ELLIPTICAL BEAM SHAPING IN A STANDARD BOW-TIE PLANAR FOUR-MIRROR OPTICAL ENHANCEMENT CAVITY USING A CYLINDRICAL LENS

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Abstract

The standard bow-tie (SBT) planar four-mirror optical enhancement cavity (OEC) can amplify laser power, circulate laser pulses, and adjust the laser beam waist. It is widely applied in fields such as steady-state micro-bunching light sources, inverse Compton scattering light sources and fusion energy. However, the different effective radii of curvature in the sagittal and tangential directions vary with the incidence angle, which in turn leads to an elliptical cavity mode. Conventional approaches suppress this effect by restricting the transverse dimensions of the cavity, but this sacrifices the transverse design freedom of the optical enhancement cavity.

This paper proposes an elliptical cavity mode correction method using a cylindrical lens, enabling the fundamental mode at the beam waist to recover a circular shape. The method utilizes a cylindrical lens to differentially compensate the wavefront curvatures in sagittal and tangential directions, eliminating elliptical spot distortion without compressing the transverse dimensions of the cavity. Consequently, it overcomes the de-sign constraints on the transverse size of the optical enhancement cavity and provides a new approach for designing large-volume, high-power optical enhancement cavities.

Introduction

The application of OECs in many fields imposes requirements on their transverse dimensions. For example, in inverse Compton scattering (ICS), a certain transverse spatial size is needed to support the collision between electrons and photons [1]. In steady-state micro-bunching (SSMB), multiple OECs are used as laser modulators, which also requires that these OECs be scalable in transverse dimensions [2].

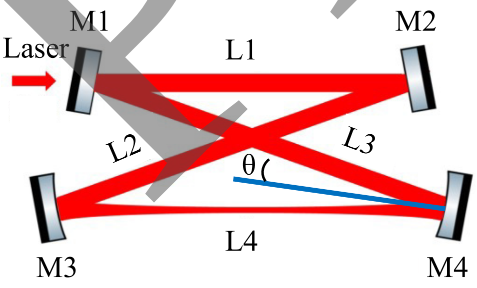

Figure 1: SBT cavity. M3 & M4: spherical mirrors with an effective radius of curvature *R*.

However, increasing the transverse dimensions leads to a larger laser incidence angle θ. For a spherical mirror with radius of curvature R, the effective radii of curvature in the sagittal and Tangential directions are $R_{e,s} = R/\cos\theta$ and $R_{e,t} = R\cos\theta$, respectively. Consequently, the radial direction reaches the stability edge first, resulting in a smaller beam waist and larger divergence angle in that direction (Fig. 2), which manifests as the major axis of the elliptical spot on the cavity mirror, and will increase the instability of the cavity mode and affect the gain of the OEC [3]. Therefore, in the applications such as SSMB and ICS, elliptical cavity modes also need to be avoided.

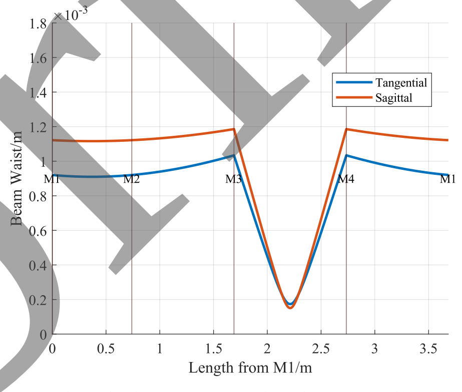

Figure 2: Simulation of gaussian beam radius evolution

Beam Shaping With a Cylindrical Lens

To suppress the elliptical cavity modes, the conventional approach is to reduce the transverse dimension of the OEC in order to decrease the incidence angle θ. However, as mentioned earlier, practical applications require a larger transverse dimension of the OEC. This paper proposes a method to correct the elliptical cavity mode using a cylindrical lens, enabling the fundamental mode at the beam waist to recover a circular shape. The cylindrical lens *CL* is inserted at the center of *L1*, adding focusing in the sagittal direction without affecting the tangential direction.

