INVESTIGATIONS OF BETATRON COUPLING AND HORIZONTAL PARTIAL SNAKE RESONANCES AT LOW ENERGIES IN THE AGS

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Abstract

Compensation of depolarizing partial snake resonances using betatron coupling has been demonstrated at the Brookhaven AGS. At the nominal acceleration rate, the depolarization of the proton beam from any one of the 82 resonances is too small to optimize the compensation each individual resonance empirically. The compensation therefore requires accurate modeling of the accelerator lattice and accounting of both known sources of such resonances (the helical dipole and applied skew quadrupole fields) and unknown sources (e.g. sextupole feed-down effects). At low energies in the AGS these efforts are complicated by the large optical effects of the helical dipoles and the near-integer vertical tune required to avoid strong vertical spin resonances. Individual resonances can be investigated by crossing the resonance more slowly, in this case at fixed energies with a slow tune ramp. We report here on progress in both the modeling and experimental investigations into these depolarizing mechanisms.

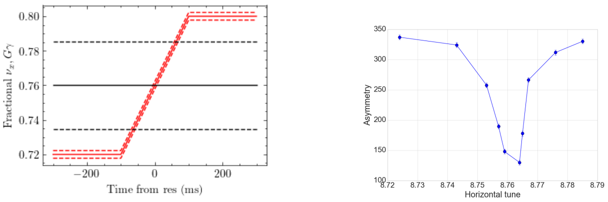
HORIZONTAL RESONANCES

Helical dipole partial snake magnets are used to avoid strong vertical depolarizing spin resonances in the AGS. These magnets help avoid depolarization from strong imperfection and intrinsic resonances associated with the vertical betatron motion. The 'partial' nature of the snakes (less than full spin flip in one transit) causes an interaction between the spin motion and the horizontal spin motion that excites weak resonance at energies corresponding to $G\gamma = N \pm \nu_x$, where G is the anomalous gyromagnetic ratio, γ the relativistic factor, ν_x the horizontal tune and N any integer. These resonances are weak, but many (82 in the AGS energy range) and result in polarization loss [1]. Since these resonances occur at the same frequencies as spin resonances from betatron coupling, they can be corrected by a set of skew quadrupoles. The design and operational experience with such a system are described in [2, 3]. The resonance compensation from the additional skew quadrupoles relies on the resulting betatron coupling producing motion in the rest of the accelerator lattice, such that the extra spin precessions exactly cancel those introduced by the snake resonances. The correction therefore relies on accurate knowledge of the lattice optics for two different reasons. First, the resulting spin resonance depends on the beta functions and phase advances around the ring and second, because unknown sources of coupling (e.g. from magnet misalignment) produce unknown drive terms that compete with the correction and reduce its efficacy.

SINGLE RESONANCE CROSSINGS

Initial commissioning of the skew quadrupole resonance compensation system was done by correcting only resonance at relatively high energy (above $\gamma \approx 8.5$). This was done to avoid complications of the optical effects of the helical dipoles and large closed orbit kicks from the skew quads interacting with relatively large horizontal closed orbit offsets at low energy in the AGS. It was noticed that as low energy resonances were gradually added to the correction, the polarization improvement was actually reduced, instead of increased, implying that there were large errors in the calculated corrections. Since the effects of any individual resonance are too small to measure ($\approx 0.1\%$ loss per resonance), investigation of individual resonances requires specialized setups.

The normal acceleration ramp is truncated and a flattop of constant energy is held near the desired resonance. In this case we chose the resonance at $G\gamma = 7.24$. This is low enough that the optical errors are still important, but far enough from injection ($G\gamma = 4.5$) to avoid interference with injection processes. The horizontal tune is set near the resonance condition ($\nu_x \approx 8.76$) and slowly varied, to reduce the resonance crossing rate and exaggerate the polarization loss. For a tune change of $\Delta\nu_x = 0.08$ over 200 ms, the resonance crossing rate $dG\gamma/d\theta = \alpha = 1.7 \times 10^{-7}$, a reduction in crossing rate by a factor of over 250 compared to crossings at the nominal AGS acceleration rate ($\alpha = 4.7 \times 10^{-5}$) (Fig. 1a). Here θ is the ring azimuth. At this rate, relative polarization losses of up to 20 % can be achieved which are easily measurable. The resonance crossing condition is determined by pausing the tune ramp for 50 ms at tunes



(a) (b)

Figure 1: (a) Schematic of nominal resonance crossing. Horizontal betatron tune is in red, energy in units of $G\gamma$ is in black. Dashed lines indicate the 4σ spread of the quantities due to the beam momentum spread (rms $dp/p = 0.001$. (b) The resonance tune is determined by pausing the tune ramp at various values for ±25 ms around the expected resonance time. Polarization is lowest when paused exactly on resonance.

