SIMULATION AND STUDY OF THE MUON COOLING DEMONSTRATOR RECTILINEAR CHANNEL IN BDSIM

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Endorsed by the International Muon Collider Collaboration

Abstract

The muon collider has great potential for enabling high-luminosity multi-TeV lepton–antilepton collisions provided low-emittance, high-intensity muon beams can be produced. Ionization cooling is the proposed technique to achieve the required muon beam emittance. The rectilinear cooling lattice used to compress the six-dimensional (6-D) phase-space volume of the beam comprises solenoids for strong focusing, dipoles to generate dispersion, wedge absorbers for differential energy loss, and RF cavities for longitudinal energy restoration. The International Muon Collider Collaboration aims to demonstrate the integration and reliable operation of a 6-D ionization cooling system, including RF acceleration in strong magnetic fields. This paper presents a full implementation of the Muon Cooling Demonstrator 6-D cooling lattice in BDSIM, together with an evaluation of its cooling performance.

INTRODUCTION

The muon collider is a promising candidate for a multi-TeV lepton collider, with the potential to reach centre-of-mass energies of up to 10 TeV at high luminosity. Realising such a machine requires beams with high brightness, and therefore low transverse and longitudinal emittances. In a proton-driven scheme, muons are produced as tertiary particles from pion decays, resulting in beams with large momentum and spatial spreads. Consequently, substantial beam cooling is required. As conventional techniques such as synchrotron radiation or stochastic cooling are ineffective for muons due to their short lifetime, ionisation cooling has been proposed as a novel and viable alternative.

Ionisation cooling involves passing the beam through an absorber, where muons lose momentum in all directions via ionisation energy loss. The longitudinal momentum is subsequently restored using RF cavities, leading to a net reduction in transverse emittance. In rectilinear (6-D) cooling channels, dipole magnets introduce dispersion, correlating particle momentum with transverse position. A wedge-shaped absorber then induces greater energy loss for higher-momentum particles, reducing energy spread at the expense of increased transverse emittance—effectively transferring emittance from the longitudinal to the transverse phase space and enabling net 6-D cooling. Performance is limited by multiple Coulomb scattering, which is mitigated by strong solenoidal focusing at the absorber and by using low-Z materials such as liquid hydrogen or lithium hydride to minimise scattering while maintaining sufficient energy loss.

MUON COOLING DEMONSTRATOR

The Muon Ionization Cooling Experiment (MICE) demonstrated transverse emittance reduction in high-emittance muon beams passing through a single, non-accelerating ionization cooling cell [1, 2]. Building on this proof-of-principle, the muon cooling demonstrator is an essential component of the muon collider R&D programme. Its aim is to demonstrate the successful integration of the cooling hardware (high-field solenoids, absorbers, dipoles, and high-gradient RF cavities), its operation with beam, and the achievement of the required beam-physics performance. To realise these objectives in a controlled and progressive manner, the programme follows a staged approach: starting with RF test stands to characterise the achievable gradients in high-field solenoids, then advancing to one-cell and multi-cell modules, and ultimately culminating in the operation of a complete cooling line with beam [3, 4].

A conceptual layout of the Muon Cooling Demonstrator is shown in Fig. 1. The muon beam is produced through the decay of pions resulting from the interaction of protons with a target. Pions emerging from the target are captured using a magnetic horn and transported through a quadrupole-based decay channel. A momentum-selecting chicane then delivers the resulting muons to a preparation system, where the beam emittance is tuned via collimation and the longitudinal phase space is rotated using RF cavities. The prepared muon beam is subsequently transported to and matched into the cooling channel.

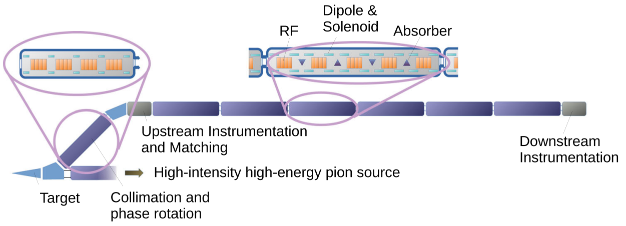

Figure 1: Conceptual layout of the Muon Cooling Demonstrator. [5]

BDSIM FOR MUON COOLING

BDSIM is a Geant4-based Monte Carlo particle-tracking code for modelling accelerators and particle–matter interactions [7]. It can be compiled and run across all major operating systems, and is also accessible through platforms such as Conda, Docker, and Apptainer. A dedicated BDSIM

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element, `muoncooler`, has been developed to enable simulation of complete 6-D muon ionisation cooling channels [8]. The element allows an entire cooling lattice to be constructed within a single component, with electromagnetic fields from solenoids, dipoles, and RF cavities superimposed to provide a continuous six-component field throughout the channel, including full fringe-field effects. A full report, along with validations, has been presented in [9].

