SPACE CHARGE EFFECTS ON SPIN POLARIZATION IN A HIGH-INTENSITY PREINJECTOR[*]

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Abstract

We are designing a high-charge, high-polarization preinjector for the EIC. Placing a Wien filter immediately after the gun as the spin rotator offers several advantages over the traditional dipole–solenoid spin rotator located after the linac. However, this is the first attempt to operate a Wien filter in a strong space-charge environment, where the energy spread and self-fields may influence spin dynamics. In this paper, we analytically study space-charge effects on electron-beam polarization in the low-energy region, from the polarized electron gun to the linac entrance. We evaluate spin degradation in the drift from the gun to the Wien filter, inside the Wien filter, and in the bunching section. A comprehensive assessment of polarization-degradation mechanisms under space-charge conditions is presented. We also perform spin tracking through the full low-energy beamline with the General Particle Tracer (GPT) code, including space charge, to benchmark the analytical model and capture higher-order effects.

INTRODUCTION

In the Electron-Ion Collider (EIC), we propose using two Wien filters as a spin rotator to transform the electron spin orientation from longitudinal to vertical in the 300–320 keV energy range [1]. Although Wien filters have been widely used in hadron and electron machines for velocity selection and spin rotation, their application to high-intensity bunches in a space-charge-dominated region is new [2–4]. Existing Wien filters typically operate at pico- or sub-picocoulomb bunch charge, where space-charge effects can be neglected. We previously studied the spin dynamics with the Zgoubi code, which does not include space charge [5]. Those studies therefore did not address the challenges associated with the intense beams required for the EIC.

The particle spin evolution is described by the Thomas–BMT equation [6].

$$\frac{d\vec{P}}{dt} = \vec{\Omega}_0 \times \vec{P} \tag{1}$$

$$\vec{\Omega}_0 = -\frac{Ze}{m\gamma}[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel + (G\gamma + \frac{\gamma}{1+\gamma})\frac{\vec{E}\times\vec{v}}{c^2}] \tag{2}$$

Here, $\vec{P}$ is defined in the particle rest frame, while $\vec{E}$ and $\vec{B}$ are defined in the laboratory frame. In addition, $\vec{B} = \vec{B}_\perp + \vec{B}_\parallel, \vec{B}_\parallel = (\vec{v}\cdot\vec{B})\vec{v}/v^2$ where $\vec{B}_\parallel$ is parallel to the beam velocity, and $G = \frac{g-2}{2}$ is the anomalous magnetic moment. For electrons, $G = 0.001159$.

SPACE CHARGE FORCES

The transverse space-charge fields E_r and B_φ contribute to the variation of the spin vector $\vec{P}$ through Eq. 2. Although the longitudinal space-charge term satisfies $\vec{E}_z \times \vec{v} = 0$, it generates an energy spread that affects the spin polarization. In the following, we discuss two contributions: (i) transverse space charge and (ii) longitudinal space charge. The detailed derivations can be found in [7].

Transverse Space Charge

The transverse space-charge fields can be derived from Gauss's law and Ampere's law:

$$\int_S \vec{E}\cdot\hat{n}dS = \frac{q}{\varepsilon_0} \tag{3}$$

$$\oint_l \vec{B}\cdot\vec{l} = \mu_0 I \tag{4}$$

The beam from the gun initially follows a truncated Gaussian distribution with a one-sigma cutoff, which approximates a uniform distribution. As the beam propagates toward the Wien filter, transverse space charge causes the transverse distribution to evolve toward a Gaussian profile. Therefore, we evaluate the electric and magnetic fields for both uniform and Gaussian distributions.

For a uniform distribution, the transverse electric field is

$$E_r = \frac{IZ_0}{2\pi\beta}\frac{r}{a^2} \tag{5}$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{c\varepsilon_0} = 377(\Omega)$

The corresponding transverse magnetic field is

$$B_\varphi = \frac{Z_0 I}{2\pi c}\frac{r}{a^2} \tag{6}$$

Equations 5 and 6 give the transverse electric and magnetic fields for the uniform distribution.