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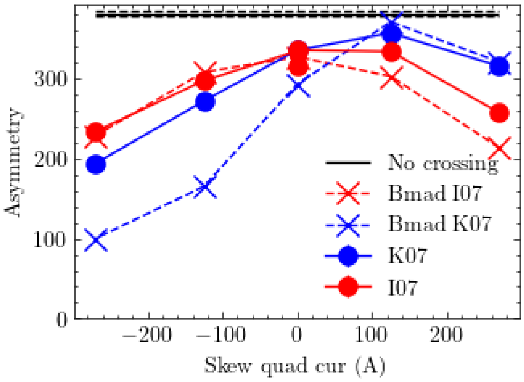


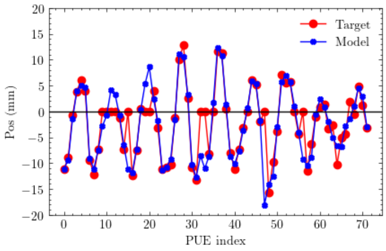
Figure 2: Scan of two skew quadrupole currents during a single slow resonance crossing with the helical dipoles on. Measurements (filled circles) are taken while scanning two skew quads are chosen to drive resonances that are orthogonal to the snake drive (red) or parallel to it (blue). The labels I07 and K07 refer to the magnet location in the lattice. Dashed lines show simulated resonance crossings using the Bmad code. The agreement deteriorates for large resonance strength. The black lines indicate the polarization without the crossing (constant tune held below the resonance).

throughout the crossing and looking for minimum polarization transmission (Fig. 1b).

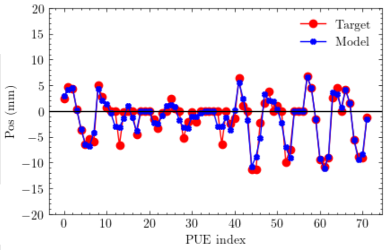
With a single, strong resonance crossing established, individual skew quadrupole currents can be scanned to measure the compensation effect. To verify the model predictions, two skew quadrupoles are chosen, one of which is predicted to produce a resonance in phase with the snake driving term and one which is predicted to be orthogonal. Figure 2 shows results of such a scan and a comparison of those results to Bmad [4] tracking runs. Without correction (applied skew quadrupole strength), the measurement polarization loss from the crossing is $16\,\%\pm1\,\%$. The peak polarization with correction occurs at the expected currents, which implies that the phasing between the skew quad and snake driving terms are correctly predicted. The disagreement for the K07 quadrupole is not well understood, but is comparable to effects observed during similar proof of principle experiments at high energy where for large correction currents, the disagreement sometimes increases. One possible cause is underlying horizontal resonance driving terms other than the snakes or the applied skew quadrupole currents, like betatron coupling error terms. This motivated measurements of the resonance in the absence of snake currents.

In the absence of helical snakes or machine errors, the horizontal resonance strength would be zero. The closed orbit in the AGS is not zero and vertical offsets in particular can cause horizontal spin resonance via betatron coupling from sextupole feed-down. An acceleration cycle was developed to the same low energy plateau at $G\gamma$=8.74, without the use of helical dipoles. Since this is below the first major depolarizing resonance at $G\gamma = 9$, significant polarization remains for measurements even without the snakes.

The measured polarization loss without snakes for the same crossing is $4\,\%\pm1\,\%$. For comparison with a Bmad model, we mock up the measured closed orbit errors of the real machine by applying corrector dipole currents to fit the model orbit to measurements. Fitted orbits are shown in Fig. 3. The modeled polarization for the same machine setup with tunes and orbit fit to measurements and as-run chromaticity sextupole currents ($I_{h,v} = 38, 31A$) and 2 µm horizontal emittance shows a 5 % loss of polarization, in good agreement with the measurement. This polarization loss is 1/3 the amplitude of the polarization loss with the snakes on, which demonstrates that sextupole feed-down is an effect competitive with the snake drive term and that it is predictable in the model. The polarization loss in the case where the sextupoles and the snakes are turned off is statistically consistent with zero.



