STUDIES ON OPTIMIZATION OF A TILTED-SOLENOID MUON IONIZATION COOLING CHANNEL IN G4BEAMLINE

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Abstract

Ionization cooling is a key requirement for future muon collider facilities. We study a cooling-channel demonstrator in which tilted solenoids are used to modify the lattice optics and induce transverse-longitudinal coupling without explicit dipole magnets. Using G4Beamline simulations for a mean momentum 200 MeV/c muon beam, we evaluate the dependence of beam cooling and transmission on solenoid tilt angle and RF phase. Tilt angles near 0.75°–1° are found to provide the most favorable overall performance, yielding strong six-dimensional emittance reduction while preserving longitudinal stability and transmission. The channel also shows useful tolerance to RF phase variation within 10°–30°. These results support tilted-solenoid lattices as a viable option for optics control in muon cooling channels.

INTRODUCTION

Muon colliders are an attractive option for future high-energy lepton collisions because they can reach multi-TeV center-of-mass energies without the severe synchrotron-radiation limitations of electron machines. Their feasibility, however, depends critically on rapid reduction of the large initial muon phase space within the short muon lifetime [1]. Ionization cooling, in which momentum is reduced in absorbers and restored by RF cavities, remains one of the practical method for achieving this emittance reduction on the required timescale [2].

Recent studies within the International Muon Collider Collaboration (IMCC) have considered cooling-channel concepts that use dipole fields to generate dispersion and enable emittance exchange [1]. In this study, we investigate an alternative approach in which the lattice optics are modified through controlled tilting of solenoidal elements. The goal is to assess whether this tilted-solenoid configuration can provide useful transverse-longitudinal coupling while preserving the cooling performance of a periodic channel composed of solenoids, RF cavities, and wedge absorbers.

The study is intentionally restricted to the cooling channel itself; upstream production, capture, and matching sections are not included. This allows the intrinsic response of the lattice to be evaluated in a controlled manner. Using G4Beamline [3], we examine the dependence of beam cooling, longitudinal stability, and transmission on both the solenoid tilt angle and the RF phase.

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SIMULATION SETUP

A low-energy μ^+ beam with mean momentum 200 MeV/c is injected directly into the cooling channel. The beam is generated with Gaussian distribution with a time spread of 0.140 ns and a longitudinal momentum spread of 15 MeV/c.

The simulated channel is based on an IMCC-inspired periodic lattice consisting of pancake-coil solenoids, RF cavities, and wedge absorbers [1]. The key modification studied here is the application of equal and opposite tilt angles to the solenoid groups on the two sides of each cell. This tilt pattern is repeated throughout the lattice and serves as the primary mechanism for perturbing the optics and generating transverse-longitudinal coupling. The principal lattice parameters are summarized in Table 1, and a representative visualization of the geometry is shown in Fig. 1.

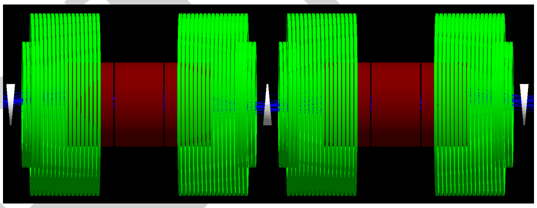

Figure 1: Visualization of two periods of the simulated cooling-channel lattice. The structure includes solenoidal focusing elements (green), RF cavities (red), and wedge absorbers (white). Equal and opposite tilt angles about the horizontal (x) axis (vertical in this view) are applied to the solenoid groups on either side of each cell.

Table 1: Key Parameters of the Simulated Cooling Channel

Parameter	Value
Cell length	1000 mm
Number of cells	50
Reference momentum	200 MeV/c
RF frequency	704 MHz
RF gradient	30 MV/m
RF phase	20 deg
Large solenoid inner/outer radius	285 mm and 355 mm
Small solenoid inner/outer radius	185 mm and 245 mm
Large/small solenoid pancakes	17 / 5
Large/small solenoid current	328 A and 300 A
Tilt angle	scan: 0.25°–2°
Absorber material	LiH
Wedge angle	10 deg

The simulations are performed with G4Beamline [3], based on Geant4 [4], using the QGSP_BERT physics list.

Particle distributions are recorded at regularly spaced longitudinal positions and processed with ECALC9 to extract derived beam quantities, including transverse, longitudinal, and six-dimensional emittances, average longitudinal momentum, and transmission. The results are presented in two stages: first, a scan of solenoid tilt angles is performed; second, an RF-phase scan is used to evaluate the sensitivity of the preferred tilt configurations to longitudinal tuning.

RESULTS

Dependence on Solenoid Tilt Angle

The performance of the cooling channel is evaluated as a function of the solenoid tilt angle, while all RF parameters are fixed to the baseline configuration given in Table 1.

