USING STRETCHING-MODULATION-COMPRESSION EFFECT TO GENERATE ISOLATED FEW-FEMTOSECOND MeV ELECTRON BUNCHES*

Q. Luo[1], W. Qin[†2,3], C.-Y. Tsai[‡1]
[1]Huazhong University of Science and Technology, Wuhan, China
[2]Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, China
[3]Spallation Neutron Source Science Center, Dongguan, China

Abstract

Femtosecond (fs) electron beams serve as effective tools for investigating ultrafast dynamic processes in matter, providing complementary capabilities to femtosecond laser beams. We propose a scheme combining an undulator with THz modulation to generate isolated few-fs electron bunches. We have developed a theoretical method that incorporates the transport dynamics of low-energy relativistic electrons interacting with the THz modulation field in the undulator and the space charge effects within the bunch itself. The results indicate that the proposed scheme can generate single isolated ultrafast electron bunches with kinetic energy 3 MeV, bunch length about 9 fs (rms) with core charge about 8 fC. We have also evaluated the influence of several relevant physical quantities on the final bunch length and arrival time. The proposed scheme and the developed theoretical model presented may provide useful insights for generating few-fs electron bunches in accelerator-based ultrafast electron diffraction (UED) facilities.

INTRODUCTION

The mechanism of our proposed scheme is conceptually similar to high-gain harmonic generation (HGHG) [1]. First we use a chicane to stretch the bunch, and then generate a shorter bunch by modulating a local portion of the beam core, thereby mitigating space-charge effects at the cost of losing some particles in the head and tail regions. Broadly speaking, these schemes share the common feature of using external electromagnetic field and undulator to modulate the electron beam. The key difference is that our proposed scheme is intended for the single-MeV energy regime relevant to UED applications. At these low electron energies, space charge effects become important during beam transport, and such dynamics have not yet been studied in detail using currently available simulation tools.

THEORETICAL MODEL

Our proposed scheme includes two chicanes and one short undulator with THz modulation. In the two chicanes, AS-

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† qinwl@ihep.ac.cn
‡ jcytsai@hust.edu.cn

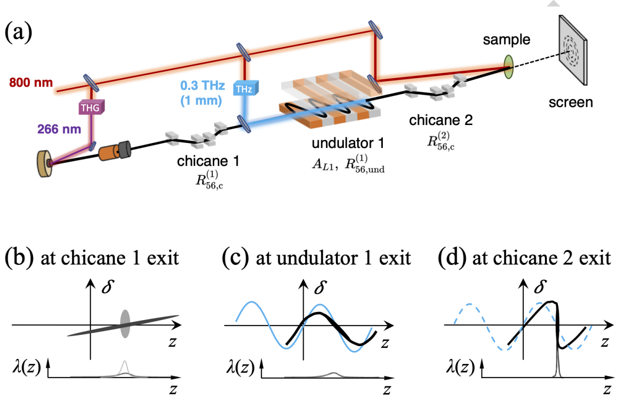

Figure 1: (a) Schematic layout of a MeV UED facility incorporating the bunch compression scheme proposed in this paper. In the figure, the 800-nm laser serves three purposes: exciting the sample, generating 266-nm UV laser through third-harmonic generation (THG) to strike the photocathode and produce photoelectrons, and generating the THz modulation wave. After acceleration by the RF cavity and focusing by the solenoid, the electron beam first passes through chicane 1 to be stretched, then modulated by the THz wave, and finally appropriately compressed in chicane 2 to achieve the possible minimum bunch length before probing the sample. (b), (c) and (d) show the bunch phase space distribution and line density at the exit of the chicane 1, the undulator 1, and the chicane 2, respectively. In the panel (b), the gray represents the initial phase space and line density distributions. The distance from the photocathode to chicane 1 is roughly 0.3 m, the length of chicane 1 itself is approximately 3.92 m, the undulator length is approximately 0.1 m and the length of chicane 2 is about 0.88 m.

TRA [2] is used to simulate beam dynamics, including 3D space charge effects. In the undulator we construct a period-by-period tracking algorithm that includes both the external THz modulation and the longitudinal and transverse space charge effects. Below we briefly summarize the period-by-period tracking equations and space charge models.

