ABSOLUTE ENERGY AND MACHINE LENGTH CALIBRATION IN THE SPS

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Abstract

This contribution presents the results of an absolute-energy and machine-length calibration of the CERN Super Proton Synchrotron (SPS) using two different particle species on the same magnetic cycle. A review of the SPS ring geometry and magnet alignments led to an update of the machine circumference used in the optics models, improving their agreement with the as-built machine. The measurement determines the effective machine circumference from absolute energy calibrations and the revolution period. It repeats a similar calibration performed in the early 2000s, using the magnetic centre of the sextupoles as a reference, and complements it with beam position monitor (BPM) centres. A comparison between new and old measurements and models is presented.

SPS RING GEOMETRY REVIEW

The SPS optics model was updated to better match the as-built machine. The machine survey was compared against the GEODE survey database [1], which represents the current installed state of the machine. All discrepancies between the two data sets were then systematically identified and analysed [2]. Each displacement identified during the comparison was projected onto the reference trajectory in the horizontal and vertical machine planes and evaluated at well-defined locations, such as the entrance or exit of each beamline element.

Bending Magnet and Starting Point Adjustments

An initial analysis, focusing on a simplified model containing only the bending dipole magnets, revealed a discrepancy between the optics model and the as-built machine reference geometry stored in the GEODE survey database. Specifically, the machine diameter in GEODE is larger than expected from the SPS as designed by 4 mm, corresponding to about 14 mm in circumference with respect to the nominal 11/7 circumference ratio to the Proton Synchrotron (PS) [2]. The discrepancy has a limited impact on the absolute beam energy, but more importantly, it makes the as-designed geometry unsuitable as a new reference for alignment. Furthermore, the present reference circumference does not fully close, leaving a gap of approximately 170 µm between the start and end points. The cause of this discrepancy was identified as a difference in the calculation of the beam trajectory path (s), also known as the beam cumulative distance (beam DCUM) [2, 3]. This fundamental difference in reference length leads to a measurable discrepancy in the bending magnet length. The magnetic length of the bending dipole is defined as the face-to-face distance, i.e., the chord length, whereas the optics calculation software defines the magnetic length of a bending magnet based on the arc path.

The difference between the two trajectory definitions (arc and chord) is given by

$$\Delta L_{\text{arc-chord}} = L_{\text{chord}} \left(\frac{\alpha}{2\sin(\alpha/2)} - 1 \right) = 18.603\,\mu\text{m}, \quad (1)$$

where $\alpha = 2\pi/744$ is the bending angle of a single dipole magnet and 744 is the total number of bending dipoles in the SPS. While a single discrepancy of 18.603 µm may appear negligible, its accumulation over all dipoles gives

$$\Delta L_{\text{total}} = 18.603\,\mu\text{m} \times 744 = 13.840\,\text{mm}. \quad (2)$$

This accumulated difference causes the SPS circumference, as recorded in the survey reference geometry, to deviate from the nominal design value, resulting in a non-conforming machine geometry. A further significant correction, which reduced the residual discrepancy, involved adjusting the position and orientation of the starting point of the SPS optics model. The applied orientation corrections were $\Delta\Theta = 0.329\,\mu$rad (yaw), $\Delta\Phi = 0.012\,\mu$rad (pitch), and $\Delta\Psi = 0.030\,\mu$rad (roll). The position corrections were $x_0 = -9.012\,\mu$m, $y_0 = -3.870\,\mu$m, and $z_0 = -0.187\,\mu$m. To fully close the machine ring in the optics model and better match the reference geometry, empirical adjustments were applied: the per-dipole arc-chord correction was increased from 18.603 µm to 19.340 µm, and the distances between elements in the straight sections were fine-tuned.

Following these adjustments, the global discrepancy between the new model and the old reference was reduced to below 10 µm. This allowed the new model to serve as a single source of truth for both alignment and beam operations.

