MODELING OF CSR AND ITS CANCELLATION IN DBA/CHICANE TYPE COMPRESSORS *

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Abstract

The most significant advances in accelerator-based light sources (i.e., x-ray free electron lasers) are driven by the production of a kiloampere-level peak current. As a prerequisite to achieve the desired high peak current, bunch compressors, like arc-type (DBA) and linear-type (chicane), are widely used to generate high-quality electron beams with several peak currents. However, a serious problem in increasing the peak current even higher is the significant degradation of beam quality caused by the coherent synchrotron radiation (CSR) effect. To tackle this, we develop a new analytical model for CSR that can describe beam transport with varying bunch lengths, establish a practical framework for analyzing CSR in both DBA and chicane-type compressors. General analytical conditions for CSR cancellation are derived for these designs. This work provides important guidance for enhancing the performance of existing accelerator facilities, as well as for the development of next-generation accelerator-based light sources.

INTRODUCTION

In modern accelerators, electron bunch compressors play a pivotal role, with wide applications in linear colliders, linacs, beam driven plasma-wakefield accelerators, and significant roles in x-ray free electron lasers (FELs) [1]. Combined with the position-energy correlation provided by the rf cavity, the following dispersive element converts the energy difference into a difference in the time of flight of the particles. This effect causes the particles at the head and tail to become closer, enabling beam compression [2].

Currently, symmetric C-chicanes and DBA-based compressors are the most commonly used bunch compressors in linac-based systems and arc systems, respectively. However, due to the high peak current required for FELs, the ability to compress an electron bunch with minimal degradation of beam quality becomes challenging because of the effects of coherent synchrotron radiation (CSR). CSR is emitted for wavelengths longer than or comparable to the length of the electron bunch, and leads to detrimental tail-head inter actions in the bends [3]. The emission of CSR results in projected transverse emittance growth [4]. In the past decades, various efforts have been stimulated to suppress CSR-induced emittance growth in chicane and DBA compressors, including analytical, numerical, and experimental studies [5–8]. Among them, the approach of manipulating beam optics has sparked continuing research interest.

The optical balance method was first proposed by Douglas [9] and was further developed by Courant-Snyder (C-S) formalism analysis [6]. Subsequently, the point-kick model was proposed and works well in transport systems where the bunch length remains constant or changes very little (hereafter referred to as the constant-σ_z point-kick model) [7, 8]. However, for magnetic bunch compressors, this model lacks self-consistency due to significant variations in bunch length within the bends.

To address this issue, in this study, we propose a self-consistent modified CSR point-kick model that accounts for variations in bunch length. Based on this model, we establish a practical framework for analyzing CSR in both DBA and chicane-type compressors, envisaging the potential designs for a CSR-immune DBA and chicane compressors.

MODIFIED CSR POINT-KICK MODEL

When considering only the steady state case and assuming the beam distribution in the dipole is Gaussian, The influence due to CSR along the bending angle can be derived based on the 1D CSR model [7].

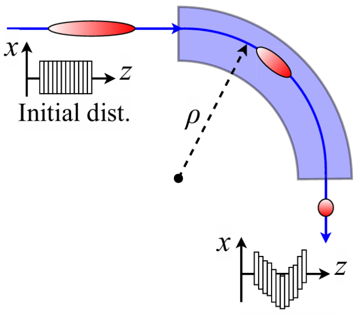

Figure 1: Schematic of the compression process after a bunch passes through a dipole, within sets of the initial beam distribution and final one affected by CSR in the $x-z$ phase plane.

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After a particle passes through a dipole magnet with a bending radius ρ and bending angle θ (see Fig. 1), its horizontal coordinate deviation and energy deviation relative to synchronous particle change under the influence of CSR as follows : [7]

$$\begin{pmatrix} x_{\mathrm{csr}} \\ x'_{\mathrm{csr}} \\ \delta_{\mathrm{csr}} \end{pmatrix} = k \begin{pmatrix} \rho^{4/3}(\theta - \sin\theta) \\ \rho^{1/3}(1-\cos\theta) \\ \rho^{1/3}\theta \end{pmatrix}, \tag{1}$$

the CSR strength parameter k corresponds to the initial bunch length and is in the form

$$k = 0.2459 \frac{N_b r_c}{\gamma \sigma_{z0}^{4/3}}. \tag{2}$$

We develop a new analytical model for CSR that can describe beam transport with varying bunch lengths (a more detailed analysis can be found in Ref. [10])