Modelling such systems is computationally demanding due to overlapping, fringe-dominated fields from multiple elements. Several enhancements have been introduced to improve flexibility and performance. Solenoids can now include transverse offsets and rotations, enabling studies of misalignment effects. Field calculations can optionally be precomputed on grids and evaluated via interpolation, significantly reducing runtime cost. In addition, a spatial indexing scheme is used to restrict field evaluations to locally overlapping elements, improving scaling with lattice size. Together, these developments enable efficient, high-fidelity simulation of realistic cooling channels. Furthermore, this study also leverages the periodicity of the magnetic field to produce a single periodic field map at the core of the cooling channel to further speed up simulations.

DEMONSTRATOR COOLING LATTICE SIMULATIONS

For this study, the latest demonstrator cooling lattice has been implemented in BDSIM. The demonstrator is based on a cooling cell concept similar to the B-5 stage of the muon collider cooling system, as described in [6, 10].

A schematic of the cooling cell together with a rendering of its BDSIM implementation are shown in Fig. 2. The cell comprises four superconducting solenoids (two at each end) providing transverse focusing, RF cavities operating at 704 MHz for reacceleration, and lithium hydride wedge absorbers at its ends for 6-D ionisation cooling. A more detailed set of the cell hardware parameters used in this study are summarised in Tables 1-4.

Table 1: Solenoid Coil Parameters

Parameter	Coil1	Coil2
z position (m)	0.211	0.081
Inner radius r_{in} (m)	0.285	0.185
Radial thickness (m)	0.070	0.060
Number of pancakes	17	5
Pancake length (m)	0.012	0.012
Pancake spacing (m)	0.004	0.003
Current density (A/mm^2)	328.43	300.00

Table 2: Dipole Parameters

Parameter	Value
Polarity	+ − − +
Field strength (T)	0.1
Length z (m)	0.1

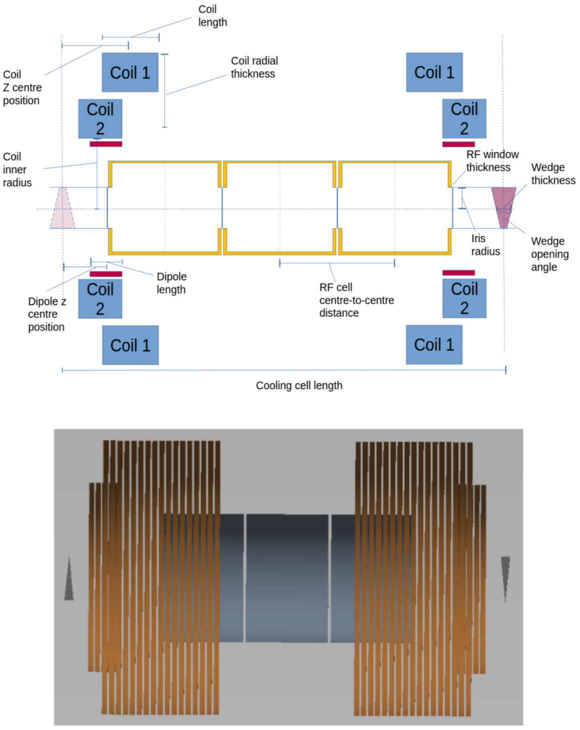

Figure 2: (top) Schematic of the Demonstrator cooling cell hardware, with labelled components and (bottom) a 3-D rendering of the cell implementation in BDSIM. [10]

Table 3: Absorber Parameters

Parameter	Value
Material	`Lithium hydride`
Wedge opening angle	10°
Wedge apex-to-base (m)	0.2286
Thickness on axis (m)	0.02

A 200 MeV/c momentum muon beam was tracked through a 100 m-long cooling lattice. The input beam was sampled from a multivariate Gaussian distribution with an initial transverse emittance of $\varepsilon_\perp \sim 1.9$ mm and a longitudinal emittance of $\varepsilon_\parallel \sim 2.1$ mm. Previous studies showed that an initial mismatch in the longitudinal phase space led to strong momentum oscillations and early emittance growth, resulting in particle losses. More recent cell optimisation studies revealed that a strong correlation between transverse amplitude and momentum develops rapidly in the channel, and that introducing a non-zero amplitude-momentum correlation in the input beam helps to mitigate this effect. The input beam used in this study can be found in [6].

Residual momentum oscillations are still observed, as shown in Fig. 3, although these are significantly damped as the beam propagates along the channel. Figure 4 shows the evolution of the transverse betatron functions at the absorber locations. After an initial mismatch, the beta functions stabilise at ~ 135 mm, indicating that the lattice provides consistent and adequate focussing at the absorbers.