For a Gaussian distribution, the transverse electric field is

$$E_r = \frac{I_{pk}Z_0}{2\pi\beta r}e^{-\frac{z^2}{2\sigma_z^2}}(1-e^{-\frac{r^2}{2\sigma_r^2}}) \tag{7}$$

Similarly, the transverse magnetic field for a Gaussian distribution is

$$B_\varphi = \frac{I_{pk}Z_0}{2\pi c r}e^{-\frac{z^2}{2\sigma_z^2}}(1-e^{-\frac{r^2}{2\sigma_r^2}}) \tag{8}$$

Longitudinal Space Charge

The beam energy at the gun exit is 320 keV [8]. The bunch experiences longitudinal space-charge effects that increase the energy spread before entering the Wien filter. Because

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the spin-rotation rate depends on beam energy, as shown in Eq. 13, we use an analytical model to evaluate the energy spread induced by longitudinal space charge and its impact on polarization degradation.

The electrostatic potential and longitudinal electric field are

$$E_z = -\frac{\partial \Phi}{\partial z} \tag{9}$$

When $r = 0$, we have

$$E_z = -\frac{IZ_0}{4\pi\beta}\left(1 + 2ln\frac{b}{a}\right)\frac{z}{\sigma_z^2}e^{-\frac{z^2}{2\sigma_z^2}} \tag{10}$$

$$E(z) = eE_z(z)L + E_0 \tag{11}$$

where L is the drift length, $E(z)$ is the electron energy as a function of position within the bunch, and E_0 is the energy of the bunch-center electron.

Initial Beam Parameters

The Wien filters are located after the gun dipole, approximately 2–3 m from the photocathode. The corresponding beam parameters in this region are listed in Table 1.

Table 1: Beam Parameters in the Wien Filter Range

Parameter	**Value**
Bunch Charge	7.5 nC
Bunch Length	1.3 ns (σ_z = 0.7 ns)
Beam Size Radius	10 mm
Beam Energy	320 keV
Peak Current for Gaussian	4.27 A

In the EIC preinjector, Wien filter is used only as a spin rotator while transmitting the electron beam with minimal deflection. The electric and magnetic fields are perpendicular, so the magnetic Lorentz force is balanced by the electrostatic force. With two Wien filters, each providing a spin rotation of $\pi/4$ with triplets placed in between to cancel the dispersion. The detailed Wien-filter parameters can be found in [9].

POLARIZATION DEGRADATION

ESP Degradation by the Energy Spread

The Electron Spin Polarization (ESP) can be calculated by

$$ESP = \frac{\sum_i^N \left|\vec{P}_i \cdot \vec{P}_0\right|}{N} \tag{12}$$

where P_0 is the desired spin direction, P_i is each electron's spin direction, and N is the number of particles in one bunch.

Using Eq. 13, the ESP can be written as

$$W = \frac{d\theta}{ds} = \frac{qB}{mv\gamma^2} \tag{13}$$

$$ESP = \int_{-\infty}^{\infty}\cos(W(E(z))L)\rho(z)dz \tag{14}$$

where W has units of (rad/m). Using the parameters in Table 1, the ESP limited by the energy spread induced by longitudinal space charge is 0.99959, with an rms energy spread of about 1.8%. This result agrees with the Parmela simulation. We can also evaluate the ESP as a function of rms energy spread, as illustrated in Fig. 1. The change in energy spread is driven by different peak-current values.

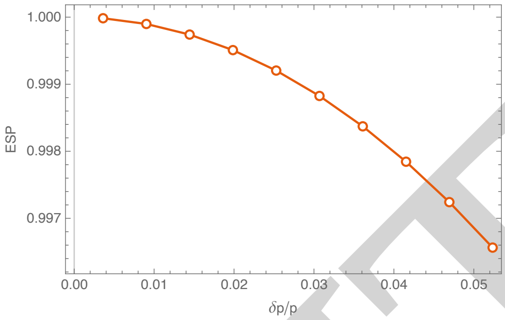

Figure 1: The ESP as the function of the RMS energy spread.