The evolution of the transverse emittance is shown in Fig. 2. All configurations exhibit a rapid initial reduction within the first ~20 m, followed by a slower cooling regime. In contrast to other observables, the transverse emittance shows only a weak dependence on the tilt angle over most of the channel. While very large tilt angles lead to some degradation at larger z, the differences among small and moderate tilt configurations remain relatively modest. This indicates that transverse cooling is primarily governed by ionization cooling and focusing strength, and is less sensitive to the precise tilt configuration.

The longitudinal emittance evolution, shown in Fig. 3, exhibits a stronger dependence on the tilt angle. For small and moderate tilts, the longitudinal phase space remains relatively stable after the initial transient region. In contrast, larger tilt angles introduce significant fluctuations and growth in ε_L, indicating increased perturbation of the longitudinal beam dynamics and a degradation of phase stability.

The combined impact on overall beam quality is reflected in the six-dimensional emittance, shown in Fig. 4. A clear separation emerges between configurations: moderate tilt angles achieve the most effective reduction in ε_{6D}, while larger tilts degrade performance due to the combined effects of longitudinal emittance growth and reduced transmission. This demonstrates that the optimal operating point is not determined by transverse cooling alone, but by the balance between transverse and longitudinal dynamics.

The evolution of the average longitudinal momentum is shown in Fig. 5. The oscillatory structure reflects the periodic acceleration and energy loss associated with the RF cavities and absorbers. The amplitude and stability of these oscillations provide a sensitive probe of longitudinal matching. For small and moderate tilt angles, the oscillations remain well-controlled and centered around a stable mean value. In contrast, larger tilt angles lead to increased oscillation amplitude and drift, indicating a mismatch between the RF re-acceleration and energy loss in the absorbers. This behavior is consistent with a degradation of longitudinal focusing and phase stability, rather than an insufficient RF gradient alone.

The particle transmission along the channel is shown in Fig. 6. All configurations exhibit an initial loss associated with injection mismatch, followed by a more gradual decrease along the channel. A clear dependence on tilt angle is observed: larger tilt angles lead to significantly reduced transmission, indicating increased particle loss and reduced transport efficiency.

Taken together, these results indicate that while transverse cooling is relatively robust against variations in tilt angle, the longitudinal dynamics and transmission are significantly affected. Moderate tilt angles, in particular in the range of 0.75° to 1°, provide a favorable balance: they preserve longitudinal stability, maintain high transmission, and achieve strong overall reduction in six-dimensional emittance. In contrast, larger tilt angles introduce optical distortions that disrupt longitudinal matching and reduce transport efficiency.

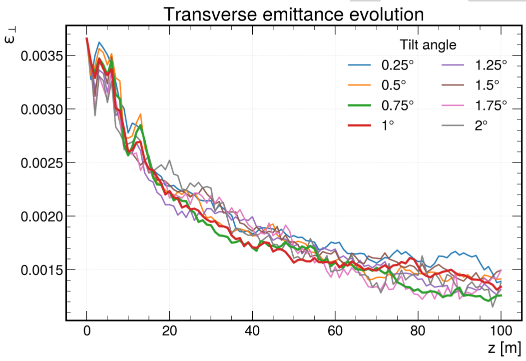

Figure 2: Evolution of the transverse emittance $\varepsilon_\perp$ along the cooling channel for different solenoid tilt angles.

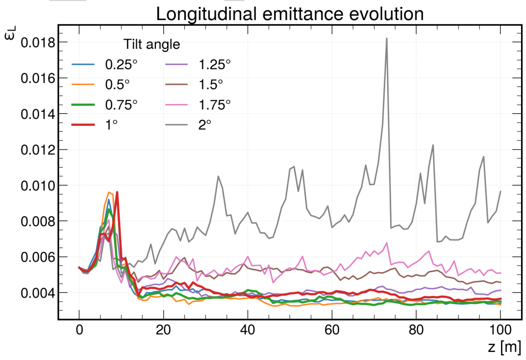

Figure 3: Evolution of the longitudinal emittance ε_L along the cooling channel.

Dependence on RF Phase

Motivated by the identification of an optimal tilt range of 0.75°–1°, a scan of the RF phase is performed while keeping all other parameters fixed, in order to evaluate the sensitivity of the cooling channel to longitudinal phase matching.

The evolution of the six-dimensional emittance and the average longitudinal momentum is shown in Fig. 7. Across the range 10°–30°, the six-dimensional emittance exhibits very similar behavior, indicating that the overall cooling performance is largely insensitive to RF phase within this region. In contrast, larger phase offsets lead to a degradation in performance, including increased emittance and reduced transmission.