Sextupole Inconsistencies

To make the model better match the as-built machine, all beamline components were analysed. Quadrupoles (QD, QF) and octupoles (LOD, LOE, LOF, LOEN) showed no significant discrepancies. However, for sextupoles (LSF, LSD), a mismatch was identified through a comparison of two historical documents on magnet characteristics [4, 5].

Around 1983, all SPS sextupoles were replaced with new models. Although they retained the same names in the databases, their characteristics differed slightly. While the mechanical lengths of the LSF and LSD were correctly updated in the database, their magnetic lengths remained unchanged from those of the original sextupoles. To ensure consistency with the latest available data [5], the magnetic lengths were updated from 0.423 m to 0.435 m for the LSF, and from 0.420 m to 0.426 m for the LSD.

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Minor Adjustments

A comprehensive length consistency check was conducted across all beamline elements. This process identified minor discrepancies in element lengths, largely attributable to rounding approximations. While not critical, these approximations required empirical corrections in the optics model to maintain accuracy. Some elements still require further investigation, particularly those exhibiting deviations greater than 1 mm. A detailed list of all optics model adjustments is provided in a dedicated technical note [2].

ENERGY AND CIRCUMFERENCE CALIBRATION

Using the updated optics model, which better represents the as-built machine, a measurement campaign was carried out to determine the SPS circumference from the revolution period of two particle species circulating with the same magnetic-cycle settings. The measurement repeats a similar calibration performed in 2003, in which proton and Pb^{53+} beams were used for an absolute energy and circumference calibration of the SPS at a momentum of 450 GeV/c [6]. That campaign observed a circumference larger than expected. In the 2025 campaign, the same calibration principle was applied and complemented by the use of beam position monitor (BPM) centres as an additional reference, alongside the sextupole magnetic centres used in the original measurement.

The measurements were performed using protons and Pb^{54+} ions at two proton-equivalent momentum points (26 GeV/c and 100 GeV/c) during the cycle. To determine the machine circumference and the beam momentum, the revolution frequencies of both species were measured while the beams circulated with the same magnetic-cycle settings, ensuring that both particles travelled on the same nominal orbit under identical magnetic rigidity conditions. The use of two particle species with different charge-to-mass ratios results in different relativistic speeds and, therefore, different revolution frequencies f_{rev}. This difference allows the machine circumference to be determined through the relation

$$\beta c = C f_{\mathrm{rev}}, \tag{3}$$

where βc is the particle speed, c is the speed of light and C is the machine circumference.

The speeds of the proton and ion ($\beta_p c$ and $\beta_i c$) are related to the proton-equivalent momentum P, the ion momentum $P_i = ZP$, and the rest masses (m_p and m_i) by:

$$\beta_p^2 = \frac{P^2}{P^2 + (m_p c)^2}, \qquad \beta_i^2 = \frac{P^2}{P^2 + (m_i c/Z)^2}. \tag{4}$$

Equation (4) can be solved for the proton beam momentum P as:

$$P = m_p c \sqrt{\frac{\kappa^2 \mu^2 - 1}{1 - \kappa^2}}, \tag{5}$$

with

$$\kappa = \frac{\beta_i}{\beta_p} = \frac{f_{\mathrm{rev}}^i}{f_{\mathrm{rev}}^p} \qquad \mu = \frac{m_i}{Z m_p}, \tag{6}$$

where $1/\mu$ is the number of charges per nucleon of the ion. For Pb^{54+} ions $\mu \simeq 3.852$.

Equation (5) can be approximated by

$$P \simeq m_p c \sqrt{\frac{f_{\mathrm{rev}}^p}{2\Delta f}(\mu^2 - 1)}, \tag{7}$$

where $\Delta f = f_{\mathrm{rev}}^p - f_{\mathrm{rev}}^i$ is the revolution frequency difference between the two particle beams. Δf should be maximized to achieve the best possible momentum accuracy for a given frequency resolution. Once momentum and f_{rev} are known, the beam path length follows from Eq. (3). Provided that the BPM reading and positioning errors are small and the orbit is well corrected, C will be close to the machine circumference.

