SYMPLECTIC INTEGRATION FOR STORAGE RING BASED FEL IN GENESIS*

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Abstract

Symplectic integration is essential for simulating the long-term stability of systems such as storage rings. In contrast, it has traditionally been considered unnecessary for single-pass free-electron laser (FEL) simulations. Consequently, standard FEL simulation codes, like Genesis, have not fully incorporated symplectic methods in their treatment of the beam dynamics. However, with the growing interest in storage ring-based FELs, the requirement for symplectic tracking in the FEL dynamics process has emerged. This paper presents a modification of the Genesis code to implement a symplectic algorithm for its dynamical part. A comparative analysis between the symplectic and non-symplectic approaches is conducted. The results demonstrate that the symplectic Genesis preserves the symplecticity of beam dynamics without compromising the accuracy of the radiation simulation, thereby providing a more reliable tool for accurate storage ring FEL modeling.

INTRODUCTION

Symplecticity is a fundamental property of long-term conservative dynamical systems, such as the single-particle dynamics in storage rings. In such systems, symplectic integration schemes are essential for suppressing nonphysical numerical artifacts during long-term tracking. By contrast, symplecticity is usually less critical for short-term dynamical processes, such as single-pass free-electron laser (FEL) amplification, where the electron beam typically experiences the FEL interaction only once and long-term accumulation effects are absent. For this reason, most FEL simulation codes do not explicitly require symplectic integration. A representative example is the widely used GENESIS code [1].

However, with the increasing demands of advanced photon science, the limited temporal resolution and moderate single-pulse photon flux of storage-ring light sources have become bottlenecks for many experiments. FELs can provide high single-pulse flux and temporal resolution down to the attosecond regime [2], but their number of user beamlines and operational stability are still difficult to match those of storage-ring light sources. Combining storage-ring operation with FEL-type radiation processes may therefore provide a promising route toward next-generation light sources [3–6].

To study the beam dynamics in such hybrid systems, a symplectic treatment of the FEL interaction becomes important. In this work, we present a symplectic improvement of the particle integration scheme based on GENESIS. The details of the proposed modification are described in the next section. Benchmark comparisons between the original and symplectic schemes are presented in Sec. 3. Finally, conclusions are given in the last section.

SYMPLECTIC MODIFICATION

In GENESIS, the FEL interaction is modeled by solving the coupled Maxwell equations and Lorentz force equations [1]. The coupling between the electromagnetic field and the particles is treated using a leapfrog integration scheme, which is symplectic. The equations of motion are separated into transverse and longitudinal parts. The transverse coordinates are advanced by first-order matrix multiplication, which also preserves symplecticity. However, the longitudinal variables, namely the particle energy and longitudinal position, or equivalently the ponderomotive phase, are integrated using a fourth-order Runge–Kutta method. Since the Runge–Kutta scheme is not symplectic in general, this part may break the symplectic structure of the full FEL tracking.

To formulate the entire FEL interaction within a symplectic framework, the Runge–Kutta integrator is replaced by a symplectic scheme. In this work, we adopt the Yoshida integrator [7], which has been widely used in accelerator tracking codes such as AT and ELEGANT [8]. The Yoshida scheme provides fourth-order accuracy by composing several second-order symplectic maps with appropriately chosen coefficients. The corresponding pseudocode is given below.

Algorithm 1 Symplectic Yoshida 4th-order Step

Require: Step size dz

$w_1 \leftarrow 1.35120719195966$
$w_2 \leftarrow -1.70241438391932$
SYMPLECTIC2ORDERSTEP($w_1 \cdot dz$)
SYMPLECTIC2ORDERSTEP($w_2 \cdot dz$)
SYMPLECTIC2ORDERSTEP($w_1 \cdot dz$)

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Algorithm 2 Symplectic 2nd-order Step

Require: Step size $delz$, variables γ, θ

{Half step for γ}
$dH/d\theta \leftarrow \partial H/\partial\theta(\gamma,\theta)$
$\gamma \leftarrow \gamma - 0.5 \cdot delz \cdot (dH/d\theta)$
{Full step for θ}
$dH/d\gamma \leftarrow \partial H/\partial\gamma(\gamma,\theta)$
$\theta \leftarrow \theta + delz \cdot (dH/d\gamma)$
{Half step for γ again}
$dH/d\theta \leftarrow \partial H/\partial\theta(\gamma,\theta)$
$\gamma \leftarrow \gamma - 0.5 \cdot delz \cdot (dH/d\theta)$

To illustrate the numerical behavior of the two integration schemes, we first consider a simple Hamiltonian system,

$$H = \frac{p^2}{2} - \cos(\theta) \ . \tag{1}$$

This system represents a nonlinear oscillator, for which the Hamiltonian should be conserved. By solving the corresponding single-particle equations of motion, the phase-space trajectories can be obtained.

As shown in Fig. 1, when a relatively large integration step size is used, the Yoshida scheme preserves the Hamiltonian more accurately than the fourth-order Runge–Kutta method. In contrast, the RK4 scheme exhibits a nonphysical damping of the momentum p, indicating the loss of the symplectic structure. When the integration step size is reduced, both schemes show good Hamiltonian conservation over the same final integration time T. However, from a theoretical point of view, if the integration time is further extended, the non-symplectic nature of RK4 will inevitably lead to long-term numerical artifacts [9].

