SymCSR: TRACKING 6D PHASE SPACE DYNAMICS OF ELECTRON BEAM WITH COHERENT SYNCHROTRON RADIATION

Z. Liu*, X. Deng†, T. Li, L. Yan‡, Tsinghua University, Beijing, China

Abstract

Coherent synchrotron radiation (CSR) is a critical effect in the design and operation of high-brightness electron accelerators, as it can lead to significant energy loss and emittance growth. In this paper, we present SymCSR, a first-principle tracking program for simulating the 6D phase space dynamics of electron beams under the influence of CSR. SymCSR computes the radiation reaction field of electrons based on its retarded and instantaneous trajectory, which efficiently reduces the requirement on macroparticle numbers. The dynamics of electron beam in various dimensions are calculated using SymCSR and are compared with theoretical models.

INTRODUCTION

Generating ultrashort electron beam to produce coherent radiation has attracted substantial interest in accelerator physics community in recent years [1, 2]. Advanced phase space manipulation methods and lattice designs have been proposed to achieve ultrashort bunch length in linear accelerators and storage rings [3,4]. Due to the high peak current of the ultrashort electron beam, when the beam traverses bending magnet or radiator, the coherent synchrotron radiation (CSR) produced by the beam itself could have significant impact on the dynamics. Meanwhile, in these scenarios the bunch length could be comparable to the transverse beam size and the 6D phase space distribution of the beam could evolve notably in the radiator. To accurately model the CSR effect, it should go beyond the 1D approximation and simulate the full 6D phase space dynamics with CSR effect.

Here, we demonstrate a first-principle tracking program, SymCSR, for calculating the 6D beam dynamics with CSR effect. The program utilizes radiation reaction (RR) field formalisim to eliminate the space-charge singularity in calculation of CSR effect, therefore largely reduce the requirement on macroparticle number for a convergent result.

RADIATION REACTION FIELD

In the simulation of CSR effect, the Lienard-Wiechert solution is frequently applied to calculate the electromagnetic field of a moving charge. However, the two terms in the Lienard-Wiechert solution both contain space-charge singularity which complicates the analysis of CSR effect.

In the previous work, we have shown that the solution of Maxwell equations can also be decomposed into three terms, namely, space-charge field, compression field and RR field [5]. It is shown that the CSR effect is induced by the RR field, and the space-charge related effects are included in the first two terms which are instantaneous. For the electric field, the decomposition can be written as

$$E = E_{\rm sc} + E_{\rm comp} + E_{\rm rr}, \tag{1}$$

with

$$\begin{aligned} E_{\rm sc} =& \frac{e}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}\frac{1-\beta^2}{(1-(\beta\times\hat{R})^2)^{3/2}}, \\ E_{\rm comp} =& -\frac{e}{4\pi\epsilon_0}\frac{1}{cR}\frac{\dot{\beta}+\left(\dot{\beta}\cdot\hat{R}\right)\hat{R}}{2(1-(\beta\times\hat{R})^2)^{3/2}} \\ &+\frac{e}{4\pi\epsilon_0}\frac{1}{cR}\frac{3(\beta\cdot\hat{R})^2\left(\dot{\beta}\cdot\hat{R}\right)\hat{R}}{2(1-(\beta\times\hat{R})^2)^{5/2}}. \end{aligned} \tag{2}$$

where $\hat{R} = R/R$, $\dot{\beta} = {\rm d}\beta\,/\,{\rm d}t$, $R = r_i - r_j$ is the distance between the source particle (j) and the test particle (i), βc is the velocity of the source particle.

The magnetic field can also be decomposed accordingly. To simulate the CSR effect, the Lorentz force from the RR field is calculated at every time step. Since the RR force does not include space-charge singularity, numerical convergence can be achieved with fewer macroparticles.

