STORAGE RING NONLINEAR OPTICS CORRECTION BASED ON CLOSED ORBIT BUMP

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Abstract

This paper presents a new method for nonlinear optics correction of storage ring lattices, which is based on locally generated orbit bumps and the corresponding response matrices. This method is conceptually similar to nonlinear optics from off-energy closed orbits (NOECO), but can be applied to the situation with harmonic sextupoles. Instead of off-energy closed orbits, it exploits the feed-down effects of sextupoles with generated local bumps to directly probe the sextupole components of a lattice. A second-order bump response matrix is constructed from the difference between orbit response matrices measured with positive and negative local bumps. A least-squares fitting procedure is then applied to minimize the difference between measured and model second-order bump response matrices, from which sextupole strength correction can be obtained. Preliminary simulations on the Hefei Advanced Light Source lattice demonstrate the theoretical viability of the method, though high sensitivity to realistic residual errors poses practical challenges that require further investigation.

INTRODUCTION

The advent of fourth-generation storage rings, characterized by multi-bend achromat (MBA) lattices, has pushed the requirement for precise tuning of nonlinear magnets to unprecedented levels. In these ultra-low emittance machines, strong sextupole and octupole magnets introduce strong nonlinearity and have a significant impact on the injection and beam lifetime [1]. Consequently, establishing an effective and precise correction scheme for nonlinear optics to restore the designed performance is of critical importance.

Currently, several established methods exist for characterizing nonlinear optics. Techniques based on extracting information from turn-by-turn beam position data, such as amplitude-dependent tune shifts, frequency maps, and resonance driving terms, can be powerful but may be limited by beam decoherence when operating at non-zero chromaticity. The LOCO method was successfully used for linear optics correction [2], while an off-energy closed orbit based method named NOECO (Nonlinear Optics from Off-Energy Closed Orbits) was proposed for the nonlinear correction [3, 4] and successfully applied at the MAX IV and NSLS II storage rings [3–5]. NOECO fits a lattice model to a measured off-energy orbit response matrix (OEORM), taking advantage of the chromatic gradient feed-down effect. However, NOECO inherently relies on dispersion to induce off-axis orbits within sextupoles. It is therefore insensitive to harmonic sextupoles located in dispersion-free regions, limiting its universality for lattices that heavily rely on harmonic sextupoles for nonlinear optimization.

Inspired by the methods of LOCO and NOECO, we propose a new nonlinear correction method, i.e., nonlinear optics from closed orbit bump, which utilizes the second-order orbit response matrix with local bumps. By introducing local closed orbit bumps at the locations of sextupoles, the feed-down effects can be exploited to calibrate the sextupole strengths regardless of the local dispersions. In this paper, the idea and framework of the nonlinear correction method are introduced. Preliminary simulations are applied to the Hefei Advanced Light Source (HALF) lattice to demonstrate the procedure, followed by a discussion of the method's limitations in the presence of realistic imperfections.

METHOD

Feed-Down Effect from Orbit Bumps

The proposed method is directly analogous to NOECO but replaces the momentum deviation δ with a spatial displacement generated by a local orbit bump. When the beam closed orbit has a transverse offset Δx from the magnetic center of a sextupole, the beam experiences a feed-down quadrupole gradient given by

$$\Delta k(s) = b_2(s)\Delta x(s), \tag{1}$$

where s is the longitudinal location, $\Delta k(s)$ is the effective quadrupole gradient, and $b_2(s)$ is the sextupole strength. Unlike the off-energy closed orbit shift $\delta x(s) = \eta(s)\delta$, where $\eta(s)$ is the dispersion and δ is the energy deviation, the local closed orbit bump $\Delta x(s)$ can be arbitrarily introduced at any sextupole location based on the orbit response matrix (ORM), granting the ability to be applied to both chromatic and harmonic sextupoles.

Provided that the linear optics has been restored, this feed-down quadrupole field perturbs the linear optics and subsequently the ORM, which can be utilized to probe the strengths of sextupoles.

Construction of Local Bump Orbit Response Matrix

Similar to the OEORM method, a symmetrization with a 2nd order orbit response matrix is utilized to isolate the linear dependence of the ORM on the sextupole strengths. The local bump orbit response matrix (LBORM) is constructed by taking the difference between two ORMs measured at two local closed orbit bumps of equal amplitude but opposite directions.