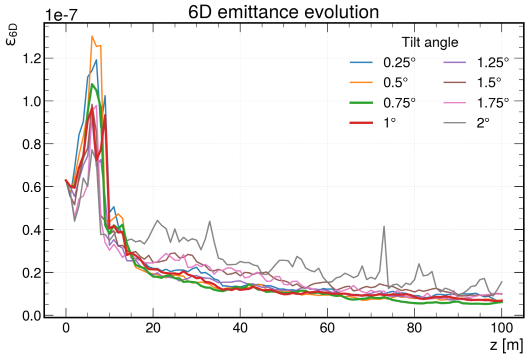

Figure 4: Evolution of the six-dimensional emittance ε_{6D} along the cooling channel.

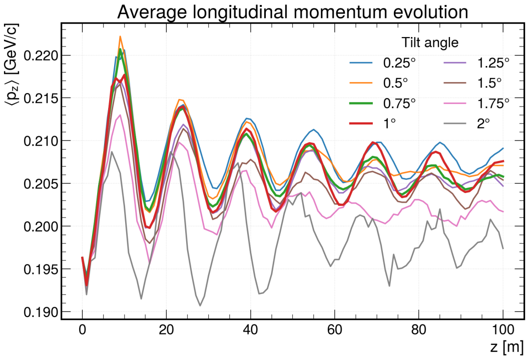

Figure 5: Evolution of the average longitudinal momentum $\langle p_z \rangle$ along the cooling channel.

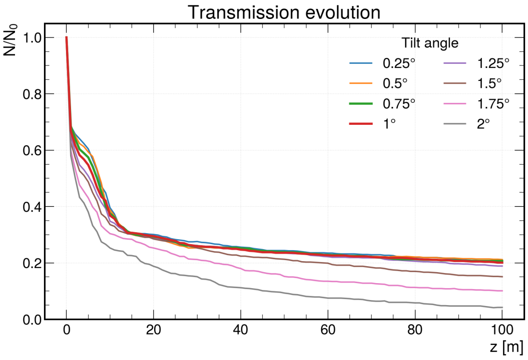

Figure 6: Particle transmission along the cooling channel for different solenoid tilt angles.

The evolution of $\langle p_z \rangle$ provides additional insight into this behavior. The oscillatory structure reflects the balance between energy loss in the absorbers and re-acceleration in the RF cavities. For phases between 10° and 30°, these oscillations remain stable and well-centered, indicating effective energy compensation and good phase matching. At larger phase offsets (e.g., 40°), the oscillations increase in amplitude and become less stable, consistent with a mismatch between the RF phase and the beam arrival time.

These results indicate that the cooling channel exhibits a relatively broad operational tolerance to RF phase, provided that longitudinal stability is maintained. A phase in the range 20°–25° is therefore selected as a representative operating point for subsequent studies.

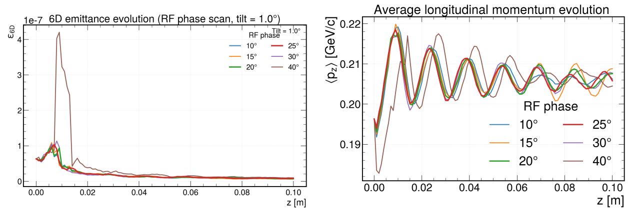

Figure 7: Dependence of cooling performance on RF phase. Left: evolution of the six-dimensional emittance ε_{6D}. Right: evolution of the average longitudinal momentum $\langle p_z \rangle$.

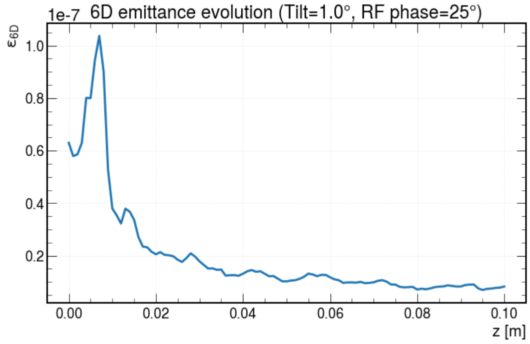

Figure 8: 6D emittance evolution for the selected point (1°, 25°).

CONCLUSION

A tilted-solenoid muon cooling channel has been studied with G4Beamline and ECALC9. Transverse cooling is found to be relatively insensitive to the tilt angle, while longitudinal stability and transmission are more strongly affected by the optics perturbation. An optimal operating region is identified around tilt angles of 0.75°–1° and RF phases of 10°–30°. A representative configuration with 1° tilt and 25° RF phase shows strong initial reduction of the six-dimensional emittance followed by a gradual approach to equilibrium, as shown in Fig. 8. Overall, tilted-solenoid configurations provide a viable mechanism for introducing transverse–longitudinal coupling in muon cooling channels. Future work will include improved beam matching, refined RF optimization, and extension to more realistic end-to-end cooling-channel studies.

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