

# THE ESSENCE FACTOR TO DETERIORATE THE CIRCULAR POLARIZATION RADIATION PERFORMANCE IN APPLE-KNOT UNDULATOR\*

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## Abstract

The APPLE-KNOT undulator forms composite magnetic fields by superimposing APPLE and KNOT fields with different period lengths. In this configuration, in which the APPLE field serves as the dominant component to approximate the target photon energy, while the KNOT field acts as an additional component to transversely deflect the electron beam off-axis. Although variable polarization modes can be realized with a low on-axis heat load, previous studies have observed a sharp reduction in flux and significant degradation of the polarization degree in the circular polarization (CP) mode. This paper discusses this phenomenon in detail from a theoretical perspective. The analysis reveals that the presence of an additional field with a longer period is the essence factor that inherently suppresses the radiation performance of CP mode. Theoretical findings are highly consistent with simulation results, demonstrating that selecting the KNOT field as the dominant component can effectively improve CP characteristics without significantly compromising the linear polarization performance.

## INTRODUCTION

Various types of undulators have been developed to meet different experimental requirements. The APPLE-type undulator is well known as the light source for generating variable polarization. The most conventional polarization-tunable undulator is the APPLE-II undulator [1]. However, low photon energy applications in high-energy storage rings lead to substantial on-axis heat load in linear polarization modes. To address this issue, the APPLE-KNOT(AK) undulator combines APPLE and KNOT arrays with a period ratio of 2:3 to achieve variable polarization while maintaining low on-axis heat load [2]. Subsequent developments including four-row merged configurations have been implemented at various facilities [3, 4]. Despite these advances, a common observation across AK undulator designs is that the circular polarization (CP) mode exhibits lower photon flux and polarization degree compared to linear modes. This phenomenon has been noted in both simulation studies and practical implementations [5], yet the underlying mechanism has not been systematically analyzed. Understanding this degradation is essential for optimizing future undulator designs for

beamline requiring high-performance circularly polarized radiation.

This paper theoretically investigates the essence factor that deteriorates CP mode radiation performance in AK undulators. Analysis of the composite magnetic field structure reveals that the presence of an additional field with a longer period is the fundamental cause. Theoretical findings are validated through numerical simulations based on Hefei Advanced Light Facility (HALF) parameters [6] and further verified using High Energy Photon Source (HEPS) parameters [7]. The end magnet design and field integral analysis for a novel APPLE-KNOT undulator developed for HALF are also presented to demonstrate its engineering feasibility. The detailed design has been reported elsewhere [8].

## PRINCIPLES

In theory, the composite magnetic field of AK undulator at circular polarization mode can be simplified as

$$\begin{aligned} B_y &= -B_{strong} \sin \frac{2\pi}{\lambda_1} z - B_{weak} \cos \frac{2\pi}{\lambda_2} z \\ B_x &= B_{strong} \cos \frac{2\pi}{\lambda_1} z - B_{weak} \sin \frac{2\pi}{\lambda_2} z \end{aligned} \quad (1)$$

where  $B_{strong}$  and  $B_{weak}$  are the peak fields corresponding to the period lengths of  $\lambda_1$  and  $\lambda_2$ , respectively. With  $B_{strong} = \alpha B_{weak} = \alpha B_0$ , and  $\lambda_1 = m\lambda_2 = m\lambda_0$  ( $\alpha > 1$ ,  $m > 0$ , and  $m \neq 1$ ), we have

$$\begin{aligned} B_y &= -B_0 \left( \alpha \sin \frac{2\pi}{m\lambda_0} z + \cos \frac{2\pi}{\lambda_0} z \right) \\ B_x &= -B_0 \left( -\alpha \cos \frac{2\pi}{m\lambda_0} z + \sin \frac{2\pi}{\lambda_0} z \right). \end{aligned} \quad (2)$$

In the traditional AK undulator (Case A), the APPLE array (shorter period) serves as the strong field and the KNOT array (longer period) as the weak field, corresponding to  $m = 2/3$ . In contrast, the new proposed configuration (Case B) selects the KNOT array as the strong field, yielding  $m = 3/2$ . The following analysis compares two cases in HALF. Using parameters listed in Table 1, with the same period and field strength ratio, to identify the origin of CP mode degradation using far-field approximation:

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} &= \frac{N^2 e^2 \omega_u^2}{(4\pi \epsilon_0) 4\pi^2 c^3} \times \\ &\left| \int_{-\frac{\lambda_u}{2\beta c}}^{\frac{\lambda_u}{2\beta c}} [n \times (n \times \beta)] e^{i\omega(t - \frac{nr}{c})} dt \right|^2 L(N\Delta\omega/\omega_1(\theta)) \end{aligned} \quad (3)$$

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Table 1: Design Parameters of the Electron Beam in the Middle of a Long Straight Section of HALF

Parameter	Specification
Beam energy (GeV)	2.2
Average current (mA)	350
Horizontal / Vertical emittance (pm-rad)	176.4/13.2
Horizontal / vertical beta function (m)	6.834/2.536
Horizontal dispersion function	0.0005

Figure 1 compares the normalized photon flux as a function of photon energy for three field strength ratios ( $\alpha=1.5$ , 2 and 4). The photon energy is scanned by changing the field strength. For all  $\alpha$  values, Case B consistently yields higher flux than Case A across the entire energy range, with the advantage becoming more pronounced as  $\alpha$  increases.

