COMPARISON OF NUMERICAL METHODS TO DETERMINE QUADRUPOLE CENTRES FOR BEAM BASED ALIGNMENT

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Abstract

A beam-based alignment procedure is used to realign the beam to the centres of the quadrupoles, enabling calibration of BPM offsets. Accurately determining these quadrupole centres is an important step that requires analyzing BPM data collected while modulating the nearby quadrupole and one or more corrector magnets in the ring. In this paper, several numerical methods to extract the quadrupole centres, with or without error analysis, are presented and their results are compared. This study is based on online experiments performed at Diamond.

INTRODUCTION

During the course of a beam-based alignment (BBA) procedure in a modern circular accelerator with many BPMs, the quadrupole centre with respect to a nearby BPM (x_c) can be extracted from the orbit variations when the quadrupole strength and one or more correct magnets (CMs) are modulated. To demonstrate the principle of BBA the acquired BPM data, in the simplest form, are arranged as depicted in Figure 1.

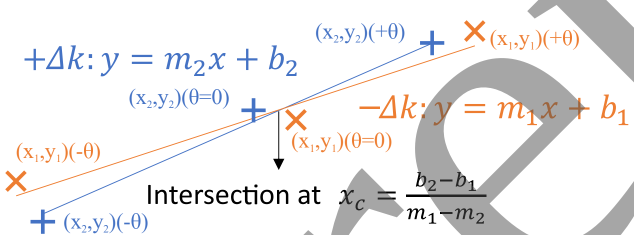

Figure 1: Schematic plot for the principle of BBA. The amplitudes of modulations of the quadrupole and CM are Δk and θ. The subscript 1 is for quadrupole modulation $-\Delta k$ and 2 for $+\Delta k$. The horizontal coordinate is the readout of the target BPM, and the vertical coordinate stands for the readout from one of the other BPMs.

The quadrupole centre with respect to the target BPM seen by the other BPM is given by the intersection coordinate, which can be calculated by the ratio of the difference of two linear fitting coefficients $x_c = (b_2 - b_1)/(m_1 - m_2)$. The absolute uncertainty is defined $u_c \equiv \delta x_c = |x_c| \cdot ru_c$, where the relative uncertainty can be derived by propagating the uncertainties from linear fitting.

$$ru_c \equiv \sqrt{\left(\frac{\delta x_c}{x_c}\right)^2} = \sqrt{\frac{\delta b_1^2 + \delta b_2^2}{(b_2 - b_1)^2} + \frac{\delta m_1^2 + \delta m_2^2}{(m_1 - m_2)^2}}$$

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FINDING QUADRUPOLE CENTRE

A global quantity can be extracted to represent the ensemble quadrupole centre from information of all BPM data, as each BPM observes the quadrupole centre differently (x_{ci}) with its own uncertainty (u_{ci}). The index i goes through all the BPMs except the target BPM. When more than one CM are varied [1], another index j which stands for the index of modulating CM is added. In this section we will review the legacy methods which are normally used in our current control room and present several alternative approaches.

Legacy Method

In the legacy method only one CM is varied. The approximated location of quadrupole centre $\bar{x}_{ci}$ is the x-intercept of the linear fitting lines of $y_2 - y_1$ vs. $(x_1 + x_2)/2$. The following series of data processes are then applied before taking average of $\bar{x}_{ci}$ as the quadrupole centre.

1. Remove those $\bar{x}_{ci}$ where the fitting residual falls outside of 20 times of the BPM resolution.
2. Remove those $\bar{x}_{ci}$ where the fitting slope is smaller than 0.25 times the max fitting slope.
3. From the remaining $\bar{x}_{ci}$ remove outliers which falls outside of 1 standard deviation range.

The parameters used in these filtering criteria are empirical and not always universal. An alternative legacy method [2] is to find the waist from a quadratic fit of a global merit function $\sum_{BPM}(y_2 - y_1)^2$ vs. $(x_1 + x_2)/2$.

