# BOOTSTRAPPING INJECTION CONDITIONING STUDY FOR FCC-EE COLLIDER

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## Abstract

Because of the strong beam–beam force at the interaction points in the FCC-*ee* collider, maintaining the charge balance between the two opposing beams is crucial. Injection of a high-charge beam introduces temporary imbalance, which can lead to beam instability. To mitigate this, bootstrapping injection has been adopted for FCC-*ee*. In this work, the conditioning of bootstrapping injection is studied using a quasi-strong-strong beam–beam scheme implemented in the SAD code. The charge threshold of injected beams for different collision modes is evaluated, and the impact of beam–beam dynamics on injection stability is discussed.

## INTRODUCTION

The FCC-*ee* collider is designed to operate in a strong beam–beam regime [1]. In such conditions, the injection process becomes sensitive to nonlinear beam–beam dynamics.

During injection, an imbalance between the two colliding beams leads to asymmetric beam–beam forces, which may induce emittance growth and particle loss. Understanding and controlling this imbalance is therefore essential.

In the present study, a simplified configuration with one bunch per beam is considered. Although the FCC-*ee* design includes four interaction points, collisions occur at two interaction points in the present simulations. Injection dynamics are investigated using a quasi-strong-strong (QSS) simulation scheme.

## QUASI-STRONG-STRONG SCHEME

The quasi-strong-strong (QSS) scheme provides an efficient framework for simulating beam–beam interaction with partial self-consistency. The beam–beam force is evaluated interaction point by interaction point along the ring, allowing the effect of individual collisions to be incorporated sequentially within a single turn, while maintaining a weak–strong tracking structure. For this evaluation, the beam parameters immediately upstream of each interaction point are used, lattice effects are naturally incorporated. The QSS scheme provides a partially self-consistent beam–beam interaction by sequentially updating the beam–beam lens using the instantaneous beam moments at each interaction point (Fig. 1).

The simulation is performed using the SAD code [2], enabling the study of injection dynamics in a realistic lattice environment with manageable computational cost.

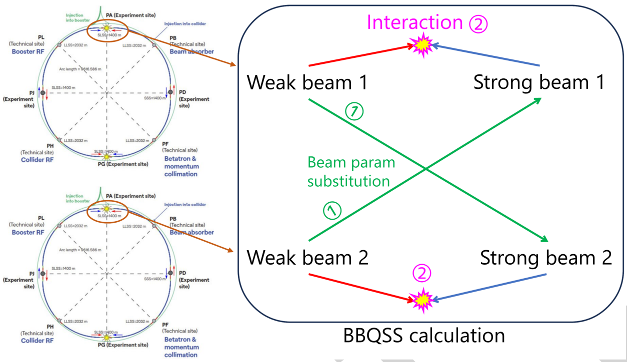

Figure 1: Schematic illustration of the quasi-strong-strong (QSS) beam–beam calculation and bootstrapping injection model. Beam–beam interactions are evaluated interaction point by interaction point within a single turn. In the present study, collisions occur at two interaction points (corresponding to IPA and IPG), although the FCC-*ee* design includes four interaction points. The effective bunch population is increased by injection, while macroparticles are not replenished and are only removed through beam loss during tracking.

### Implementation Concept

In the present implementation, QSS is realized by coupling two weak–strong beam–beam simulations that represent the two counter-rotating beams. At each interaction point, the parameters of the opposing strong beam are substituted using the moments evaluated from the corresponding weak-beam tracking immediately before the collision. This substitution is performed at every interaction point, thereby updating the beam–beam elements sequentially along the ring while avoiding the computational cost of full strong–strong tracking.

The updated strong-beam parameters include the beam centroid coordinates and second moments relevant to transverse–longitudinal coupling (e.g. $(x, y, x', y', z, \delta)$, $(\beta_{x,y}, \varepsilon_{x,y})$, $(\sigma_{z,\delta})$, and selected cross-moments such as $xz$, $x'z$, $yz$ and $y'z$), enabling the study of multi-dimensional effects within the full-lattice framework. Here, standard accelerator coordinates and beam moments are used, following conventional definitions in accelerator physics.

