OPTIMISATION TECHNIQUES FOR INTEGRATED LUMINOSITY WITH AND WITHOUT β^* LEVELLING FOR A CIRCULAR COLLIDER WITH EXAMPLES FROM THE CERN LHC

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Abstract

The main performance indicator of a particle collider is the integrated luminosity. It depends not only on operational efficiency, but also on a range of beam parameters to be optimised to enhance performance. It is common to operate a collider with decaying luminosity, due to beam burn-off. However, the planned luminosity upgrade of the LHC (HL-LHC) is based on luminosity levelling: a time variation of colliding-beam offset, crossing angle, and β^* is used to keep luminosity constant over a certain period of time. Operational experience with luminosity levelling is being acquired at the LHC using different levelling strategies. In this work, we investigate optimisation strategies to maximise integrated luminosity without and with β^* levelling. Monte Carlo simulations of years of physics runs have been performed with and without optimisation approaches. The key physical parameters are derived from a detailed analysis of the data collected at the LHC during the Run 2 and Run 3 periods. These findings provide valuable information for improving future LHC operational strategies and preparing for forthcoming collider configurations in the HL-LHC era.

INTRODUCTION

The performance of a particle collider is characterised by the luminosity, quantifying the rate of particle interactions delivered to experiments [1]. Instantaneous luminosity $L(t)$ relates the event rate to the total cross-section σ_{int} of particle interaction. The luminosity depends on various beam and optical parameters as given below:

$$L(t) = \frac{\Xi N_\text{i}^2}{(1+\sigma_\text{int}\, n_\text{c}\, \Xi N_\text{i}\, t)^2}\,, \quad \Xi = \frac{\gamma_\text{r} f_\text{rev} F(\theta_\text{c}, \sigma_z, \sigma^*)}{4\pi\epsilon^* \beta^*\, k_\text{b}}\,. \tag{1}$$

Here, N_i is the initial beam intensity, n_c is the number of collision points, γ_r is the Lorentz factor, f_rev is the revolution frequency, θ_c is the crossing angle, σ^* and σ_z are the transverse and longitudinal RMS bunch dimensions, respectively, ϵ^* is the normalised RMS emittance, and k_b is the number of bunches [2]. The primary quantity of interest is the integrated luminosity $\mathcal{L}_\text{int}$, defined as the time integral of $L(t)$ over a data collection period. This determines the recorded physics dataset. Maximising $\mathcal{L}_\text{int}$ requires balancing the duration of each collision phase with the interval between a beam dump and the start of the next data collection period. This time interval, known as the turnaround time, includes the operational overhead of preparing a new fill, such as the injection, ramp, and setup phases, as well as the recovery from faults and other inefficiencies until subsequent beams are brought into collision [3]. The duration of the fill has a random component: a fill may end with a programmed beam dump or a protection dump triggered by the machine protection system [4]. The probability that a fill survives up to a given time is described by the survival probability function $S(t)$ and must be considered in optimisation strategies [3,5]. Failures affect the expected integrated luminosity delivered over a run and must therefore be accounted for in any optimisation approach.

Whenever β^* and ϵ^* remain constant, the luminosity decreases during a collision phase due to beam burn-off and other beam dynamics effects [2,6]. This is the case for the CERN Large Hadron Collider (LHC) [7] during Run 2 [8,9]. Luminosity levelling was introduced in the LHC as early as Run 1 for the ALICE and LHCb experiments, by varying transverse beam separation [10]. From Run 3, it was also applied to the ATLAS and CMS high-luminosity experiments, due to constraints from the cryogenic system and pile-up level in the detectors. Levelling is then implemented by varying beam separation, β^*, and, more recently, the crossing angle to maintain approximately constant luminosity before natural decay resumes [11–14]. This operational mode changes how luminosity evolves during a collision phase and modifies the strategy to optimise integrated luminosity compared to head-on collisions at constant β^*. The High Luminosity (HL) LHC upgrade [15], scheduled to begin operation around 2030, anticipates levelling as the baseline scenario. In this study, we present optimisation approaches to maximise integrated luminosity in both scenarios, with and without β^* levelling, including or omitting the survival probability of the fill.