Figure 2 shows an example of the raw data from all BPMs from a BBA measurement, in which the most effective for the target BPM is varied. In this example a strong nonlinear response in the horizontal axis is observed when varying the quadrupole. This nonlinearity may systematically degrade the accuracy of the approximated solution $\bar{x}_{ci}$. Therefore, the direct solution x_{ci} should be used instead.

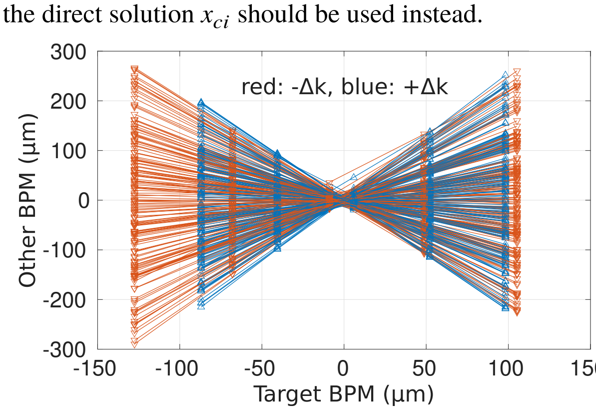

Figure 2: Raw data for the BBA. This measurement is taken from a horizontal BBA on Quadrupole (2, 2) at Diamond.

Single-CM BBA

Simply taking the average of x_{ci} risks biasing the results because of outliers. A simple but not rigorous way to avoid the impact of extremes from outliers is to take the median of all x_{ci}. One can also find the location of the most probable value in a fitted distribution function of x_{ci} [3].

It is better to take uncertainties into consideration to judge the quadrupole centres. To elaborate this we can take a look at the plot of all fitting coefficient differences $(b_2 - b_1)$ vs. $(m_1 - m_2)$ in the same measurement, as shown in Figure 3. The ensemble slope can then be identified as the quadrupole centre by the new methods provided here.

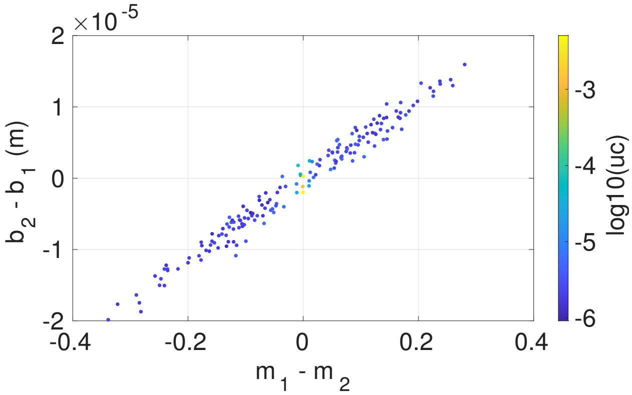

Figure 3: Relation of fitting coefficient differences. Each point represents a datum seen by a BPM. The colour indicates its absolute uncertainty of their slopes, in log scale.

A first method is to utilise reliability weights when taking the average of x_{ci} [1]. The reciprocal of uncertainties raised to a power p can be used to calculate the reliability weight. $w_i \equiv u_{ci}^{-p}$ or ru_{ci}^{-p}. Figure 4 shows the relation of many u_c and ru_c vs. x_c in log scales. Larger uncertainties are bounded by asymptotic lines and their slopes suggest the power $p = 2$ for u_c and $p = 1$ for ru_c.

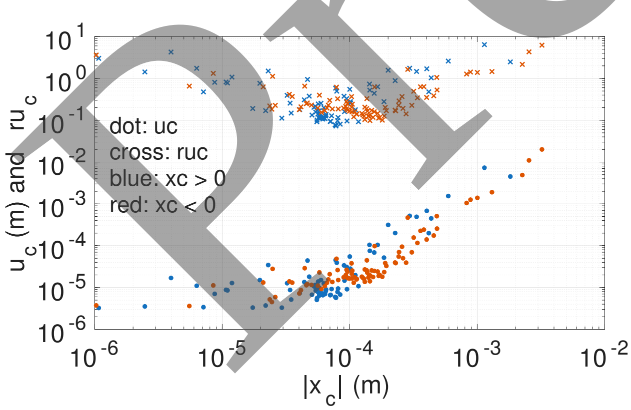

Figure 4: Uncertainties vs. absolute values of quadrupole centre. Different signs of x_c are painted differently. The data is taken from a single-CM BBA measurement on Quadrupole (21, 7) horizontally.