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To avoid the introduction of coupling and simplify the analysis, only the horizontal closed orbit bump is considered. Let $M_{ORM,+\Delta x}$ be the ORM measured when a positive local orbit bump $+\Delta x$ is applied for a specific sextupole (or a family of sextupoles), and $M_{ORM,-\Delta x}$ be the ORM measured with a symmetric negative bump $-\Delta x$. The LBORM is then defined as

$$M_{LBORM} = \frac{M_{ORM,+\Delta x} - M_{ORM,-\Delta x}}{2\Delta x}, \quad (2)$$

or equivalently

$$(x_{co,+\Delta x} - x_{co,-\Delta x}) = 2\Delta x \cdot M_{LBORM} \cdot d\theta_x, \quad (3)$$

where M_{LBORM} is the local bump orbit response matrix, Δx is the magnitude of the local bump, and $x_{co,\pm\Delta x}$ are the orbit responses when a kick $d\theta_x$ is applied in the positive bump and negative bump cases, respectively.

By taking this difference, contributions independent of the spatial offset cancel out, leaving a second-order response matrix that is approximately linear with respect to the sextupole field strengths.

Fitting Procedure

To determine the actual sextupole strengths in the machine, a lattice model is iteratively fitted to the measured LBORM. This is achieved via a least-squares minimization utilizing a penalty function

$$\chi^2 = \sum_{i,j} \frac{(M_{LBORM,meas,ij} - M_{LBORM,model,ij})^2}{\sigma_i^2}, \quad (4)$$

where σ_i represents the noise level or weighting factor of the i-th BPM.

The minimization can be performed using a non-linear least-squares algorithm, with the sextupole strengths as the fitting parameters. The method updates the simulated sextupole settings until the model's LBORM matches the measured LBORM, indicating that they share the same linear and nonlinear optics. The difference between the nominal and fitted sextupole values yields the requisite correction factors to restore the machine's nonlinear optics.

SIMULATION

To validate the viability of the LBORM method, simulations were performed on the HALF storage ring lattice [6,7]. HALF is a fourth-generation synchrotron light source under construction, featuring an H6BA lattice with strong chromatic sextupoles. The linear optics and magnet layout of a lattice cell of the HALF storage ring are shown in Fig. 1.

Ideal Case Verification

Initial tests were performed in the HALF lattice with ideal linear optics and without measurement noise and component alignment errors. A set of random relative strength errors with a normal distribution with $2\sigma = 0.01$ was intentionally introduced into all sextupoles of the HALF lattice. Then the nonlinear fitting and correction procedure was carried out as follows:

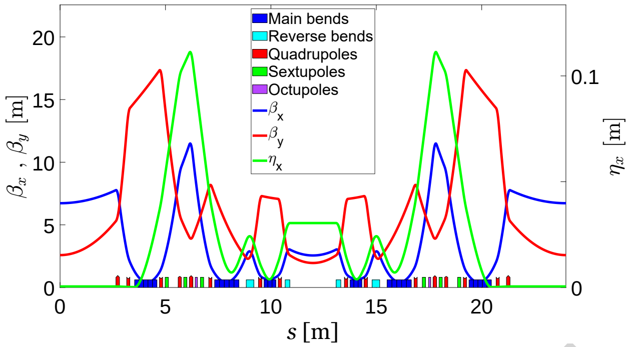

Figure 1: Linear optics and magnet layout of a lattice cell of the HALF storage ring.

- Use the nominal ORM to create a local orbit bump in the horizontal direction with a maximum amplitude of 100 μm at each sextupole location in the model lattice, and record the horizontal kick settings θ_x used. The closed orbit bump in one lattice cell is shown in Fig. 2.
- Calculate $M_{LBORM,model}$ with the two closed orbit bumps introduced by θ_x and $-\theta_x$.
- Calculate $M_{LBORM,meas}$ of the error lattice, with the closed bumps introduced by the same kicks θ_x and $-\theta_x$.
- Minimize the difference between $M_{LBORM,meas}$ and $M_{LBORM,model}$, with sextupole strengths as the fitting parameters. The Levenberg-Marquardt method was used in this procedure.
- Apply the fitted corrections to the sextupole strengths in the error lattice.