Machine Settings and Closed Orbit Analysis

The measurement was performed during several Machine Development (MD) sessions in 2025. The proton and Pb^{54+} beams were accelerated using the same magnetic-cycle settings, with a timing offset of 725 ms between the two species due to technical constraints in beam production in the pre-injectors. Both proton and ion cycles were injected at 26 GeV/c (proton equivalent) and accelerated to 100 GeV/c. Both cycles were operated with the SPS Q26 optics configuration, with nominal transverse tunes of $Q_H = 26.13$ and $Q_V = 26.18$. The settings of all magnets were kept identical for the proton and ion cycles, except for the chromaticity correction settings, which were changed during the sextupole crossing-point measurements. Prior to the energy calibration measurements, the radial steering of both beams was carefully optimised (i.e. the radio-frequency (RF) system frequency was adjusted) to ensure that both species were circulating on the same nominal closed orbit. For the proton beam, the radial steering was trimmed until $\delta p/p$ converged to zero at the measurement plateau. This was confirmed by the BPM closed-orbit analysis performed at the injection flat bottom (26 GeV/c, 1051 ms) and at an intermediate plateau in the momentum ramp (100 GeV/c, 3501 ms). At $t = 1051$ ms and 26 GeV/c, the horizontal RMS closed-orbit values had converged to 1.80 mm and 1.77 mm for protons and ions, respectively, indicating that the two beams were circulating on the same orbit. The vertical RMS closed-orbit values were similarly consistent at this time and momentum point (1.25 mm and 1.26 mm), confirming symmetric beam conditions in both planes at 26 GeV/c.

Sextupole Check

The sextupole crossing-point method was applied at 100 GeV/c. The method consists of varying the horizontal chromaticity across a range of settings and identifying the $\delta p/p$ value at which the betatron tune no longer depends on chromaticity, meaning the beam is centred in the machine sextupoles.

Six (normalised) chromaticity settings were applied sequentially: $\xi_H = +0.5, -2.5, -2.0, -3.0, -1.5$, and -0.5. The beam remained stable throughout the full $\delta p/p$ sweep only for $\xi_H = +0.5$ and $\xi_H = -0.5$. The setting $\xi_H = -1.5$

Table 1: Circumference Results from the Revolution Frequency Method at Two Proton-Equivalent Momentum Points

f_{rev}^{p} [Hz]	$f_{\mathrm{rev}}^{\mathrm{Pb}}$ [Hz]	P [GeV/c]	$\delta p/p$	C [m]	$\Delta C_{\text{as-built}}$ [m]
43347.256	42967.967	25.987	-5.2×10^{-4}	6911.56176	0.04357
43373.600	43347.704	100.158	1.6×10^{-3}	6911.56150	0.04331

provided marginal stability and was retained for the crossing-point analysis with increased uncertainty. The instabilities limited the number of usable data points and should be investigated in future MD sessions to improve the precision of the crossing-point determination. A strong asymmetry was observed between $\xi_H = +0.5$ and $\xi_H = -0.5$: the negative chromaticity setting produced a noticeably larger tune excursion, as shown in Fig. 1. The tune dependence on $\delta p/p$ was analysed for the three chromaticity settings ($\xi_H = +0.5$, -0.5, and -1.5). The crossing point of the three lines (shown in Fig. 2 and expected to be at the centre of the sweep) corresponds to a $\delta p/p$ value of -1.35×10^{-4}. This represents the momentum offset relative to the case in which the proton beam is centred in the BPMs, corresponding to a transverse offset of 0.3 mm.