Another way is to perform a least square fit to find a slope for all. This method can be further improved when the reliability weight based on the uncertainties is applied. The ensemble slope can also be identified as the slope of the major axis found by the principal component analysis (PCA). In this case the uncertainty is not used.

Multiple-CM BBA

In the previous study [1], the MCM-BBA method is proposed to improve the accuracy of BBA when nonlinearity is encountered. In this method more than one CM are modulated and the weighted average or a weighted least square fit can still be applied to find an ensemble slope from all data points. However there are sometimes large discrepancies between results obtained from different CMs. An example of this case is shown in Figure 5, where x_{cij} and its u_{cij} are calculated from a 10-CM BBA measurement.

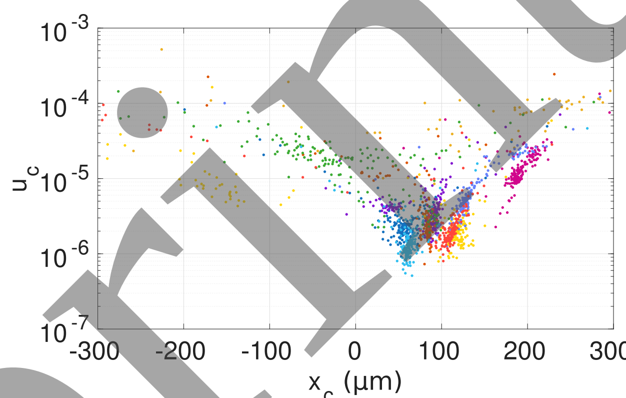

Figure 5: Quadrupole centres and their uncertainties from a MCM BBA measurement. Results from different CMs are painted in different colours.

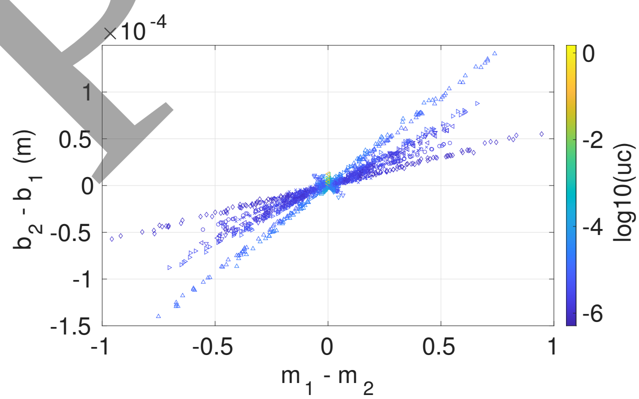

Figure 6: The fitting coefficient differences for a 10-CM BBA measurement. Data from different modulating CMs are marked in different symbols.

Figure 6 shows more information about the detailed plot of the fitting coefficient differences. For a data set from each individual CM modulation, the PCA method can determine its own major axes. A balanced solution can then be obtained by applying the weighted average technique again to combine the fitted slopes. Mathematically the quadrupole centre from this method is expressed as $\sum_j w_j x_{cj} / \sum_j w_j$, where x_{cj} is the slope of the principal component for data from CM j and the weight w_j is the ratio of magnitudes of first and second principal components.