SYMPLECTIC INTEGRATOR

During each field evaluation time step, a symmetric second-order symplectic integrator is employed to update the 6D phasespace coordinates of the particles. The use of symplectic integrator instead of Runge-Kutta integrator preserves the phase space volume and at the same time reduces the computation time which is essential for simulating CSR effects in storage rings.

For simplicity, assuming only the x component of the vector potential of the external field is non-zero, the Hamiltonian of an electron under the external field can be written as [6]

$$\begin{aligned} &\mathcal{K}(x,p_x,y,p_y,s,p_s,-l,\delta;\sigma) = K_1 + K_2 \\ =&\frac{(p_x-a_x)^2}{2D}+\frac{p_y^2+p_s^2-(1+\delta)^2}{2D} \end{aligned} \tag{3}$$

where $p = (p_x, p_y, p_s)$ is the electron momentum, $r = (x, y, s)$ is the electron position, δ is the momentum deviation, l is the electron path length and

$$D = \sqrt{(1+\delta)^2+\frac{1}{\beta_0^2\gamma_0^2}} \tag{4}$$

where β_0, γ_0 is the velocity and Lorentz factor of the reference particle.

The independent variable of this Hamiltonian is σ, satisfying ${\rm d}\sigma = c\,{\rm d}t$. Since the Hamiltonian does not contain σ

* liu-zy20@mails.tsinghua.edu.cn
† dengxiujie@mail.tsinghua.edu.cn
‡ yanlx@mail.tsinghua.edu.cn

explicitly, the corresponding second-order symplectic integrator can be directly written as

$$\mathcal{M}_2(\Delta\sigma) = e^{-:\frac{\Delta\sigma}{2}K_1:} e^{-:\Delta\sigma K_2:} e^{-:\frac{\Delta\sigma}{2}K_1:} \tag{5}$$

where

$$\begin{aligned} e^{-:\frac{\Delta\sigma}{2}K_1:} &= e^{-:\frac{\Delta\sigma}{2}\frac{(p_x-a_x)^2}{2D}:} = \mathcal{A}_x^{-1} e^{-:\frac{\Delta\sigma}{2}\frac{p_x^2}{2D}:}\mathcal{A}_x, \\ \mathcal{A}_x &= e^{:\int a_x(x,y,s)\mathrm{d}x:}. \end{aligned} \tag{6}$$

The motion of the particles are updated using Eq. (5).

RETARDATION RELATION

To calculate the CSR effect, SymCSR split the simulation time window into a number of field evaluation time steps. In each step it calculates the (point-to-point) RR field based on the instantaneous phasespace coordinates and past phasespace trajectories of the electrons. The Lorentz force from the RR field is used to update the beam phasespace coordinates. Between every two field evaluation steps a number of particle motion updates are performed using symplectic integrator.

In evaluating the RR field, the instantaneous space-charge and compression field can be directly calculated using the current phasespace coordinates. For the retarded Lienard-Wiechert field, SymCSR saves the history of particle trajectory and solves the root of the retardation relation for each pair of source and test particle to compute the retarded time. To improve memory efficiency, SymCSR uses chunked ring buffer to record the past particle trajectory. It only records the phasespace trajectory in a fixed time window and shift the data after a certain number of time steps. In this way, the memory is contiguous and further computation can be accelerated.

In solving the retardation relation, SymCSR calculate the retarded phasespace coordinates by quadratic interpolating the discrete particle history. To accelerate the root finding, Chanrupatla algorithm [7] is implemented which is an optimized algorithm for shifting between bisect and secant method (inverse quadratic interpolation) to ensure fast covergence. To further accelerate the calculation, previous result of the retarded time is used as an initial guess for the root finding interval. MPI is used to split the calculation of N^2 interactions to all threads at each field evaluation step. The result of the RR field are then gathered and broadcasted to all threads for the update of phasespace coordinates.