LUMINOSITY OPTIMISATION

Optimisation Without Levelling

In this work, the optimisation of the integrated luminosity over a period of time, such as the number of allocated physics days in a year, requires determining the optimal duration of the current fill while accounting for both past fills and the predicted performance of future ones. The total integrated luminosity can then be expressed as the sum of the integrated luminosities of the past i fills, that of the current fill, $\mathcal{L}_{\mathrm{c}}$, and an estimate of that of the future fills, $\mathcal{L}_{\mathrm{f}}$:

$$\mathcal{L}_{\mathrm{tot}} = \sum_{j=1}^{i} \mathcal{L}_{\mathrm{tot}}(t_{\mathrm{fill},\,j}) + \mathcal{L}_{\mathrm{c}}(t_{\mathrm{fill,\,c}}) + \mathcal{L}_{\mathrm{f}}(t_{\mathrm{fill,\,f}})\,. \tag{2}$$

In the case of no levelling and no failures, the current and future integrated luminosities are defined as:

$$\begin{aligned} \mathcal{L}_{\mathrm{c}}(t_{\mathrm{fill,\,c}}) &= \int_0^{t_{\mathrm{fill,\,c}}} \mathrm{d}t L(t) \\ \mathcal{L}_{\mathrm{f}}(t_{\mathrm{fill,\,f}}) &= \frac{T_{\mathrm{tot}} - T_{\mathrm{past}} - t_{\mathrm{ta}} - t_{\mathrm{fill,\,c}}}{\langle t_{\mathrm{ta}}\rangle + t_{\mathrm{fill,\,f}}} \int_0^{t_{\mathrm{fill,\,f}}} \mathrm{d}t L_{\mathrm{mp}}(t)\,, \end{aligned} \tag{3}$$

where $\mathcal{L}_{\mathrm{f}}$ is evaluated assuming that future fills are all equal and $L_{\mathrm{mp}}(t)$ represents the most probable value of luminosity. The numerical coefficient is the estimated number of remaining fills, based on the total time for physics in a year T_{tot}, the time already passed T_{past}, and the turnaround time t_{ta}, together with the length of the current fill $t_{\mathrm{fill,\,c}}$, subject to the expected length of a future fill (the average turnaround time $\langle t_{\mathrm{ta}}\rangle$, and the typical duration of future fills $t_{\mathrm{fill,\,f}}$). Here, t_{fill} is equivalent to the decay time t_{dec}, since in this case we only consider a pure burn-off decay. Optimisation is performed with respect to $t_{\mathrm{fill},c}$ and $t_{\mathrm{fill},f}$. The resulting analytical expressions show that $t_{\mathrm{fill},c}$ depends primarily on the average turnaround time and the intensity of the beam [3].

In the case where fills may terminate prematurely due to protection dumps (failures), Eq. (3) must be modified to include the fill survival probability $S(t) = e^{-\mu_{\mathrm{f}}^k t^k}$, derived from the Weibull distribution of the failure rate given as follows:

$$f(t) = k\mu_{\mathrm{f}}^k\, t^{k-1} e^{-\mu_{\mathrm{f}}^k\, t^k}\,, \tag{4}$$

where k represents the ageing process and μ_{f} is the inverse of the mean time between failures (MTBF). Both expected fill time, t_{e}, and the expected integrated luminosity, $\mathcal{L}_{\mathrm{e}}$, are then weighted by $S(t)$ according to

$$t_{\mathrm{e}}(t_{\mathrm{fill}}) = \int_0^{t_{\mathrm{fill}}} \mathrm{d}\tau S(\tau)\,;\; \mathcal{L}_{\mathrm{e}}(t_{\mathrm{fill}}) = \int_0^{t_{\mathrm{fill}}} \mathrm{d}\tau L(\tau)\, S(\tau)\,, \tag{5}$$

the resulting expressions are to be solved numerically [3].

Optimisation With Levelling

In Run 3, the levelling of luminosity introduces a constant luminosity plateau at L_ℓ maintained for a levelling time t_ℓ, followed by the regular decay profile given in Eq. (1) for a time t_{dec}. Accordingly, $t_{\mathrm{fill}} = t_\ell + t_{\mathrm{dec}}$ and the per-fill integrated luminosity becomes:

$$\mathcal{L}_{\mathrm{c}}(t_\ell, t_{\mathrm{dec,\,c}}) = L_\ell t_\ell + \int_{t_\ell}^{t_\ell + t_{\mathrm{dec,\,c}}} \mathrm{d}t L(t)\,. \tag{6}$$