ESP Impacted by the Transverse Space Charge

In this section, we evaluate the independent effects of E_r and B_φ on the spin motion in the Wien filter.

At the front end of the injector, the space-charge force is strong. Using Eq. 2 and Eq. 8, the spin-direction change can be written as a function of B_φ:

$$\frac{d\vec{P}}{ds} = \frac{e}{\beta cm\gamma}\left[\left(1 - \frac{\gamma}{1+\gamma}\right)\frac{\vec{E}_r \times \vec{v}}{c^2} \times \vec{P}\right] \tag{15}$$

B_φ depends on the particle coordinates r and z, as shown in Eq. 8. We now evaluate the ESP degradation through the Wien filter, taking into account the dependence on φ and $\vec{P}$.

Because the spin vector $\vec{P}$ changes throughout the Wien filter, we express the spin direction as a function of the path length inside the filter. From Eq. 13, the spin angle is $\theta = \frac{l}{L}\frac{\pi}{2}$, where l is the particle path length and L is the total Wien-filter length. We then have

$$\vec{P} = \cos(\theta)\,\hat{z} - \sin(\theta)\,\hat{y} \tag{16}$$

$$\vec{B}_\varphi = B_\varphi\left[\cos(\varphi)\,\hat{x} - \sin(\varphi)\,\hat{y}\right] \tag{17}$$

$$\vec{B}_\varphi \times \vec{P} = B_\varphi \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\varphi & -\sin\varphi & 0 \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \tag{18}$$

$$\tag{19}$$

Then we can integrate $d\vec{P}$ through the entire Wien filter using Eq. 15 as

$$\Delta\vec{P} = \int_0^L \frac{e}{\beta cm\gamma}\left[\left(1 - \frac{\gamma}{1+\gamma}\right)\vec{B}_\varphi \times \vec{P}\right] ds \tag{20}$$

$$\tag{21}$$

Let

$$\frac{e}{\beta cm\gamma}\left(1 - \frac{\gamma}{1+\gamma}\right)\frac{2L}{\pi} = k, \tag{22}$$

where k has units of $\frac{\mathrm{C\cdot s}}{\mathrm{kg}}$. Using the parameters in Table 1, we obtain $k = 110.9\,\frac{\mathrm{C\cdot s}}{\mathrm{kg}}$. Because B_φ is on the order of 10^{-5} T, $kB_\varphi \ll 1$. At the exit of the Wien filter, $\vec{P}_0 = \hat{y}$, and the ESP can be calculated using Eq. 12.

$$\mathrm{ESP} = \int \frac{(\vec{P}_0 + \Delta\vec{P}(r,z)) \cdot \vec{P}_0}{\left|\vec{P}_0 + \Delta\vec{P}(r,z)\right|\left|\vec{P}_0\right|}\,\rho(r,z)r\,d\varphi\,dr\,dz \qquad (23)$$

$$\approx 2\pi \int_0^\infty \int_{-\infty}^\infty \left(1 - \frac{\left(kB_\varphi\right)^2}{2}\right) \rho(r,z)\,r\,dr\,dz \qquad (24)$$

In the front end of the preinjector, downstream of the Wien filter, the spin direction is approximately vertical, $\vec{P} = \hat{y}$, except for small oscillations inside the solenoids. Because the solenoids occupy only about 5% of the total front-end length, we neglect their contribution here. We divide the front end into the three major sections listed in Table 2.

Table 2: Front-End Preinjector Beam Parameters Split into Three Major Sections

Section	Energy	Bunch Length	Polarization
Sec. 1	320 keV	1.3 ns	0.99961
Sec. 2	3 MeV	10 ps	0.998872
Sec. 3	55 MeV	2.9 ps	0.999997

From the gun to the end of the Wien filter (Section 1), the total spin degradation due to longitudinal and transverse space-charge effects is about 0.0008.