$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} \rho^{4/3}k[g_x\theta\cos(\frac{\theta}{2}) - 2\sin(\frac{\theta}{2})(g_x\cos^2(\frac{\theta}{2}) + g_{xp}\sin^2(\frac{\theta}{2})] \\ \sin(\frac{\theta}{2})[2\delta + \rho^{1/3}k(g_x\theta + (g_{xp} - g_x)\sin\theta)] \end{pmatrix}, \tag{3}$$

where g_x, g_{xp} and g_δ are the dimensionless correction functions, which can be expressed as

$$\begin{aligned} g_x([a,b],\xi) &= 3\int_0^1 \left[1 + \xi\frac{a\tau + b\tau^2 + \tau^3}{a+b+1}\right]^{-4/3}(1-\tau)^2\,d\tau, \\ g_{xp}([a,b],\xi) &= 2\int_0^1 \left[1 + \xi\frac{a\tau + b\tau^2 + \tau^3}{a+b+1}\right]^{-4/3}(1-\tau)\,d\tau, \\ g_\delta([a,b],\xi) &= \int_0^1 \left[1 + \xi\frac{a\tau + b\tau^2 + \tau^3}{a+b+1}\right]^{-4/3} d\tau. \end{aligned} \tag{4}$$

where the variable $\xi \equiv \sigma_{zf}/\sigma_{z0} - 1$ represents the rate of bunch length variation after passing through the dipole, the parameters a and b are related to η_0 and η'_0, respectively, and can be written as

$$a = 6\frac{\eta_0}{\rho\theta^2}, \qquad b = 3\frac{\eta'_0}{\theta}. \tag{5}$$

They reflect the impact of the dipole on the dispersion function. By setting all correction functions are equal to 1, the resulting CSR kick in Eq. (4) can be reduced to the constant-σ_z point-kick model:

$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} \rho^{\frac{4}{3}}k\left[\theta\cos\left(\frac{\theta}{2}\right) - 2\sin\left(\frac{\theta}{2}\right)\right] \\ \sin\left(\frac{\theta}{2}\right)\left(2\delta + k\rho^{\frac{1}{3}}\theta\right) \end{pmatrix}.$$

This constant-σ_z model is more appropriate to the transport system where the bunch length has a small variation.

APPLICATION TO A DBA COMPRESSOR

Next we try to apply the varying-σ_z CSR model to the physical design of DBA (Fig. 2). For the dispersion function of DBA, the dipole with zero entrance dispersion and dispersion gradient, where $a = b = 0$. In this case, the correction functions can be expressed in the following analytical form: [10]

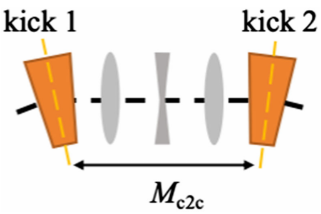

Figure 2: Schematic of a DBA and the physical model for the CSR point-kick analysis in the DBA. The lattice section between two dipoles containing two quadrupoles and one skew quadrupole. The kicks point to the centers of the bends.

$$\begin{aligned} g_x([0,0],\xi) &= 3\frac{-1 + (1+\xi)^{1/3}}{\xi(1+\xi)^{1/3}} \\ &\quad - 3\frac{\xi\left[{}_2F_1\left(\frac{1}{3},1,\frac{5}{3},-\xi\right) - 1\right]}{\xi(1+\xi)^{1/3}}, \\ g_{xp}([0,0],\xi) &= \frac{2 - 2\,{}_2F_1\left(\frac{1}{3},1,\frac{5}{3},-\xi\right)}{(1+\xi)^{1/3}}, \\ g_\delta([0,0],\xi) &\approx \frac{1}{(1+\xi)^{1/3}}. \end{aligned} \tag{6}$$

where ${}_2F_1$ denotes the standard hypergeometric function. The dispersion and dispersion gradient at the dipole exit are zero. In this case, $a = 3$ and $b = -3$ can be derived. The corresponding correction functions can be expressed analytically as [10]

$$\begin{aligned} g_x([3,-3],\xi) &= 3\frac{(1+\xi)^{1/3} - 1}{\xi(1+\xi)^{1/3}}, \\ g_{xp}([3,-3],\xi) &\approx \frac{1}{(1+\xi)^{3/4}}, \\ g_\delta([3,-3],\xi) &\approx \frac{1}{1+\xi}. \end{aligned} \tag{7}$$

For the g_{xp} and g_δ functions, we utilized approximate forms to avoid overly complex expressions. We calculate the corresponding θ_2/θ_1 and r_{21} for a given C: (A more detailed analysis can be found in Ref. [10])

$$\begin{aligned} \frac{\theta_2}{\theta_1} &\equiv \zeta = \frac{(3g_{xp1} - 2g_{x1})^{1/4}}{(3g_{xp2} - 2g_{x2})^{1/4}} C_1^{-\frac{1}{3}}, \\ r_{21} &= \frac{6\zeta(2g_{\delta 1} - g_{xp1} + g_{xp2}\zeta C_1^{4/3})}{(3g_{xp1} - 2g_{x1})\rho\theta_1}. \end{aligned} \tag{8}$$

These results can be reduced to the cancellation conditions using constant-σ_z model by setting $g_x = 1, g_{xp} = 1, g_\delta = 1$:

$$\frac{\theta_2}{\theta_1} = C_1^{-\frac{1}{3}}, \qquad r_{21} = \frac{6(1 + C_1)}{C_1^{1/3}\rho\theta_1}. \tag{9}$$