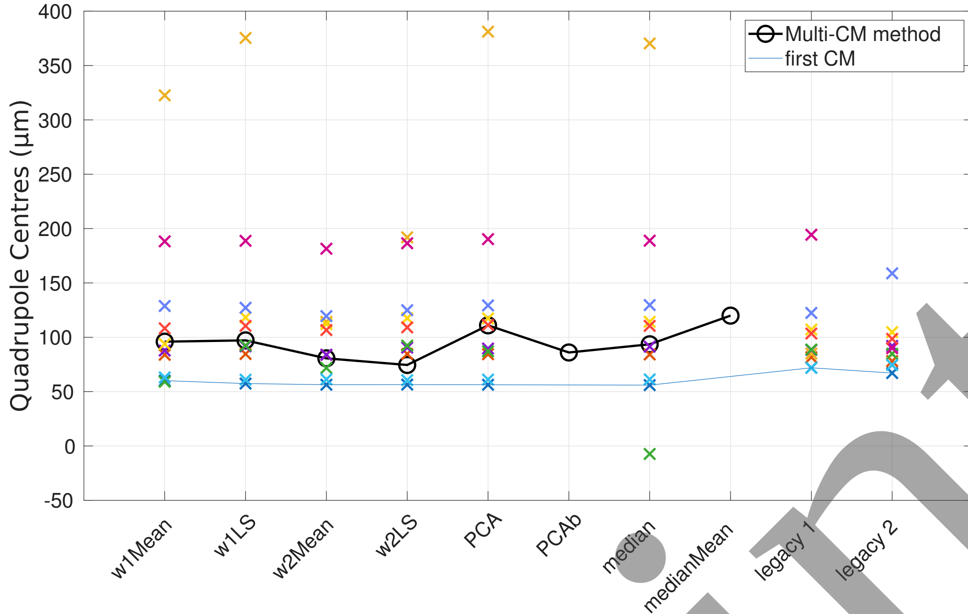

Figure 7: Result comparisons for quadrupole centre estimations. Single-CM BBA results are marked in crosses in different colours while MCM-BBA results are marked in circles and connected in a black line. The centres found by first CM modulation are connected in a blue line.

RESULT COMPARISON

An overall comparison of the results obtained from 10 different algorithms is shown in Figure 7. The results are extracted from the shared data collected in a 10-CM BBA measurement on the Quadrupole (2, 2) horizontally at Diamond. The first modulating CM is deliberately chosen to be the most effective CM for the target BPM. For reference the names and algorithms are summarised as follows.

1. **w1Mean**: weighted average $w_{ij} = ru_{cij}^{-1}$
2. **w1LS**: least square fit with weights $w_{ij} = ru_{cij}^{-1}$
3. **w2Mean**: weighted average $w_{ij} = u_{cij}^{-2}$
4. **w2LS**: least square fit with weights $w_{ij} = u_{cij}^{-2}$
5. **PCA**: slope of principal component of all x_{cij}
6. **PCAb**: balance PCA results x_{cj} with weights w_j
7. **median**: median of all x_{cij}
8. **medianMean**: $\text{mean}_j(\text{median}_i(x_{cij}))$
9. **legacy 1**: mean of $\bar{x}_{ci}$ excluding outliers
10. **legacy 2**: waist of a quadratic fit of a merit function excluding outliers

Variations among the results from individual CM results can be large but the MCM-BBA always gives a more balanced solution. Our preferred method for MCM-BBA is **w2Mean**, as the original recipe provided in Ref [1].

DISCUSSION

In preparation for Diamond-II commissioning, several additional BBA improvements have been developed. These are:

- Option of a CM scheme to control the orbit variations within the orbit interlock,
- BBA using local bump methods,
- An application to display the relevant data and compare results.

Further topics about the BBA performance are to be studied, including:

- Accuracy with different beam currents especially low currents,
- Effective CM scheme including number of CMs used, choice of CM, step sizes, and number of steps,
- Sensible amplitude of quadrupole modulations,
- Balanced waiting time for measurement speed and the hysteresis effects,
- Proper algorithm to sort the sequence of BPM for iterative BBA.

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[3] Y. K. Wu, J. Li, V. Litvinenko, and P. Wang, "BPM and Orbit Correction Systems at the Duke Storage Ring", in *Proc. PAC'03*, Portland, OR, USA, May 2003, pp. 2479–2481.