(a) Horizontal



(b) Vertical

Figure 3: Measured and model closed orbits. Model orbits are determined by fitting orbit correctors to match measured target. Positions shown as identically zero are bad measurements excluded from the fit. The vertical closed orbit can cause spin depolarization at horizontal resonance via sextupole feed-down.

The effects of an individual sextupole can be measured with a local vertical bump at the sextupole location (Fig. 4). A 5 mm displacement in a single sextupole can cause $\pm10\,\%$ relative change in the polarization transmission.

An estimate can be made of the closed orbit impact on the polarization transmission of the nominal acceleration cycle by fitting measured orbits for every resonance and recalculating the resonance strengths. The closed orbit deviations are largest at injection, but are largely constant after $\gamma \approx 8$. The resonance strengths on a zero design orbit (only snake drive term) and for a model with an orbit fit to measurements (includes feed-down effects) are shown in Fig. 5. Because the relative phasing of the resonance drive terms from the snakes and the individual sextupoles is energy dependent, the impact on the net drive term varies strongly during the

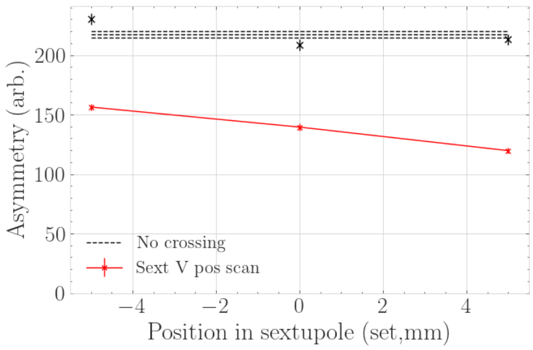


Figure 4: Scan of a local vertical orbit bump at a single chromaticity sextupole for the $G\gamma = 8.74$ crossing.

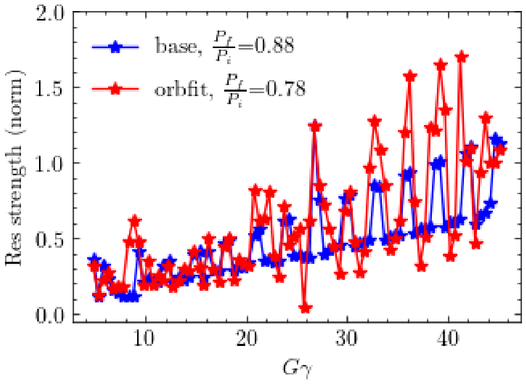


Figure 5: Depolarizing horizontal resonance strengths (normalized to the rms emittance), calculated using Bmad for the AGS acceleration cycle. The ‘base’ case (blue) has a zero closed orbit and includes only the snake drive terms. The ‘orbfit’ case (red) has the model orbit fit to the measured machine orbits at each resonance. The impact of the closed orbit is energy dependent and results in a net relative polarization loss of ≈10 % over the base snake-only case

ramp. Some resonances are weakened, some strengthened by the closed orbit offsets, even when the orbit is relatively constant. The model estimates that at the nominal acceleration rate, the closed orbit effects are responsible for an additional 10 % relative polarization loss over that caused by the snakes.

DISCUSSION AND CONCLUSION

One of the remaining sources of polarization loss in the AGS is horizontal spin resonances excited by the horizontal betatron motion in the snakes. A pulsed skew quad system has been implemented to combat these resonances. A series of measurements of single resonance crossings at strongly reduced crossing rate have been carried out to investigate why this correction system does not yield the full expected correction. We have found that closed orbit errors, especially vertical offsets in the sextupoles are responsible for at least some of the correction inefficiency. Skew quadrupole based correction in the presence of even horizontal closed orbit errors is complicated by the fact that the skew quad pulses themselves change the closed orbit and therefore the underlying resonance strength. The present orbit correction steerers are too weak to fully correct the residual closed orbit errors to zero. Two improvement efforts are underway in order to address the correction inefficiency. First, all the AGS magnets will be surveyed and realigned (including all 240 combined function gradient dipoles). This will bring the residual closed orbit closer to the reach of the orbit correction system. Turn-by-turn BPMs electronics will also be added to all the AGS BPMs, which can presently only measure the average orbit. This will help identify residual sources of unknown coupling that may be degrading the polarization. These improvements will take place during the present AGS shutdown period between the end of RHIC operations in Feb. 2026 and resumption of AGS beam operation for polarization development in a few years (exact dates to be determined).

ACKNOWLEDGEMENTS

Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy and in part by the grants DE-SC0024287 and DE-SC0025351.

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