As shown in Fig. 3, the discrepancy between the constant-σ_z model and the modified model is notable, emphasizing the importance of accounting for the bunch length

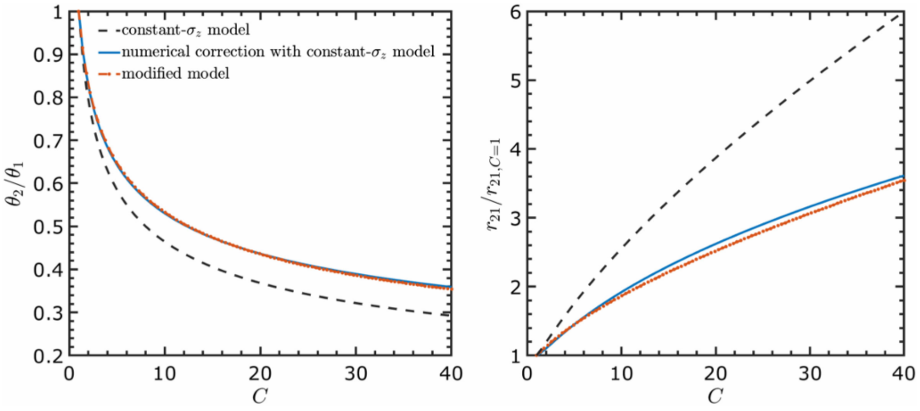

Figure 3: The bending angle ratio and matrix element r_{21} as functions of the total compression ratio C.

variation within the dipole. The numerical corrections and the modified model demonstrate excellent agreement for C up to 40, with a relative discrepancy of less than 1.5% in θ_2/θ_1 and less than 4% in r_{21}, respectively. Notably, the numerical fit coefficients used in the numerical corrections were originally derived for C less than 15, which illustrates the strong generalizability of the numerical corrections from Ref. [11].

APPLICATION TO A CHICANE COMPRESSOR

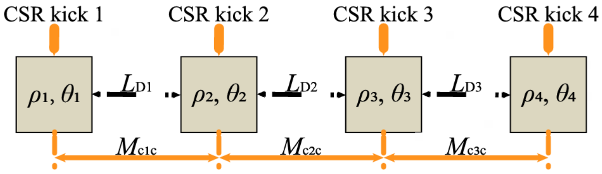

Figure 4: Schematic of the point-kick analysis of the CSR ef-fects.

We consider a general chicane model consisting of four bends separated by three adjustable drifts (see Fig. 4). The length of each bend is L_B, while the bending angles of the four bends are not subject to a specific constraint. We derive the analytical expressions of net CSR kick at the chicane exit and further the ss-CSR cancellation conditions using a modified CSR-kick model (see Ref. [12]). The calculation result show that the unique way of canceling the CSR kicks in a four-bend chicane is to reshape the chicane to a non-symmetric S-shape layout (see Fig. 5).

By setting $L_B = 0$ and assuming that all correction functions are equal to 1, the ss-CSR cancellation conditions can be reduced to the constant-σ_z point-kick model. Using the modified CSR-kick model, the parameters for a ss-CSR-immune chicane are plotted in Fig. 6, compared to the simplified results (dot-dashed curves). It can be observed that these angle parameters (q_3, q_4) exhibit relatively small errors, while there are relatively larger errors in the drift parameters (ℓ_2, ℓ_3).

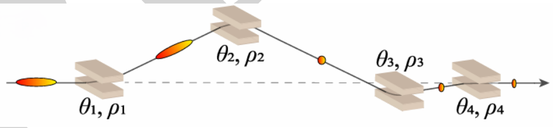

Figure 5: Schematic of a generic four-bend chicane (not to scale).

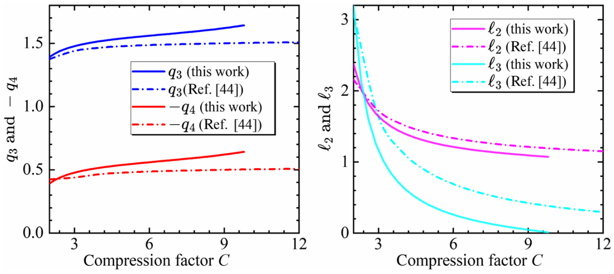

Figure 6: Calculated $(q_3, q_4, \ell_2, \ell_3)$ for $\Lambda = 0.81 : 5$ for a partial-CSR-immune chicane and ss-CSR-immune chicane.

CONCLUSION

We have proposed a self-consistent modified CSR point-kick model that accounts for variations in bunch length. Then we presented a solvable model of the CSR effects in a generic four-bend chicane and DBA, which provides us insights on CSR compensation during beam compression. The design of the CSR-immune DBA and chicane compressors are successfully completed. For CSR-immune DBA, the discrepancy between the constant-σ_z model and the modified model is notable, emphasizing the importance of accounting for the bunch length variation within the dipole. For the CSR-immune chicane, our results demonstrate that the modified model exhibits only minor deviations, with the constant-σ_z model yielding better performance.

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