## INJECTION MODEL

The injection process is modeled by increasing the effective bunch population,

$$r = \frac{N}{N_{\mathrm{nom}}}, \tag{1}$$

in discrete increments $\Delta r_I$.

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The number of macroparticles is not replenished during injection. Macroparticles are only removed through beam loss during tracking, such that the effective particle weight increases as the bunch population grows. This modeling allows the injection process to be studied without disturbing the phase-space matching of the stored beam.

## RESULTS

In addition to the bunch population, the macroparticle survival rate and the specific luminosity are evaluated as complementary diagnostic quantities. The specific luminosity corresponds to the luminosity per bunch and is calculated independently at the two interaction points, denoted as IPA and IPG. These diagnostics provide direct insight into beam loss and collision performance during the injection process.

### *Stable Injection in W and H Modes*

Figure 2 shows the evolution of the normalized bunch population as a function of turn number $n$ in the W and H modes. Stable injection paths are observed in both cases, reaching the nominal population without significant particle loss. The survival rate remains high and the specific luminosity at IPA and IPG increases monotonically, indicating that the beam–beam interaction remains well controlled during injection.

### *Injection–Loss Balance in the Z Mode*

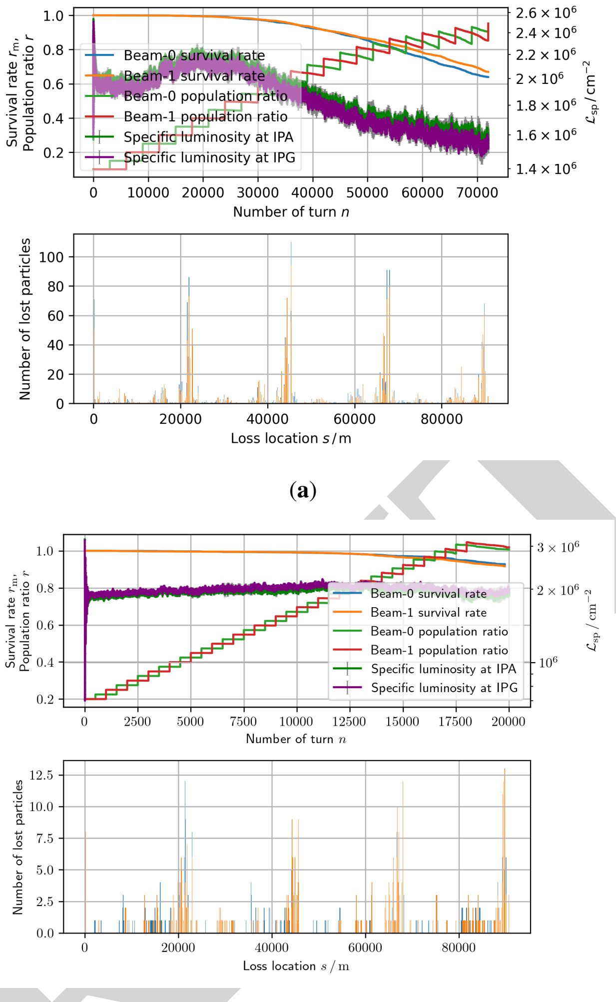
In contrast, the Z mode exhibits a different behavior. As shown in Fig. 3, the bunch population saturates below the nominal value, indicating an injection–loss balance where particle loss compensates the injected charge. The onset of saturation coincides with a rapid decrease in the survival rate and the saturation of the specific luminosity, indicating that beam loss is triggered by beam–beam-induced instability rather than by insufficient injection rate.

### *Multi-Dimensional Beam Dynamics*

Further insight into the Z mode behavior is obtained from Fig. 4. The emittance evolution and the development of $x$–$z$ correlation indicate the emergence of coupled transverse–longitudinal dynamics.

The observed $x$–$z$ coupling suggests that longitudinal motion becomes dynamically linked to transverse dynamics under strong beam–beam interaction. Such multi-dimensional effects can enhance emittance growth and accelerate particle loss, particularly in the weakly damped Z mode.

These results indicate that injection stability in the strong beam–beam regime is strongly influenced by inter-beam dynamics. This behavior is consistent with the development of coupled transverse–longitudinal dynamics under weak radiation damping. Recent studies suggest that the relative population between the two beams plays a key role in determining stability, pointing to possible strategies for improved control.