The evolution of the beam emittances is shown in Fig. 5, where transverse, longitudinal and 6-D cooling are all observed. The influence of the residual momentum oscillations is visible in the evolution of the longitudinal emittance,

Table 4: RF Cavity Parameters

Parameter	Value
Number of RF cavities per cell	3
Frequency (MHz)	704
Voltage (MV)	30
Phase	20°
Length (m)	0.18856
Cavity radius (m)	0.1630
Iris radius (m)	0.0816

which exhibits an initial increase followed by a reduction consistent with cooling. Overall, the 6-D emittance decreases by a factor of ~ 2.3 across the 100 m lattice. The beam transmission is shown in Fig. 6, indicating that ~ 93% reach the end of the channel. Muon decays were disabled for this study.

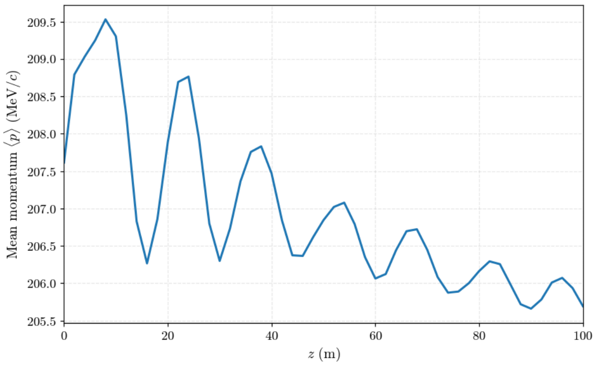

Figure 3: Mean longitudinal momentum $\langle p_z \rangle$ of the beam along the cooling channel.

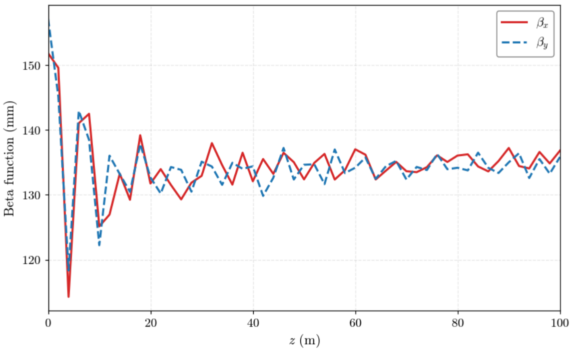

Figure 4: Transverse beta functions β_x and β_y along the cooling channel, characterising the transverse focusing of the beam envelope at the absorbers.

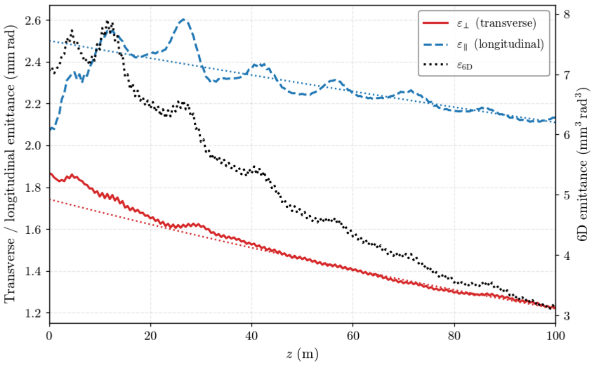

Figure 5: Evolution of the (red) transverse, (blue) longitudinal, and (black) 6-D normalised emittances along the cooling channel. The transverse and longitudinal emittances are shown on the left axis; the 6-D emittance is shown on the right axis. Dotted lines indicate exponential fits to the transverse and longitudinal emittances, to highlight the cooling rate.

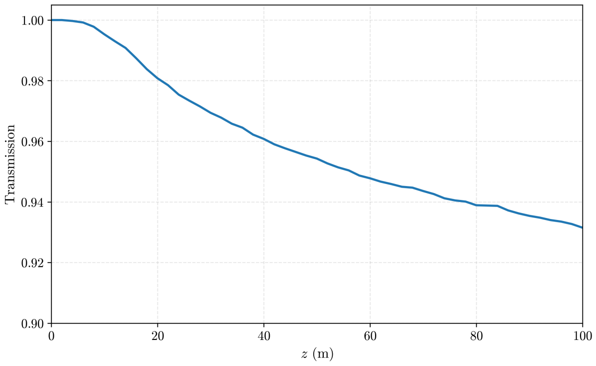

Figure 6: Beam transmission along the cooling channel, defined as the fraction of the initial particles surviving at each longitudinal position. The vertical axis is restricted to the 0.9–1.0 range to better resolve the loss profile.

CONCLUSION

A dedicated `muoncooler` beamline element in BDSIM has been used to simulate a 100 m muon cooling lattice based on the IMCC demonstrator cooling cell. Tracking a 200 MeV/c muon beam through the channel yielded a 6-D emittance reduction by a factor of ~ 2.3, with an overall reduction in both the transverse and longitudinal emittances, despite a modest initial growth in the longitudinal plane, in line with similar G4Beamline studies. Seeding the input beam with a non-zero amplitude–momentum correlation was found to yield transmission of ~ 93%. Future work will focus on further optimisation of the input beam matching to the cooling channel, as well as tolerance studies to quantify the impact of variations in key hardware parameters on beam performance.

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