in the FFA-based dispersion creator for different relative momenta δ.

Matching Cell II

The second matching cell adapts the beam exiting the dispersion creator to the TGU requirements. This section is again based on HTS periodic quadrupoles. It consists of four quadrupoles, providing gradients in the range of 60–180 T/m at a reference energy of 300 MeV, enabling strong and flexible focusing within a compact footprint. In particular, the beta functions and dispersion are adjusted to reach the target values at the undulator entrance, while maintaining a small dispersion slope ($D' \approx 0$). This is essential to preserve the energy-position correlation required for efficient interaction with the transverse gradient field in the TGU. The resulting optical functions are shown in Fig. 7, demonstrating matching of the beam parameters to the downstream requirements for the reference energy.

CONCLUSION

A compact transport line for laser–plasma accelerated (LPA) electron beams has been presented, combining HTS-based periodic quadrupoles with a scaling FFA cell for dispersion generation. Within a length of about 2.8 m, the system provides strong focusing, controlled dispersion, and large momentum acceptance for beams with substantial energy spread. This work demonstrates the potential of FFA-based dispersion management for compact LPA-driven light sources, while future studies may explore FFA-based matching optics to further improve broadband transport performance.

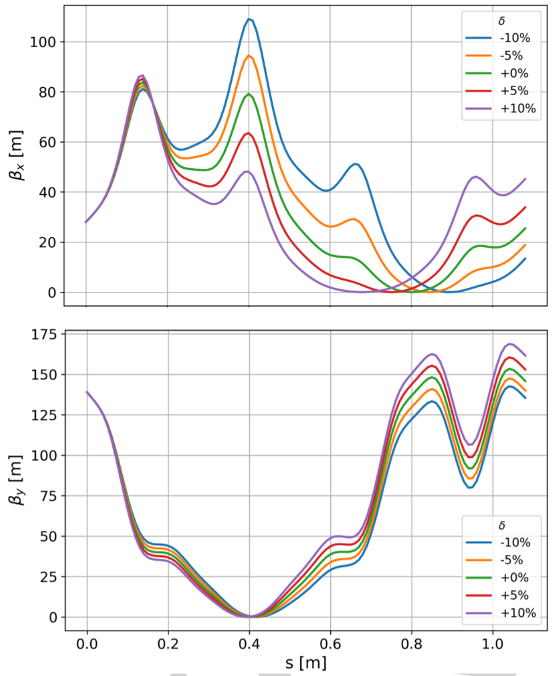

Figure 6: Evolution of beta functions along the FFA dispersion section.

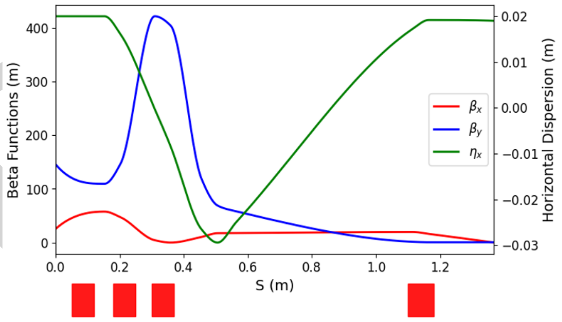

Figure 7: Optical functions along Matching Cell II.

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