LONGITUDINAL MOMENTUM HALOS – RISK OF THE FRACTIONAL VELOCITY PARTICLES IN HIGH-CURRENT HADRON LINACS

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Abstract

Recent high-intensity hadron linacs, such as the IFMIF accelerator and linacs under development for nuclear waste transmutation, aim to deliver average beam currents exceeding 100 mA and up to 1 A in continuous operation. In such machines, even sub-percent beam losses can cause significant activation of accelerator components. Studies of beam halo dynamics with accuracies far beyond conventional approaches are required.

In this work, we investigate momentum halos formed by fractional velocity particles (FVPs). Using a discrete dynamical model of a simplified drift-tube linac (DTL), we demonstrate that these particles can be stably captured in fractional RF buckets and continuously accelerated. Furthermore, the stability analysis of periodic focusing channels indicates that such FVPs can propagate downstream.

INTRODUCTION

The 100% capture rate of the injected particles by RF is ideally achievable in RF bunching mechanism, where a coasting beam is adiabatically trapped in gradually expanding RF buckets to formulate the bunch structure. However, this is not true in a real situation, especially in RF linacs such as RFQ or DTL. The practical constraints on the accelerator size does not allow this. In most of realistic designs, the capture efficiency falls within 90% to 95%. It is well known that 5 to 10 % of injected particles are lost upstream of the RF linacs and other fractional components are lost somewhere before their exits.

In this study, we focus on the longitudinal dynamics of particles that fail to be captured in the main RF buckets, including FVPs. We introduce a concept of fractional synchronous phase conditions and discuss the behavior of fractional velocity bunches captured by the corresponding fractional RF buckets during acceleration. Here, it is noted that FVPs are mixed in the 5-10 % lost particles pointed out above and fractional particles leaking from the main RF buckets on the way to exit. This study identifies FVPs as a potential source of longitudinal halo components in high-intensity hadron linacs, because the fractional velocity bunches are continuously accelerated downstream.

THEORETICAL MODEL

We define a simplified discrete dynamical model based on an Alvarez-type DTL [1]. The accelerating voltage is assumed to be applied instantaneously at each gap, and the gap length is ignored. The model focuses on the evolution of the longitudinal phase space, while transverse particle motion is not included. A schematic illustration of the longitudinal model is shown in Fig. 1.

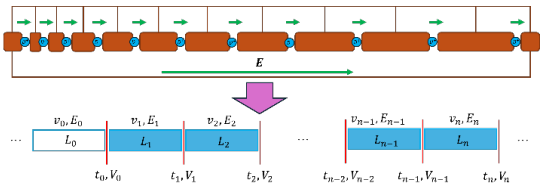


Figure 1: Schematic illustration of the DTL model employed in this study. The lower panel illustrates the relationship between the DTL structure and the parameterization of particles.

At each gap, the particle energy and arrival time are updated, and the particle motion in the DTL is described by the following discrete mapping. Here, E_n, t_n, L_n, and v_n denote the particle energy, time, cell length, and particle velocity after the n-th iteration, respectively. Q is the particle charge, V is the effective accelerating voltage at the gap, ω_{rf} is the angular frequency of the RF field, A is the mass number of the particle, and m is the proton rest mass:

$$\begin{cases} E_{n+1} = E_n + QeV \cdot sin(\omega_{rf} \cdot t_n) \\ t_{n+1} = t_n + \dfrac{L_{n+1}}{v_{n+1}} \left(v_{n+1} = \sqrt{\dfrac{2E_{n+1}}{A \cdot m}} \right) \end{cases} \quad (1)$$

In Eq. (1), the effective accelerating voltage V includes the transit-time factor and is expressed as $V = E_0TL$. The lengths of successive cells are designed to synchronize with a particle having a specific velocity, referred to as the synchronous velocity at the n-th cell v_n^s. By replacing the time variable with the RF phase, the longitudinal phase of each particle is introduced as $\phi_n = \omega_{rf} \cdot t_n$, which represents the phase of the RF voltage at the moment the particle reaches the gap position. Here, v_n denotes the particle velocity after the n-th iteration.

$$\begin{cases} E_{n+1} = E_n + QeV \cdot \sin \phi_n \\ \phi_{n+1} = \left\{ \phi_n + 2\pi \cdot \left(\dfrac{v_{n+1}^s}{v_{n+1}} \right) \right\}_{(\mathrm{mod}\ 2\pi)} \end{cases} \quad (2)$$