Period-by-period Tracking Equations

Assume the phase space coordinates can be written as $[x\ x'\ y\ y'\ z\ \delta]$, where x and x' are the transverse horizontal position and divergence coordinates, y and y' are the transverse vertical counterparts, z is the bunch coordinate ($z > 0$ for bunch head and $z < 0$ for bunch tail) and $\delta = (E - E_0)/E_0$

is the energy deviation. To appropriately describe the low-energy particle dynamics in the undulator, following the "1/2-drift-kick-1/2-drift" principle, we construct the period-by-period tracking from the p-th period to the $(p+1)$-th period in the undulator as

$$\begin{cases} z_{p+1/2} = z_p + \frac{1}{2} R_{56,p} \delta_p \\ \delta_{p+1} = \delta_p + A_{L,p} \sin\left(k_L z_{p+1/2}\right) + \delta_{\rm LSC}\left(z_{p+1/2}, \delta_p\right), \\ z_{p+1} = z_{p+1/2} + \frac{1}{2} R_{56,p} \delta_{p+1} \end{cases} \tag{1}$$

where $R_{56,p} = 2\lambda_L$ is the longitudinal dispersion strength for each undulator period, λ_L is the modulation wavelength, and $A_{L,p} = A_L/N_u$ is the relative modulation depth for each undulator period with N_u the number of undulator periods. Here A_L is the modulation amplitude or depth $A_L = \frac{\Delta\gamma}{\gamma_0} = \frac{eK_u[\mathrm{JJ}]}{\gamma_0^2 mc^2}\sqrt{\frac{4P_L Z_0}{\pi w_0^2}}\frac{L_u}{2}$, with the charge unit $e = 1.6\times10^{-19}$ C, the dimensionless undulator parameter $K_u = 0.934 B_0[\mathrm{T}]\lambda_u[\mathrm{cm}]$ with B_0 the peak magnetic field and λ_u the undulator period, $\gamma_0 = E/mc^2$ being the Lorentz relativistic factor in unit of electron rest mass energy ($mc^2 \approx 0.511$ MeV), P_L the THz modulation power, $Z_0 \approx 377\ \Omega$ the free-space impedance, w_0 the THz beam waist, $L_u = N_u\lambda_u$ the length of the undulator and the coupling coefficient $[\mathrm{JJ}] = J_0\left(\frac{K_u^2}{4+2K_u^2}\right) - J_1\left(\frac{K_u^2}{4+2K_u^2}\right)$, with $J_{0,1}(\cdot)$ the Bessel functions of the zeroth and first order.

The total longitudinal dispersion of an undulator is $R_{56,\mathrm{und}} = N_u R_{56,p}$. Here the $R_{56,\mathrm{und}}$ has already included the effect of wiggling motion and the velocity drift. Following our notation, the undulator entrance corresponds to $p = 0$ (i.e., $z_i = z_0, \delta_i = \delta_0$), and the undulator exit corresponds to $p = N_u$ (i.e., $z_f = z_{N_u+1}, \delta_f = \delta_{N_u+1}$). Here δ_{LSC} is the particle energy deviation caused by the longitudinal space charge (LSC) field per period

$$\delta_{\rm LSC} = -\frac{eE_z\left(z_{p+1/2}\right)\lambda_u}{\gamma_0 mc^2}, \tag{2}$$

where the specific expression for E_z will be summarized below. For the transverse dynamics, we write the mapping equations as (the subscripts "i, f, m" denote, respectively, the initial, intermediate, and final locations in the undulator)

$$\begin{cases} x_m = x_i + \frac{L_u}{2} x_i' \\ x_m' = x_i' + \frac{eE_x}{\gamma_0\beta^2 mc^2} L_u \, . \\ x_f = x_m + \frac{L_u}{2} x_m' \end{cases} \tag{3}$$

An analogous expression also applies to the y-direction. Here, E_x (E_y) denotes the transverse space charge (TSC) field, as will be given below.