Here, L_ℓ is the luminosity levelling value, t_ℓ is the time for which levelling is maintained, and $t_{\mathrm{dec,\,c}}$ is the decay time of the current fill. This expression and the corresponding expression for $\mathcal{L}_{\mathrm{f}}$, can be substituted accordingly in both parts of Eq. (3) to obtain an expression for integrated luminosity in the presence of levelling [16]. Subsequently, when failures are also included, the survival probability weighting must cover both the levelling and decay phases:

$$\begin{aligned} t_{\mathrm{e}}(t_\ell, t_{\mathrm{dec}}) &= \int_0^{t_\ell} \mathrm{d}\tau S(\tau) + \int_{t_\ell}^{t_\ell + t_{\mathrm{dec}}} \mathrm{d}\tau S(\tau)\,; \\ \mathcal{L}_{\mathrm{e}}(t_\ell, t_{\mathrm{dec}}) &= L_\ell \int_0^{t_\ell} \mathrm{d}\tau S(\tau) + \int_{t_\ell}^{t_\ell + t_{\mathrm{dec}}} \mathrm{d}\tau L(\tau) S(\tau)\,. \end{aligned} \tag{7}$$

As before, these are substituted in Eq. (3) for the modified expression of luminosity with levelling, including failures.

In all four cases, without/with levelling, without/with failures, the optimised solution is obtained by equating the partial derivatives of $\mathcal{L}_{\mathrm{tot}}$ with respect to the current and future fill decay parameters to zero. Note that in this approach, L_ℓ is a parameter that should be fixed prior to optimisation, as it is typically determined by external constraints from machine cryogenics or pile-up in experiments [17].

SIMULATION RESULTS

Monte Carlo Configuration

A Monte Carlo (MC) approach is used to simulate a full year of collider operations based on parameter distributions derived from LHC operational data. The number of fills satisfying nominal operations condition (such as proton physics fills with collisions at top energy with maximal bunch numbers) in a single operational year is insufficient for a statistically robust baseline, a bootstrap procedure is used to generate a large pool of synthetic fills based only on nominal operations data, excluding non-standard fills. The 2018 and 2024 datasets have been selected as they provide the largest data samples with nominal beam parameter values, thus representing well Run 2 and Run 3 configurations, respectively.

The key per-fill parameters, namely initial beam intensity N_{i}, levelling luminosity L_ℓ, decay time t_{dec}, and the decay curve fitting parameters ρ and ϵ (from an extended burn-off model including pseudo-diffusion losses [2]), are sampled from their empirical, correlated distributions. L_ℓ is sampled uniformly in the interval $[2.05, 2.15] \times 10^4\,\mathrm{Hz\,\mu b^{-1}}$, matching the luminosity levelling values of 2024 [18]. For Run 2, only the (N_{i}, ρ) correlation is included, while for Run 3, the correlations are calculated across all five parameters. The turnaround time t_{ta} is modelled using a shifted exponential distribution, while the failure rate follows the Weibull distribution of Eq. (4), with parameters μ_{f} and k deduced from the operational data. The fill survival probability $S(t)$ is applied only when failures are considered. The simulation is repeated 10^4 times to ensure statistical robustness in the results.

Simulations Without Levelling

The Run 2 (2018) configuration, which comprises 3480 h of operations, is used to evaluate the optimisation strategy in the absence of levelling, corresponding to the decay of pure burn-off from the beam [19]. Figure 1 (left panels) shows the integrated annual luminosity distributions from the MC simulation and from simulations applying the optimal decay time per fill, both without (top) and with (bottom) failures, with peak kernel density estimate values reported in each panel. The downward shift between the cases without and with failures is compatible with the loss of efficiency caused by protection dumps. The relative gain from the optimisation in each case is shown in Fig. 2 (left panel).

Simulations With Levelling

For the Run 3 (2024) optimisation configuration, which comprises 2254 h of operations, L_ℓ is fixed at $2.1 \times 10^4\,\mathrm{Hz\,\mu b^{-1}}$ [20]. Here, β^* levelling substantially changes the sensitivity of the result to the choice of fill duration. As shown in the right panels of Fig. 1, the integrated annual luminosity is significantly higher, in the range $120\,\mathrm{fb}^{-1}$ to $135\,\mathrm{fb}^{-1}$, due to the improved beam parameters made possible by the LHC Injectors Upgrade (LIU) project [21] implemented during the Long Shutdown 2. In contrast to the no-failure case (Fig. 1, top-right), failures (Fig. 1, bottom-right) broaden both distributions and reduce the separation between optimised and MC cases. In particular, the MC distribution now peaks at $126.1\,\mathrm{fb}^{-1}$, almost replicating the 2024 LHC case [18, 22].