We now introduce the fractional synchronous velocity: ${}_l v_n^s = v_n^s / l$, $(l \in \mathbb{N})$, which corresponds to a particle velocity reduced by a fractional factor: l, relative to the synchronous velocity of the leading particle ($l = 1$). For such a fractional synchronous particle, the phase advance per cell becomes

$${}_l\phi_n^s = \{ {}_l\phi_{n-1}^s + 2\pi \cdot l \} (\mathrm{mod}\ 2\pi) = {}_l\phi_{n-1}^s \quad (3)$$

indicating that the phase remains constant modulo 2π. From Eqs. (2) and (3), the energy evolution of the

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synchronous particle with fractional velocity can be derived recursively as

$$ {}_lE_n^s = {}_lE_0^s + \mathrm{n} \cdot QeV \cdot \sin\left({}_l\phi_0^s\right) \tag{4} $$

The corresponding synchronous fractional velocity is then obtained as

$$ {}_lv_n^s = \sqrt{({}_lv_0^s)^2 + n\left(\frac{2QeV}{A \cdot m}\right) \cdot \sin({}_l\phi_0^s)} \tag{5} $$

Consequently, the synchronous conditions for particles with fractional velocities are summarized as

$$ \begin{cases} {}_lv_n^s = \dfrac{v_n^s}{\ell} \\ {}_l\phi_0^s = \sin^{-1}\left(\dfrac{sin\,(\phi_0^s)}{\ell^2}\right) \simeq \sin^{-1}\left(\dfrac{\phi_0^s}{\ell^2}\right) \end{cases} \tag{6} $$

where the last approximation holds for small synchronous phases.

As shown in Fig. 2, the synchronous phase approaches zero with increasing fractional factor. This phase corresponds to a stable fixed point in the RF bucket, indicating stable longitudinal motion for FVPs.

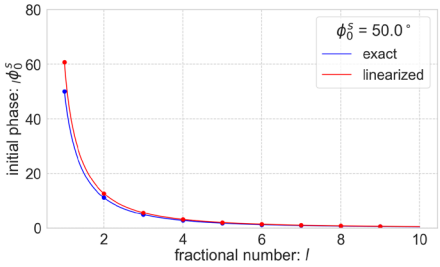


Figure 2: Synchronous phase conditions of fractional RF buckets determined by Eq. (6) for a leading synchronous phase of 50°, illustrating the approach of fractional synchronous phases toward zero.

NUMERICAL RESULTS

Figure 3 presents numerical simulation results obtained using the discrete model derived in Eq. (2). The simulation code was developed in Python 3. As an initial condition, one million particles were uniformly distributed in phase over the range [−π, π). The particles were tracked through 1000 accelerating gaps with a constant effective accelerating voltage of 100 kV and a fixed synchronous phase of 20°. The RF frequency and the input and output beam energies were chosen with reference to the IFMIF accelerator parameters [2].

In the left-hand panel, particles initially distributed near the leading synchronous velocity show a small capture into fractional velocity buckets, with a capture ratio of about 3% of the total population. In contrast, for initial distributions corresponding to fractional synchronous velocities (middle and right-hand panels), capture ratios increase to about 70–75%, while the leading velocity bucket is limited to approximately 38%. This behaviour reflects the fractional synchronous-phase condition given by Eq. (6) and Fig. 2.

A more detailed analysis and further discussion of the underlying dynamics are presented in our submitted paper [3].

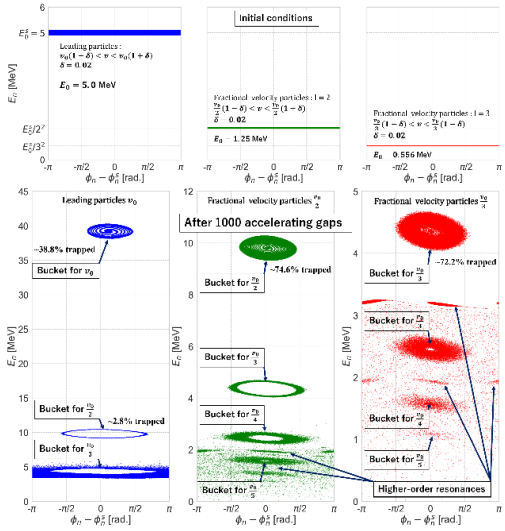


Figure 3: Numerical simulation results of the discrete model. Upper panels show initial distributions, and lower panels show distributions after acceleration through 1000 gaps. Capture ratios are indicated near the corresponding RF buckets.