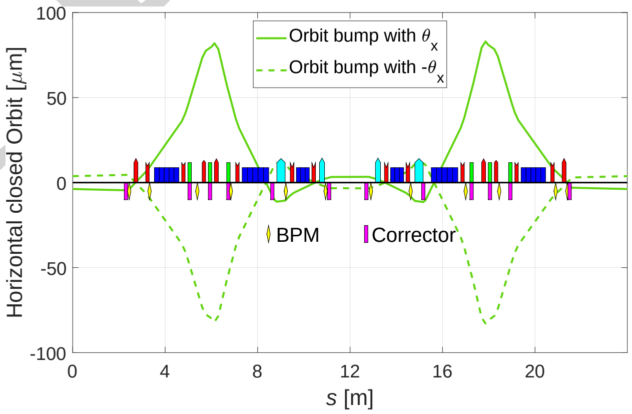

Figure 2: Horizontal local closed orbit bumps created with $\pm\theta_x$ in one lattice cell. The other lattice cells are symmetric as this one.

After two iterations with 30 error seeds, the sextupole strengths converged back to their nominal values quite well, as shown in Fig. 3. Under these ideal conditions, the method demonstrated excellent convergence, effectively proving that this framework of the nonlinear correction method is theoretically viable.

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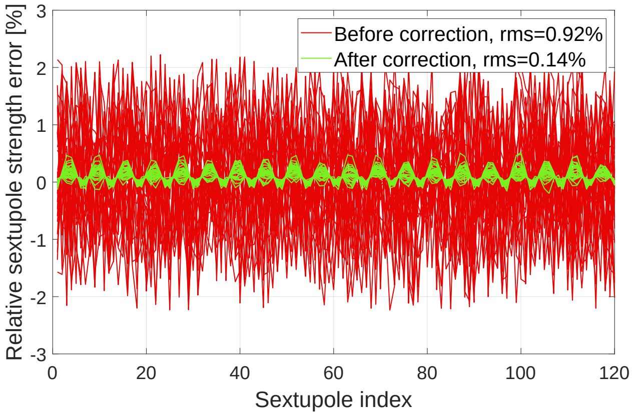

Figure 3: Residual errors in sextupole strengths after the fitting and correction procedure for two iterations with 30 error seeds.

Impact of Practical Imperfections

Despite the theoretical effectiveness of this method, its implementation in a real machine is constrained by the precision of orbit bump generation and measurement accuracy. To emulate realistic machine conditions, the alignment errors, magnet strength errors and BPM reading errors were all introduced in the subsequent simulations. The sextupole strengths after fitting and correction failed to converge.

The results with practical imperfections indicated some limitations of the method. The main factor affecting the fitting result is the precision of the created local bumps. With all magnet and alignment errors introduced, the local closed orbit bumps in the lattice model and the imperfect machine can be significantly different. The created bump Δx at the locations of sextupoles can deviate from the expected values in the model, especially when there is a comparable residual closed orbit distortion. This difference in orbit bumps can result in a discrepancy in M_{LBORM}, and thus break the fitting convergence. Although it was found that the impact of residual errors could be somewhat mitigated by increasing the magnitude of the orbit bump (Δx), a large closed orbit bump will drastically increase the nonlinearity of the lattice and break the linearity needed for the fitting. Furthermore, the strong feed-down effects from the large closed orbit can also affect the precision of the created orbit bump.

This indicates that the success of the new method strictly relies on a well-calibrated linear lattice and highly accurate beam-based alignment of the BPMs to the sextupole magnetic centers prior to the nonlinear optics measurement.

CONCLUSION

In this paper, a new method for characterizing and correcting nonlinear optics was introduced using the local bump orbit response matrix. Conceptually inheriting the idea of the NOECO method, the new method constructs the second-order orbit response matrix by introducing local orbit bumps instead of energy deviations, which can probe and correct both chromatic and harmonic sextupoles.

Simulations applied to the HALF lattice confirmed that under ideal conditions, the method can precisely fit and correct sextupole field errors, proving its potential to help restore the nonlinear performance of MBA lattices. However, preliminary simulations with realistic imperfections revealed a significant vulnerability to residual errors in the previous orbit and linear optics corrections, which can distort the actual amplitude of the closed orbit bumps at the sextupole locations and induce fitting failures. Ongoing and future work will focus on a more comprehensive consideration of the signal-to-noise ratio and more tailored parameter choices for the appropriate bump generation and robust least-squares fitting.

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