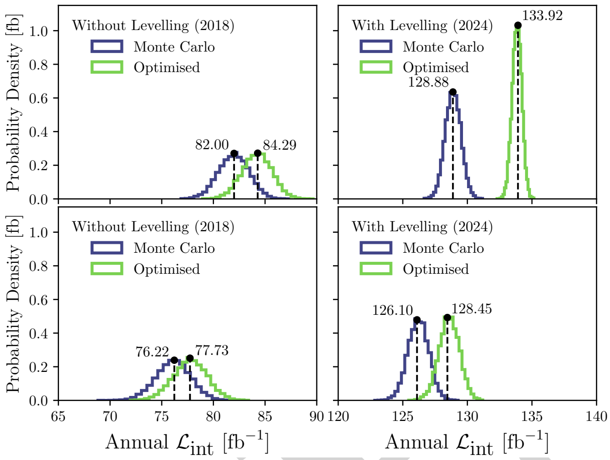

Figure 1: Annual integrated luminosity distributions without (top row) and with (bottom row) failures, comparing the results of MC (purple) and of the optimised strategy (green), for the Run 2 (left) and Run 3 (right) configurations.

Finally, Fig. 3 shows the distribution of the yearly averages of the optimised decay time for all the cases considered. Without levelling, the optimiser increases the decay time with respect to the MC, reflecting a strategy where longer fills are preferred to maximise integrated luminosity. However, in the presence of levelling, the optimiser consistently selects shorter decay times, favouring an early dump and refill, which results in a much sharper distribution. Such a contrasting behaviour highlights how levelling fundamentally changes the optimal fill length strategy.

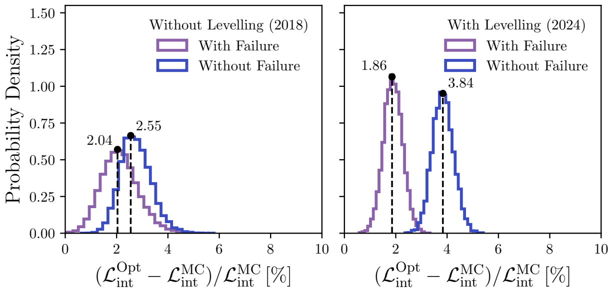

Figure 2: Distributions of the relative difference in annual integrated luminosity between the optimised and MC scenarios, comparing the cases with (purple) and without failure (blue), for the Run 2 (left) and Run 3 (right) configurations.

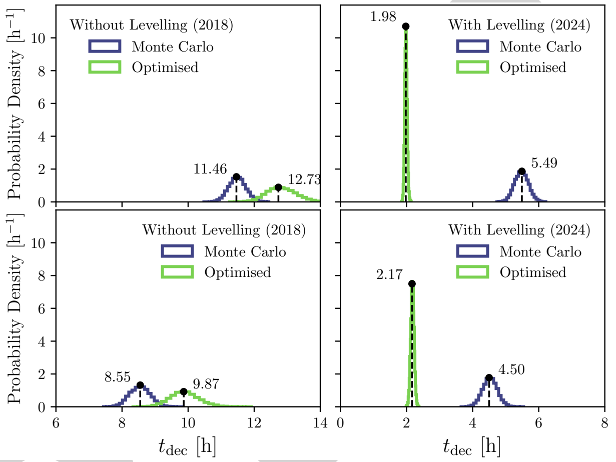

Figure 3: Average decay time distributions without (top row) and with (bottom row) failures, comparing the results of MC (purple) and of the optimised strategy (green), for the Run 2 (left) and Run 3 (right) configurations.

CONCLUSIONS

An analytical framework for fill-length optimisation has been developed and applied to operational scenarios representative of LHC Run 2 and Run 3 corresponding to natural luminosity decay and levelling using β^* variation, respectively, with and without failures. The results are validated by Monte Carlo simulations seeded from LHC operational data. Although in the absence of levelling, the optimiser chooses longer decay times than those applied in Run 2, the opposite strategy is applied for Run 3, where the decay times are reduced. The gains in annual integrated luminosity reach 2.0 % or 2.6 % for Run 2 (with and without failures, respectively) while gains of 1.9 % or 3.8 % are obtained for Run 3 (with and without failures, respectively). The benefits of the proposed optimisation schemes are evident. These findings show that the use of an analytical approach to optimise fill length can significantly improve the performance of a collider. The framework developed is directly relevant for the HL-LHC, where levelling will be implemented. Future work [17] will focus on the analysis of a four-parameter optimisation algorithm, which will determine not only the optimal decay time, but also a theoretical optimal value at which levelling should occur.

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