Longitudinal Space Charge Field Calculation

According to the Lorentz transformation, the LSC field in the beam rest frame can be converted to that in the lab frame, given by [3]

$$E_z(z) = \frac{Q}{4\pi\epsilon_0\gamma_0^2}\int_{-\infty}^{\infty}\lambda(\zeta)\frac{z-\zeta}{|z-\zeta|^3}d\zeta, \tag{4}$$

where Q is the total bunch charge and λ is the line density satisfying the normalization condition $\int_{-\infty}^{\infty}\lambda(\zeta)d\zeta = 1$. To avoid the divergence as $\zeta \to z$ in the denominator we can introduce the finite transverse distribution of the bunch, assuming a uniform distribution with radius a, then the above expression can be written as

$$E_z(z) = -\frac{Q}{2\pi\epsilon_0 a^2}\int_{-\infty}^{\infty}\lambda(\zeta)\left(\frac{z-\zeta}{\sqrt{\frac{a^2}{\gamma_0^2}+(z-\zeta)^2}} - \frac{z-\zeta}{|z-\zeta|}\right)d\zeta. \tag{5}$$

In this way, when ζ is close to z, the integrand no longer has a singularity. Before numerically calculating $E_z(z)$, evaluating the line density function λ will require to histogram all simulation particles using reasonable bins. The obtained λ must satisfy the normalization condition. In our simulations all particles within the same bin experience the same $E_z(z)$ from neighboring bins.

Transverse Space Charge Field Calculation

Following similar spirit, assuming the cylindrical symmetry, the TSC field can be written as [3]

$$E_\rho(z,\rho) = \frac{Q\rho}{4\pi\epsilon_0\gamma^2}\int_{-\infty}^{\infty}\frac{\lambda(z')\,dz'}{\left[(z-z')^2+\frac{\rho^2}{\gamma^2}\right]^{3/2}}, \tag{6}$$

where $\rho = \sqrt{x^2+y^2}$. We note that, depending on the transverse position of a particle within the bunch, the corresponding x- and y-components are calculated as $E_x = E_\rho\cos\theta, E_y = E_\rho\sin\theta$, with $\theta = \tan^{-1} y/x$.

EXAMPLE

In this section we will apply our developed theoretical model to demonstrate a scheme for generating an isolated few-fs electron beam and explore its phase space dynamics. Taking the HUST MeV UED as an example [4–9], a 1.4-cell photocathode RF electron gun is employed. The RF cavity provides an accelerating gradient of 76.6 MV/m, with ≈ 65° relative to the driver laser, accelerating the electron beam to a kinetic energy of 3 MeV. Before entering the proposed stretching-modulation-compression module, the electron beam is assumed 50 fs (rms), with a bunch charge of approximately 0.05 pC and a transverse rms beam size of 1 mm. Bunch compression will proceed through the combination of undulator and chicanes as described previously. Moreover we assume an undulator parameter $K_u \approx 2.34$, the corresponding transverse wiggling amplitude of the electron beam is approximately $\pm K_u/\gamma k_u \approx \pm1.4$ mm with $k_u = 2\pi/\lambda_u$. Considering the effective transverse beam size, taken as $\pm3\sigma$ (approximately 3 mm) and combining the two contributions in quadrature, the total transverse extent is about 3.3 mm (half-range), smaller than the THz beam waist (0.5 cm), indicating that the transverse variation of the modulation field can be neglected. The bunch and phase space distributions at various locations can be seen in Fig. 1.

To evaluate how the compressed bunch characteristics fluctuate shot-to-shot, when there are possible errors in THz modulation power and phase, arrival time of electron bunches to the undulator, while simultaneously the bunch charge also fluctuates from every shot. Using Monte-Carlo simulation, we simulated 100 independent shots, where for each shot the modulation power P_L is generated with $\pm 3\%$ rms fluctuation around the nominal value of 0.1 MW. Meanwhile, the centroid position $\bar{z}_0$ of the initial bunch distribution also has $\pm 2^\circ$ rms fluctuation around the THz modulation wavelength (1 mm or 0.3 THz). The generated initial bunch charge is also assigned 3% (rms) fluctuation around 0.05 pC. Figure 2 records the simulation results of the 100 shots, indicating that the averaged final compressed bunch length is 9.18 ± 2.51 fs (rms), while the deviation in arrival time is approximately 18.8 fs. As the simulation presented here is not a start-to-end simulation of a complete UED beamline, we do not explore the timing jitter systematically.