For the case of large macroparticle number or long bunch length, allocating the history buffer will require a large amount of memory. To reduce the memory consumption, a sparse history sampling method can be applied. In this method, the particle phasespace coordinates are recorded in a much longer time step. A coarse retarded time is calculated based on this sparse particle history, and then the source particle is updated using the symplectic integrator around this coarse retarded time to generate a temporary particle history. A fine retarded time calculation is performed on this history to get the accurate retarded time.

RESULTS

First we discuss the convergence of SymCSR in integration step size and macroparticle number. Consider an electron beam with central energy $E_0 = 250\,\mathrm{MeV}$, $Q = 1.6\,\mathrm{fC}$, transverse emittance $\epsilon_\perp = 0.3\,\mathrm{nmrad}$, transverse angular spread $\sigma'_\perp = 20\,\mu\mathrm{rad}$, bunch length $\sigma_z = 12\,\mathrm{nm}$, energy spread $\sigma_\delta = 1 \times 10^{-3}$ and an undulator with undulator parameter $K = 0.6$, period $\lambda_u = 0.2\,\mathrm{m}$ and number of periods $N_u = 2$. Using macroparticle number $N_m = 600$, we calculate the emittance increase in three directions with total field evaluation steps $100, 200, 400, 600, 900, 1200$. The result is shown in Fig. 1.

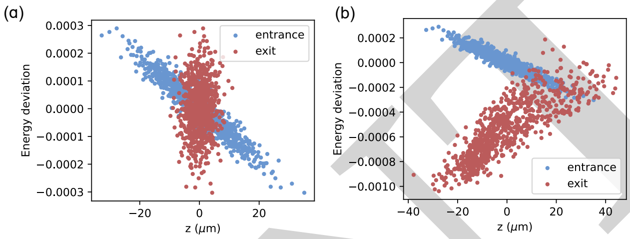

Figure 1: Beam emittance growth induced by CSR at the exit of undulator with various number of field evaluation time steps.

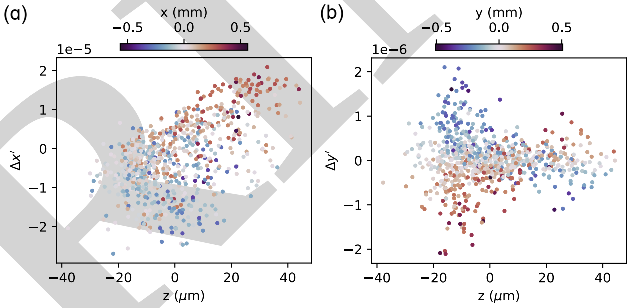

Figure 2: Beam emittance growth at the exit of undulator with different macroparticle number for RR field.

It can be seen that for number of time steps larger than 600 the results are convergent. Next, we fix the number of field evaluation steps to be 600, and calculate the emittance change for macroparticle number of $100, 300, 600, 900, 1200$. The results are shown in Fig. 2. For macroparticle number larger than 600, the results are close to convergent and the fluctuation of the results comes from the fluctuation of the macroparticle density distribution.

Using SymCSR, we can also calculate the emittance change considering the full Lienard-Wiechert field. The results are shown in Fig. 3. The fluctuation is approximately two orders of magnitude larger than the results of RR field, which is due to the space-charge singularity.

Next, we validate our calculation by comparing with CSR theory. To compare with 1D CSR theory, we consider a beam with $E_0 = 250\,\mathrm{MeV}$, $Q = 0.32\,\mathrm{fC}$, $N_m = 500$, $\sigma_\perp = 1\,\mu\mathrm{m}$, $\sigma'_\perp = 1\,\mathrm{nrad}$, $\sigma_\delta = 1 \times 10^{-10}$ and two different bunch lengths $\sigma_z = 1\,\mu\mathrm{m}, 10\,\mathrm{nm}$. For a dipole with bending radius $\rho = 10\,\mathrm{m}$, the calculated normalized radiation loss is shown

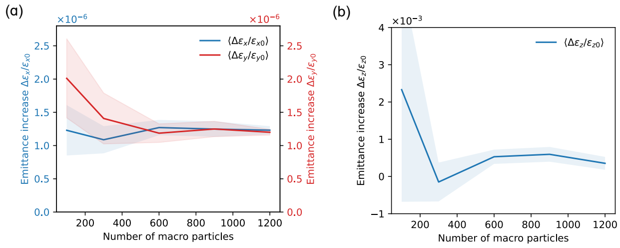

Figure 3: Beam emittance growth at the exit of undulator with different macroparticle number for Lienard-Wiechert field.

in Fig. 4. For comparison, the result using 1D theory is shown in Fig. 5.