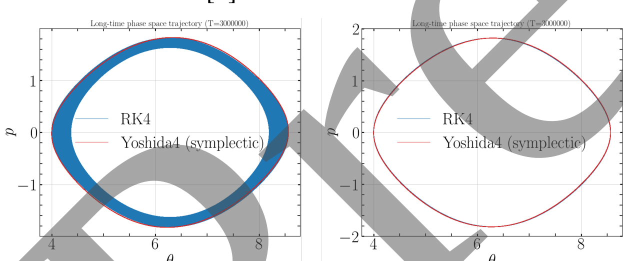

Figure 1: Comparison of phase-space trajectories for different integration step sizes. The left plot uses a step size of 0.1, while the right plot uses a step size of 0.03. Both cases are tracked to the same final time T.

In GENESIS4 [10], the standard simulation workflow is based on a single pass through a specified undulator. To evaluate the long-term behavior of different integration schemes, however, the beam needs to be tracked through the undulator repeatedly. A straightforward approach is to export the particle distribution after each pass and then restart a new GENESIS4 simulation using the exported beam file. However, this procedure is computationally inefficient and introduces significant input/output overhead.

To enable more convenient multi-pass tracking, we take advantage of the modular structure of GENESIS4 and its ability to execute all modules sequentially as defined in the input file. A Python script is developed to automatically generate a GENESIS4 input file containing multiple repeated simulation blocks. In this way, the beam can be tracked through the same undulator section for many passes within a single GENESIS4 run. The corresponding pseudocode is shown below.

Algorithm 3 Generate Genesis Multi-pass Input File

Input: filename, beam_folder, beam_interval, num_steps, save_field, reload_field_each_step
Output: Genesis input file
Open file `filename` for writing
Write setup, lattice, and initial beam import
if *reload_field_each_step* = *false* **then**
  Write initial field block
end
for $i \leftarrow 1$ **to** *num_steps* **do**
  if *reload_field_each_step* = *true* **then**
    Write field block for step i
  end
  Write track block for step i
  if (i mod $beam\_interval = 0$) ***or*** ($i = num\_steps$) **then**
    Write beam output: `beam_folder/beam_pass_`i
    if *save_field* = *true* **then**
      Write field output: `field_pass_`i
    end
  end
end
Close file
Print "Genesis input file generated"

This input file enables multi-pass tracking through the undulator within a single GENESIS4 run, without repeatedly exporting and re-importing the particle distribution.

RESULTS COMPARISON

The Yoshida algorithm described in the previous section was implemented as a new Yoshida function and used to replace the RungeKutta function in the GENESIS4 source file BeamSolver.h. This modification enables symplectic longitudinal particle tracking in GENESIS4.

For each pass through the undulator, the radiation field was regenerated using the `&field ... &end` module. This treatment ensures consistent boundary conditions, so that the electron beam interacts with the same initial radiation field in each undulator pass.

The results obtained with the symplectic and non-symplectic schemes are compared for two different integration step sizes. As shown in Fig. 2, when a relatively large step size is used, the advantage of the Yoshida scheme becomes evident: the particle energy does not exhibit the nonphysical damping observed with the original Runge–Kutta method. When a smaller step size is used, both schemes maintain good Hamiltonian conservation over the simulated tracking range.

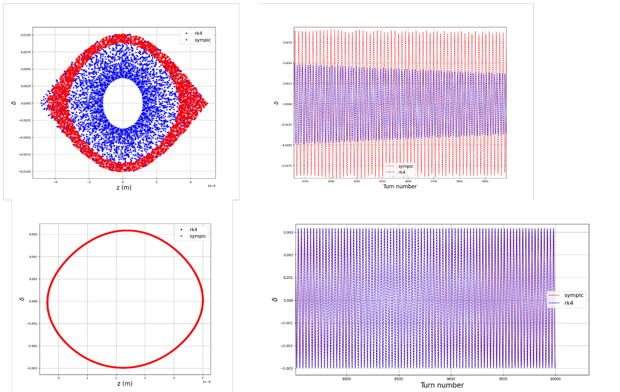

Figure 2: Comparison of the symplectic Yoshida scheme and the original RK4 scheme using different integration step sizes. The upper two panels correspond to the case with a larger step size, while the lower two panels correspond to the case with a smaller step size.

We further compare the radiation output obtained with the original RK4 scheme and the fourth-order symplectic scheme for a single pass through a 57-m-long undulator. As shown in Fig. 3, the output radiation power and related quantities are consistent in order of magnitude for the two algorithms. Minor differences are observed in the temporal profiles and spectra.

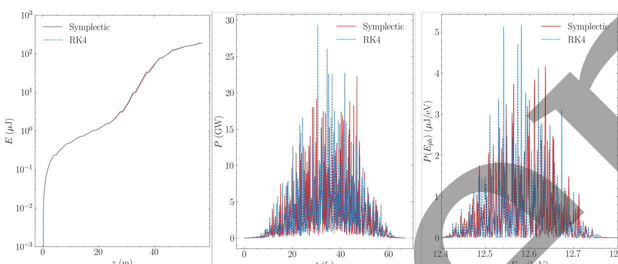

Figure 3: Comparison of FEL radiation output obtained with the RK4 and fourth-order symplectic schemes.

CONCLUSION

A fourth-order symplectic integration scheme has been implemented in GENESIS4 for storage-ring-based FEL simulations. The original Runge–Kutta integrator for the longitudinal particle motion was replaced by a Yoshida symplectic integrator. Benchmark studies with a simple Hamiltonian system show that the symplectic scheme provides improved long-term conservation properties and suppresses the non-physical damping observed with RK4 for relatively large integration step sizes.

A multi-pass tracking method was also developed by automatically generating repeated GENESIS4 input modules, avoiding repeated export and import of particle distributions between successive undulator passes. The GENESIS4 benchmark confirms that the symplectic scheme improves the long-term tracking behavior, while giving single-pass FEL radiation outputs comparable to those obtained with the original RK4 scheme. These results demonstrate that the proposed modification provides a useful numerical framework for studying storage-ring-based FEL processes.

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