SPIN TRACKING

The analysis discussed above focuses primarily on space-charge effects and does not include beam emittance or other effects such as Wien-filter field non-uniformity and solenoid or quadrupole focusing. Therefore, we also performed a more comprehensive simulation with GPT (v3.5) [10], including both space charge and spin tracking.

The gun, solenoid, quadrupole, and Wien-filter fields were generated with external codes, including Poisson and Opera 3D.

The GPT output includes each particle's 6D phase space and spin direction at both time steps and position detectors defined in the GPT input file. Initially, all electrons had spins aligned with the z-axis; after passing through the Wien filters, the spins rotated toward the y-axis.

As illustrated in Fig. 2, the beam spin rotates from the z-axis to the y-axis as it passes through the two Wien filters, with each filter contributing a 45° rotation. The ESP was calculated using Eq. 12. From the gun to the exit of the Wien filter, the ESP decreases by 0.0014 as shown in Fig. 3. The analytical model predicts a total polarization degradation of 0.0008, so the simulated degradation is about 80% larger than the analytical prediction.

To investigate this discrepancy, we repeated the simulations without space charge. In that case, the polarization

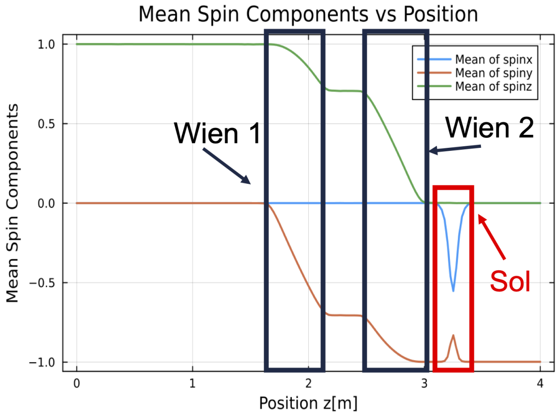

Figure 2: The spin angle from the gun to the exit of the Wien filter relative to the beam velocity direction.

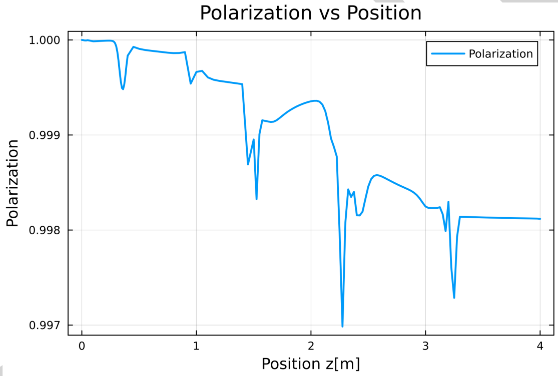

Figure 3: The ESP evolving from the gun to the exit of the Wien filter.

degradation was at the 10^{-5} level. In addition, the beam emittance was significantly reduced and the beam size became much smaller, so the beam experienced a more uniform field. These findings suggest that the additional 80% spin degradation seen in the full simulation arises from the combined effects of space-charge-driven emittance growth and the realistic 3D field map.

Overall, the contribution of space charge effects to spin degradation remains small, as confirmed by the simulations.

CONCLUSION

Vertical polarization electron bunch with high-charge, low-energy was a concern because of the strong space-charge force. In this paper, we evaluated the impact of space charge on ESP both inside the Wien-filter section and downstream of it. The analytical model predicts a polarization degradation of about 0.0008 in the front end, while the spin-tracking simulation gives an ESP degradation of 0.0014. The larger simulated value is attributed to the inclusion of realistic 3D fields and beam-emittance effects. Overall, both the analytical study and the simulations show that the impact of space charge on spin-polarization degradation is small. Therefore, the EIC will use Wien-filter as the spin rotator in the preinjector.

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