TRANSVERSE STABILITY

Magnetic FOFO Lattice: Solenoid Channel

The solenoid focusing channel shown in Fig. 4 is considered in this subsection. Here, L_1 and L_2 denote the lengths of the drift section and the solenoid, respectively. The hard-edge model is assumed for magnetic fields, and no accelerating structures are included in the system considered in this section.

The transverse stability of the system can be evaluated using the transfer matrix formalism. The equation of motion for transverse FVPs motion in the periodic solenoid channel is written as

$$ \begin{cases} \dfrac{d^2x}{ds^2} + \kappa(s) \cdot x = 0 \\ \kappa(s) = \begin{cases} l^2K & (0 \le s < L_1) \\ 0 & (L_1 \le s < L_1 + L_2) \end{cases} \end{cases} \tag{7} $$

where s is the longitudinal coordinate and K represents the focusing strength of the solenoid field. Here, the focusing strength is scaled as $\kappa(s) = l^2K$ to account for the increased focusing strength resulting from the reduced velocity of FVPs. The transfer matrix for one lattice period, consisting of a drift section and a solenoid, can be constructed from this equation. From the resulting transfer matrix, the stability condition of the periodic channel [3] is obtained as

$$ \left|\cos(l\psi_1) - \left(\frac{l\psi_1}{2}\right)\left(\frac{L_1}{L_2}\right)\sin(l\psi_1)\right| \le 1 \tag{8} $$

where ψ_1 is the phase advance associated with the solenoid section.

Figure 5 shows coloured stability maps for various values of ψ_1 and L_2/L_1. FVPs exhibit stability properties distinct from those of leading velocity particles and are generally unstable unless the lattice phase advance is properly chosen. As discussed in Ref. [3], however, the effective betatron tune of FVPs is largely reduced when they are passed through by leading velocity bunches. The betatron tune of FVPs (at least $l = 2$) may fall into a stable region, enabling stable propagation through the solenoid channel.

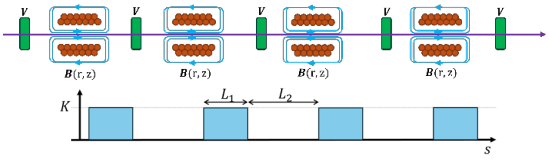


Figure 4: Illustration of the periodic solenoid channel. The upper drawing includes the accelerating section, which is not considered in this model.

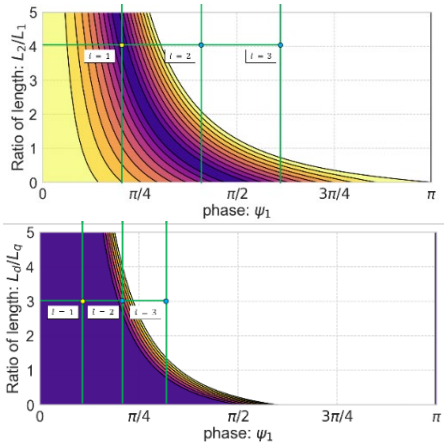


Figure 5: Stability diagram of the solenoid channel (upper) and the quadrupole channel (lower). The coloured area indicates the stable region; the yellow dot represents the leading velocity particle, and the blue dots represent (FVPs).

Magnetic FODO Lattice: Quadrupole Channel

The magnetic FODO lattice, a periodic quadrupole focusing channel, has stability characteristics similar to those of the solenoid channel. The transverse particle motion in the horizontal and vertical directions (u = x, y) obeys:

$$\frac{d^2}{ds^2}u(s) + \sigma_u \kappa_q(s) u(s) = 0 \qquad (9)$$

where $\sigma_u = (1, -1)$ for (x, y), $\kappa_q(s)$ represents the quadrupole focusing strength. The transverse stability can be evaluated using the transfer matrix formalism. Defining $\psi_1 \equiv \sqrt{\kappa_q} L_q$, the stability diagram can be constructed as shown in Fig. 5, where $\psi_1 = \pi/9$ is chosen for the leading velocity particle.

CONCLUSION

In this study, we introduced the concept of synchronous phases for fractional velocity particles (FVPs) and demonstrated the formation of fractional velocity RF buckets based on a simplified discrete dynamical model. The results show that FVPs can be stably captured and continue to be accelerated, indicating that FVPs can survive transport through focusing channels. These findings identify FVPs as a previously overlooked source of longitudinal halos in high-intensity hadron linacs [4, 5].

The present study summarizes the essential part of results reported in Ref. [3] and extends the analysis to a simplified RFQ structure.

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