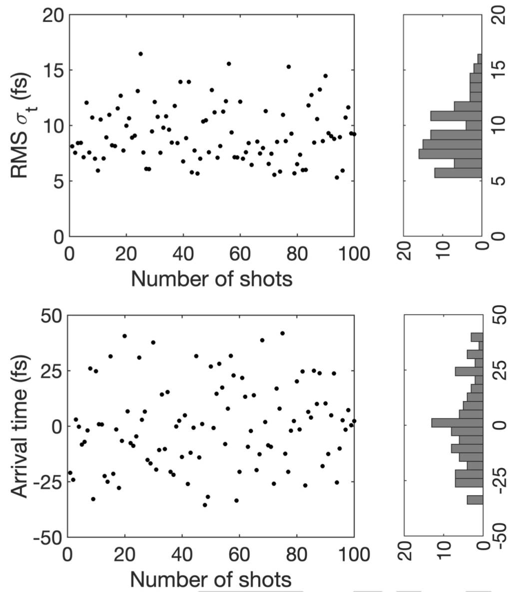

Figure 2: (a) Final compressed bunch length and (b) arrival time at the exit of chicane 2 for various shots with ± 3 % (rms) fluctuating amplitude of the THz power around 0.1 MW, $\pm 2^\circ$ (rms) around π, and ± 3 % (rms) charge fluctuation around 0.05 pC. Averaged results over 100 shots: bunch length approximately $1.8 \times 10^3 \pm 487$ nm or 9.18 ± 2.51 fs (rms), arrival time fluctuation approximately 18.8 fs (rms).

Figure 3 shows the achievable compressed bunch length, core charge, effective charge ratio, and the arrival time jitter for different charges in the range of 0.01 pC to 0.2 pC. For each case, after considering the possible errors mentioned above, we summarize some interesting observations. Note that for the above simulation results within the 0.01 pC to 0.2 pC range, except for the initial bunch charge, all other parameters are set to the nominal values. i) The shortest compressible bunch length within the charge range is around 8 fs, see Fig. 3 (a); ii) The final core charge is approximately linear proportional to the initial bunch charge, with the proportionality (the slope) about 16%, see Fig. 3 (b) and (d); iii) The arrival time jitter remains about 18 ~ 20 fs within the 0.01 pC to 0.2 pC range.

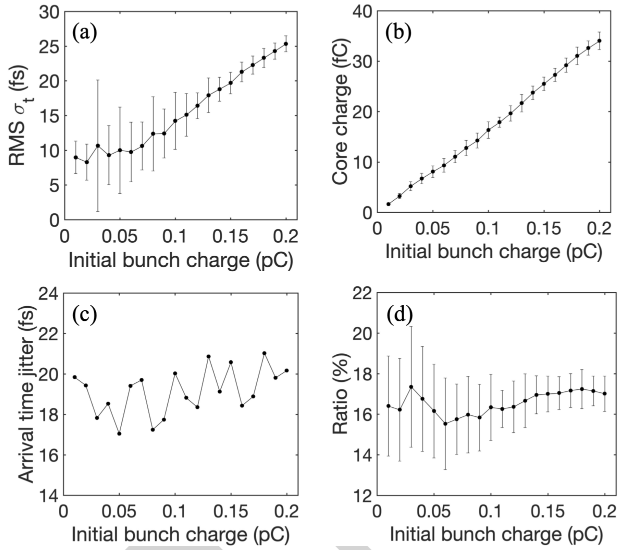

Figure 3: (a) Final rms bunch length; (b) the corresponding core charge within the bunch at FWHM; (c) the arrival time jitter at the exit of chicane 2; and (d) the ratio of the bunch charge within the beam core (FWHM). The error bars represent the standard deviation obtained from 100 independent simulation runs with the $\pm 3\%$ (rms) variation in modulation power and bunch charge and $\pm 2^\circ$ (rms) variation in the modulation phase.

SUMMARY

In this paper we have proposed and quantitatively verified the feasibility of a scheme combining an undulator with THz modulation and two chicanes to generate isolated few-fs electron bunches. To analyze the transport dynamics of low-energy relativistic electrons modulated by external electromagnetic fields in the undulator, we constructed period-by-period tracking equations that include both the longitudinal and transverse space charge effects generated by the low-energy, high-charge bunch itself. In the two chicane sections, we perform ASTRA simulations for beam dynamics, including 3D space charge effects. The simulation results indicate that under currently achievable conditions in laser technology and undulator engineering, this stretching-modulation-compression scheme can generate single isolated ultrafast electron bunches with kinetic energy of approximately 3 MeV, final bunch length of about 9 fs (rms), and core charge about 8 fC. We have also evaluated the potential influence of several relevant physical quantities on the final bunch length and arrival time for the initial bunch charge within 0.01 pC to 0.2 pC.

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