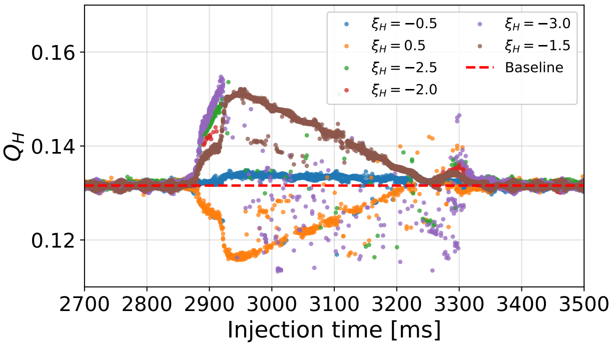

Figure 1: Evolution of the horizontal tune Q_H along the cycle during the $\delta p/p$ ramp for six chromaticity settings.

CONCLUSION

Table 1 summarizes the circumference results from the revolution frequency method at two proton-equivalent momentum points. The two momentum points yield consistent circumference values, differing by less than 0.3 mm, which demonstrates the internal consistency of the measurement. The measured circumference, $C \approx 6911.562$ m, exceeds the nominal design value ($C_{\text{nom}} = 1100 \times 2\pi = 6911.504$ m) by approximately 58 mm and the as-built model value ($C_{\text{as-built}} = 6911.518$ m) by approximately 43 mm. Thus, approximately 15 mm of the original discrepancy between the nominal and modelled circumference has been resolved through the improved geometrical description of the as-built machine, bringing the optics model into closer agreement with the installed machine. The result obtained in 2003 at 450 GeV/c, $C = 6911.5662(24)$ m [6], is about 5 mm larger than, but still consistent with, the present circumference estimate. The sextupole crossing-point method gave a result consistent with the 2003 measurement, with the proton beam centred in the sextupoles at $\delta p/p = -1.35 \times 10^{-4}$. This sextupole-based result provides an independent measurement of the reference orbit, consistent with the BPM-based analysis. The observed asymmetry in the tune dependence between chromaticity settings should be further investigated to improve the precision of the crossing-point determination in future measurements.

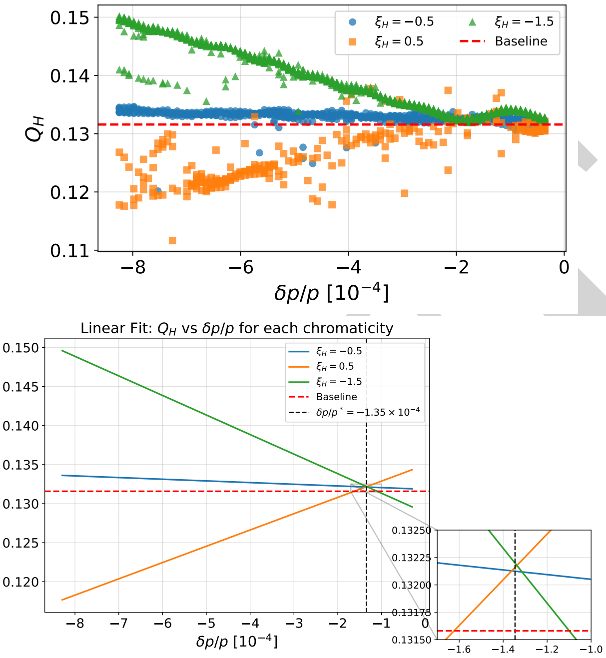

Figure 2: Top: horizontal tune Q_H as a function of $\delta p/p$ for the three chromaticity settings with stable beam conditions ($\xi_H = -0.5$, $+0.5$, and -1.5); the baseline fractional tune $Q_H = 0.13158$ is shown as a red dashed line. Bottom: linear fits of Q_H as a function of $\delta p/p$, extrapolated to the crossing point at $\delta p/p = -1.35 \times 10^{-4}$ (inset), corresponding to the momentum offset at which the beam is centred in the machine sextupoles.

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