(**a**)

(**b**)

Figure 2: Evolution of the normalized bunch population, macroparticle survival rate, and specific luminosity at the two interaction points. (**a**) W mode. (**b**) H mode.

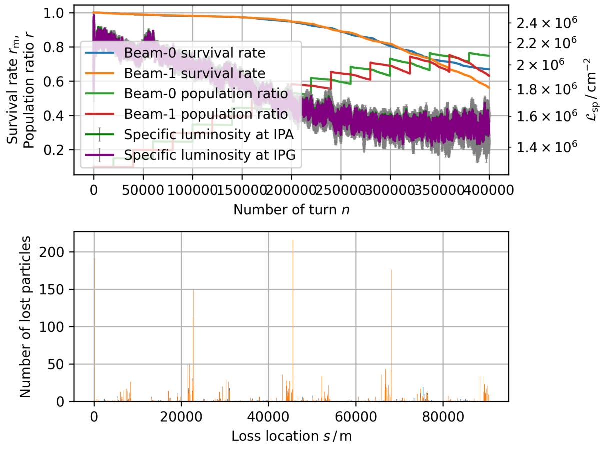
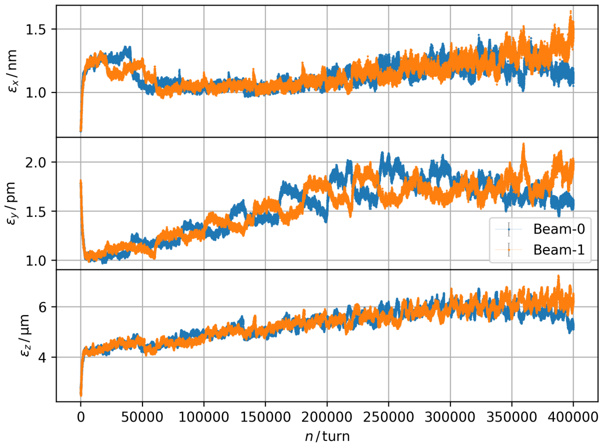


Figure 3: Evolution of the normalized bunch population, macroparticle survival rate, and specific luminosity in the Z mode, showing the emergence of injection–loss balance.



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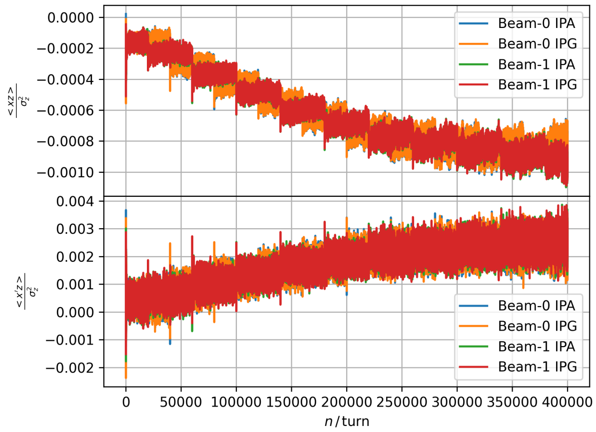

(**b**)

Figure 4: Multi-dimensional beam dynamics in the Z mode. (**a**) Evolution of transverse and longitudinal emittances. (**b**) Development of $x$–$z$ correlation.

## CONCLUSION

The quasi-strong-strong scheme has been applied to study bootstrapping injection in the FCC-*ee* collider.

Stable injection up to the nominal population is demonstrated in the W and H modes. In contrast, the Z mode exhibits injection–loss balance associated with multi-dimensional beam dynamics.

These results highlight the importance of beam–beam effects and inter-beam dynamics in determining injection stability in future circular colliders.

## ACKNOWLEDGMENT

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## REFERENCES

[1] A. Abada *et al.*, “Fcc-ee: the lepton collider”, *European Physical Journal Special Topics*, vol. 228, pp. 261–623, 2019. `doi:10.1140/epjst/e2019-900045-4`

[2] Sad home page, `https://acc-physics.kek.jp/SAD/`