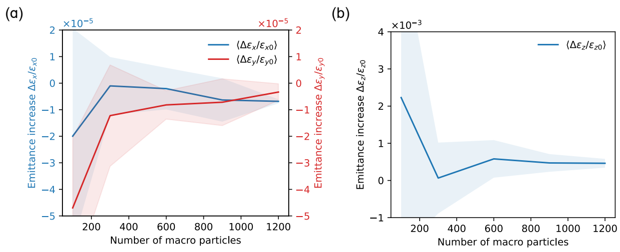

Figure 4: Normalized radiation loss in dipole calculated by SymCSR for beam with bunch length (a)1 µm and (b)10 nm.

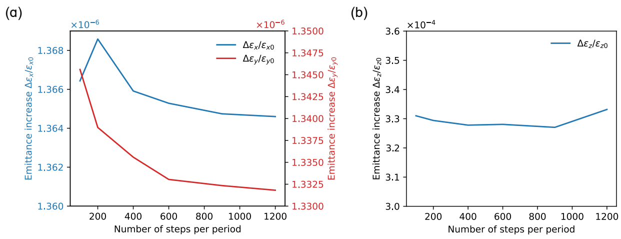

Figure 5: Normalized radiation loss in dipole calculated by 1D CSR theory for beam with bunch length (a)1 µm and (b)10 nm.

It can be seen that the results from SymCSR and 1D theory are exactly the same for the long bunch case. For short bunch with $\sigma_z = 10$ nm, the deviation of radiation energy loss compared to the 1D theory comes from the fact that the beam can no longer be regarded as a 1D line charge.

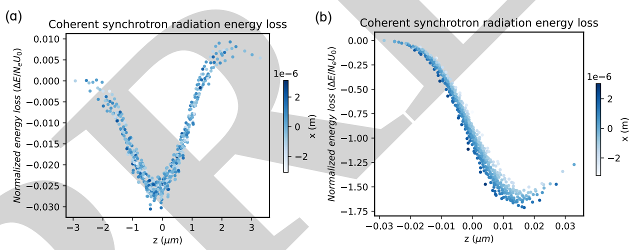

Figure 6: Longitudinal phasespace distribution of the beam at the entrance and exit of the chicane (a) without CSR effect and (b) with 6D CSR effect.

Finally, we calculate the 6D coherent synchrotron radiation effect of symmetric C-shape chicane. Consider the beam with $E_0 = 250$ MeV, $Q = 3.1$ pC, $N_m = 800$, $\sigma_\perp = 160$ µm, $\sigma'_\perp = 16$ µrad, $\sigma_z = 10$ µm, $\sigma_\delta = 3 \times 10^{-5}$ and energy chirp $h = 9\,\mathrm{m}^{-1}$. The chicane consists of four dipoles with bending angle $\theta = 0.1125$. The drift length between first two dipole is $L_{D1} = 3.55$ m, and the drift length between the second and third dipole is $L_{D3} = 0.5$ m. The length of the rectangular dipole is $L_B = 0.6$ m. The longitudinal phasespace of the beam at the entrance and exit of the chicane is shown in Fig. 6. Figure 6(a) shows the result without CSR effect considered, while Fig. 6(b) is the longitudinal phasespace distribution with 6D CSR effect. It can be seen that the CSR effect leads to a significant energy loss which under the effect of dispersion leads to the over-compression of the beam.