Figure 2 further illustrates the flux dependence on  $\alpha$  at a fixed photon energy of 6 eV. The flux increases monotonically with  $\alpha$ . When  $\alpha$  exceeds 10, the AK undulator asymptotically approaches an APPLE-II undulator, where

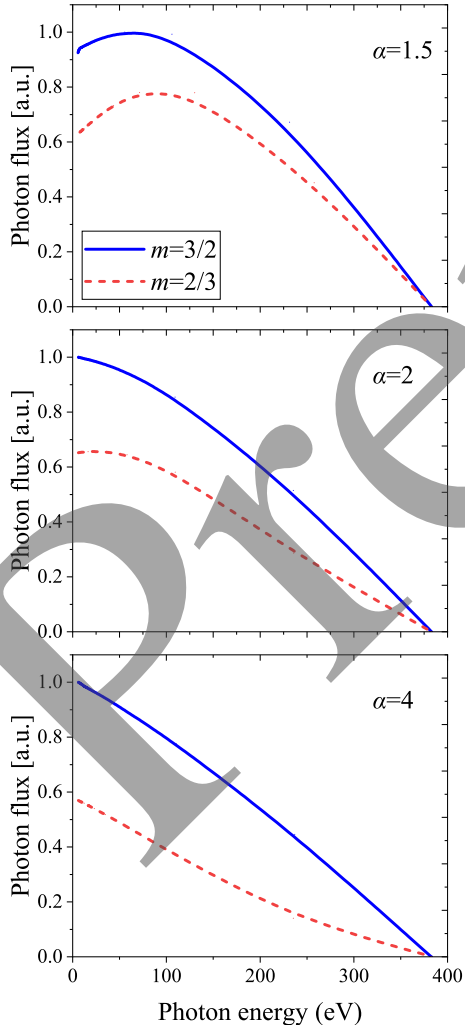


Figure 1: Normalized photon flux at different resonance photon energy for  $m = 3/2$  (solid) and  $m = 2/3$  (dashed), with  $\lambda_u = 240$  mm and the electron beam energy of 2.2 GeV.

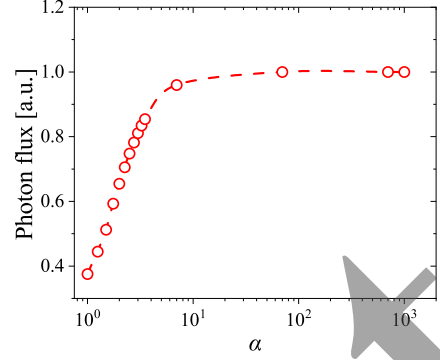


Figure 2: Normalized photon flux depends on field strength ratio  $\alpha$  with  $m = 3/2$ , the photon energy of 6 eV and the electron beam energy of 2.2 GeV.

the additional field effect becomes negligible. Conversely, selecting the longer-period field as the dominant component effectively mitigates this effect.

The heat load suppression capability of an AK undulator relies on its long period to steer radiation power away from the axis. In the short-period regime, the heat load gradually converges toward the undulator axis. This diminishes the suppression effectiveness and reduces the photon flux. Moreover, in the short-period regime the AK structure may introduce on-axis heat load in CP mode, a feature intrinsically absent in APPLE-II undulators.

Nevertheless, short-period AK undulators retain a degree of heat load suppression in linear polarization modes. This makes them potentially applicable for high-energy beamline with stringent on-axis heat load constraints, where a moderate reduction in photon flux is acceptable. To quantitatively evaluate this high-energy applicability, we maintain the field strength ratio  $\alpha = 1.5$  and the electron beam energy of 2.2 GeV, but reduce the period to  $\lambda_u = 60$  mm. As illustrated in Fig. 3, the photon energy shifts to the 700–1500 eV range.

Although AK undulators are not originally developed for short-period or high-energy applications, this discussion

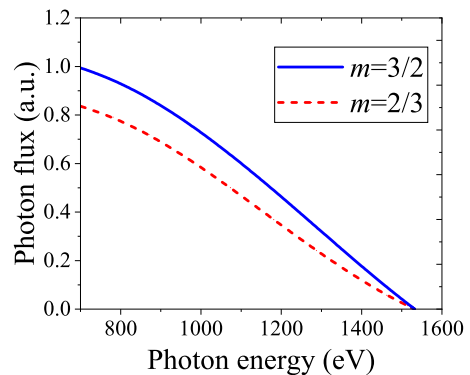


Figure 3: Normalized photon flux at different resonance photon energy for  $m = 3/2$  (solid) and  $m = 2/3$  (dashed), with  $\lambda_u = 60$  mm and the electron beam energy of 2.2 GeV.

extends their potential operational scope. When an AK configuration is considered in such regimes, the ratio between the dominant and additional field components should be evaluated to determine the optimal scheme.

## UNDULATOR DESIGN

Based on this principle, a novel AK undulator configuration has been proposed, where the longer-period KNOT field is enhanced to serve as the dominant component. The magnetic structure and radiation performance of this design, optimized for the HALF parameters, have been validated through simulations [8].

Table 2: Parameters for Undulators in Fig. 4

Undulator	HEPS AK	Novel AK
Period $3\lambda_u$ (mm)	256.8	220.8
APPLE period $\lambda_u$ (mm)	85.6	73.6
KNOT period $1.5\lambda_u$ (mm)	128.4	110.4
Number of periods	18	21
APPLE:Knot	5:2	3:10

To verify the generality of our theoretical conclusion beyond the specific parameters, we perform an independent validation based on HEPS. As shown in Table 2, another novel AK undulator is designed to match the same minimum photon energy as the HEPS traditional design. Figure 4 compares the performance of the two designs in CP mode, showing photon flux, polarization degree and on-axis power as functions of photon energy. The novel design consistently delivers higher flux and polarization degree across the entire energy range, confirming that the advantage of selecting the longer-period field as dominant is independent of storage ring parameters.

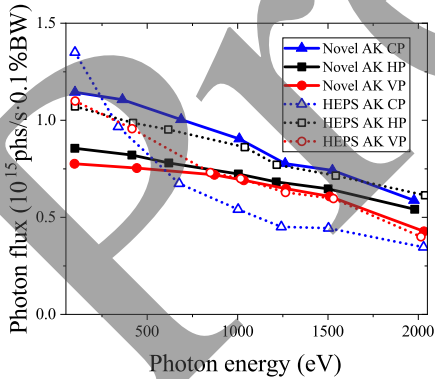


Figure 4: The comparison of radiation performance between the novel AK undulator and the HEPS AK undulator for three polarization states, with the electron beam energy of 6 GeV. For consistency, the acceptance angle at different photon energy always keeps two multiples of divergence.

For practical implementation, proper termination design is essential to minimize field integrals. Taking the HALF-based novel AK undulator as an example, we adopted an antisymmetric end structure with optimized air gaps and

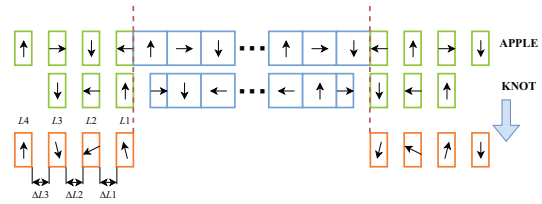


Figure 5: The concept of the end design.

end magnet dimensions, as shown in Fig. 5. By tuning the lengths of the air gaps and the magnetization directions of the end magnets, the first field integrals are naturally canceled for all three polarization modes. Figure 6 shows the corresponding second field integrals at the minimum working gap for horizontal, vertical and circular polarization modes, which remain within acceptable ranges.

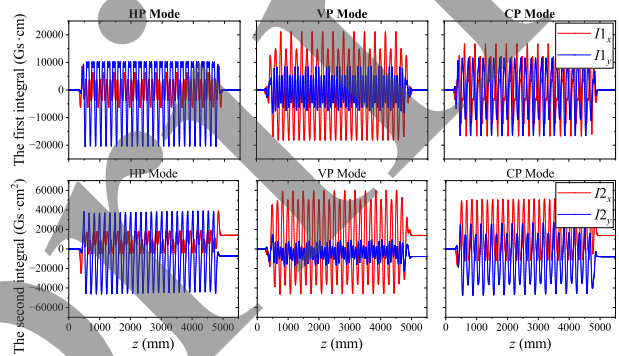


Figure 6: The corresponding first and second field integral distributions at the minimum working gap.

The residual second field integrals exhibit a straight distribution along the undulator and can be further corrected by corrector coils if needed. These results demonstrate the engineering feasibility of the proposed configuration.

## SUMMARY

This paper identifies the longer-period additional field as the essence factor governing circular polarization degradation in APPLE-KNOT undulators. Theoretical analysis and simulations based on HALF and HEPS parameters demonstrate that selecting the KNOT field as the dominant component improves CP radiation performance. A preliminary end design confirms the engineering feasibility of this configuration.

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