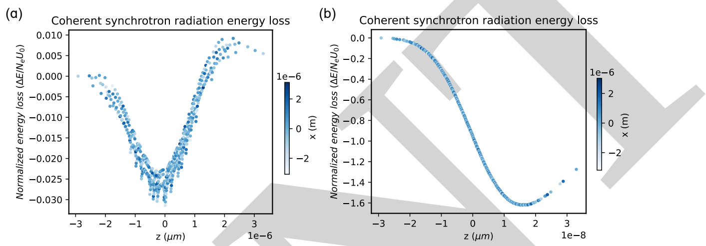

Figure 7: The relation between angular divergence change and the longitudinal and transverse position of the particle at the exit of chicane. (a) horizontal angular divergence change and (b) vertical angular divergence change.

Here, we also present the result of the change of transverse angular distribution of the beam induced by CSR effect. The result is shown in Fig. 7. The change in transverse angular divergence is given by the difference between the angular divergence at the chicane exit with and without CSR effect. The overall horizontal emittance growth is 38.2% and the vertical emittance growth is 0.20%.

CONCLUSION AND OUTLOOK

The simulation of ultrashort electron beams with coherent synchrotron radiation is a major challenge in the development of future light sources. In this paper, we introduced SymCSR, a first-principle tracking program for calculating the 6D phase space dynamics of electron beams with CSR effects. By calculating the RR field, SymCSR greatly reduce the computational demand. It supports the definition of lattices with arbitrary combinations of drift, dipole, and undulator by declaring the lattice file. Components such as quadrupole will be implemented in future versions.

ACKNOWLEDGEMENTS

This work was supported by National Key Research and Development Program of China (Grant No. 2022YFA1603403), Beijing Outstanding Young Scientist Program (No. JWZQ20240101006), National Natural Science Foundation of China (Grant No. 12522512, 124B2104) and center of High performance computing, Tsinghua University.

REFERENCES

[1] D. F. Ratner and A. W. Chao, "Steady-state microbunching in a storage ring for generating coherent radiation", *Phys. Rev. Lett.*, vol. 105, no. 15, p. 154801, Oct. 2010. doi:10.1103/PhysRevLett.105.154801

[2] X. Deng *et al.*, "Experimental demonstration of the mechanism of steady-state microbunching", *Nature*, vol. 590, no. 7847, pp. 576–579, Feb. 2021. doi:10.1038/s41586-021-03203-0

[3] Z. Pan, J. Zhao, C. Tang, X. Deng, and A. W. Chao, "Isochronous lattice design with time-of-flight fluctuation reaching subattoseconds based on high-order achromat", *Phys. Rev. Accel. Beams*, vol. 28, no. 12, p. 124001, Dec. 2025. doi:10.1103/3v8n-84fr

[4] Z. Li, X. Deng, Z. Pan, C. Tang, and A. Chao, "Generalized longitudinal strong focusing in a steady-state microbunching storage ring", *Phys. Rev. Accel. Beams*, vol. 26, no. 11, p. 110701, Nov. 2023. doi:10.1103/PhysRevAccelBeams.26.110701

[5] Z. Liu, X. Deng, T. Li, and L. Yan, "Instantaneous and retarded interactions in coherent radiation", *Phys. Rev. Lett.*, vol. 133, no. 16, p. 165001, Oct. 2024. doi:10.1103/PhysRevLett.133.165001

[6] Y. K. Wu, E. Forest, and D. S. Robin, "Explicit symplectic integrator for s-dependent static magnetic field", *Phys. Rev. E*, vol. 68, no. 4, p. 046502, Oct. 2003. doi:10.1103/PhysRevE.68.046502

[7] T. R. Chandrupatla, "A new hybrid quadratic/bisection algorithm for finding the zero of a nonlinear function without using derivatives", *Adv. Eng. Software*, vol. 28, no. 3, pp. 145–149, Apr. 1997. doi:10.1016/S0965-9978(96)00051-8