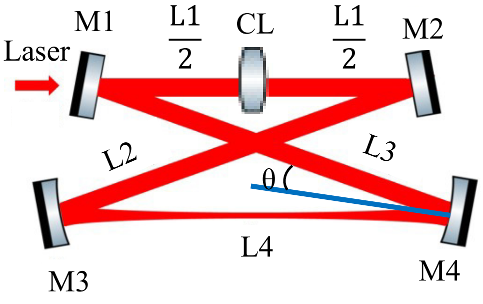

Figure 3: SBT cavity with a cylindrical lens *CL*

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Our goal is to correct the cavity mode at the beam waist to a circular shape. Let the round-trip length of the laser inside the SBT cavity (Fig. 1) be *L=L1+L2+L3+L4*, and the round-trip ABCD matrix starting from the beam waist between the cavity mirrors *M3* and *M4* is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & \frac{L_3}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & L-L_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L_3}{2} \\ 0 & 1 \end{bmatrix} \quad (1)$$

Where:

$$A = 1 - \frac{L}{f} + \frac{(L-L_3)L_3}{2f^2},$$
$$B = L + \frac{L_3(L_3-2L)}{2f} + \frac{(L-L_3)L_3^2}{4f^2},$$
$$C = -\frac{2}{f} + \frac{L-L_3}{f^2},$$
$$D = 1 - \frac{L}{f} + \frac{(L-L_3)L_3}{2f^2} \quad (2)$$

The beam waist ω_0 between the spherical mirrors of the optical cavity is [4]:

$$\omega_0 = \sqrt{\frac{\lambda}{\pi} \cdot \frac{2|B|}{\sqrt{4-(A+D)^2}}} \quad (3)$$

Substituting (2) into (3):

$$\omega_0 = \left\{ \left(\frac{\lambda}{2\pi}\right)^2 \cdot \frac{(2f-L_3)[(2f-L_3)L+L_3^2]}{2f+L_3-L} \right\}^{\frac{1}{4}} \quad (4)$$

Consider $R_{e,s} = R/\cos\theta$ and $R_{e,t} = R\cos\theta$, we get $f_t = \frac{R\cos\theta}{2}$, $f_s = \frac{R}{2\cos\theta}$, so the beam waists of the sagittal and tangential directions are:

$$\omega_{0,s} = \left\{ \left(\frac{\lambda}{2\pi}\right)^2 \cdot \frac{\left(\frac{R}{\cos\theta} - L_3\right)^2 \left[\frac{R}{\cos\theta} \cdot 4L_2\cos^2\theta - L_3(4L_2\cos^2\theta - L_3)\right]}{\left(\frac{R}{\cos\theta}\right)^2 - \frac{R}{\cos\theta} \cdot 4L_2\cos^2\theta + L_3(4L_2\cos^2\theta - L_3)} \right\}^{\frac{1}{4}} \quad (5)$$

$$\omega_{0,t} = \left\{ \left(\frac{\lambda}{2\pi}\right)^2 \cdot \frac{(\mathrm{R}\cos\theta - L_3)^2[\mathrm{R}\cos\theta \cdot 4L_2\cos^2\theta - L_3(4L_2\cos^2\theta - L_3)]}{(\mathrm{R}\cos\theta)^2 - \mathrm{R}\cos\theta \cdot 4L_2\cos^2\theta + L_3(4L_2\cos^2\theta - L_3)} \right\}^{\frac{1}{4}} \quad (6)$$

Now we need to calculate how does inserting the *CL* effect on the cavity mode in the sagittal direction. For the cavity with *CL* (Fig. 3), the round-trip ABCD matrix starting from the beam waist between the cavity mirrors *M3* and *M4* is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_{cy}} & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \quad (7)$$

Where:

$$A = 1 + \left(\frac{b-f-2f_{cy}}{f \cdot f_{cy}}\right)\left(a+b-\frac{a \cdot b}{f}\right),$$
$$B = \left[2\left(1-\frac{a}{f}\right) - \frac{1}{f_{cy}}\left(a+b-\frac{a \cdot b}{f}\right)\right]\left(a+b-\frac{a \cdot b}{f}\right),$$
$$C = -\left(1-\frac{b}{f}\right)\left[-\frac{2}{f} - \frac{1}{f_{cy}}\left(1-\frac{b}{f}\right)\right],$$
$$D = 1 + \left(\frac{b-f-2f_{cy}}{f \cdot f_{cy}}\right)\left(a+b-\frac{a \cdot b}{f}\right) \quad (8)$$

Substituting (8) into (3), also considering $R_{e,s} = R/\cos\theta$:

$$\omega'_{0,s} = \left\{ \left(\frac{\lambda}{2\pi}\right)^2 \cdot \frac{\left[4f_{CL}\left(2\frac{R}{\cos\theta} - L_3\right) - g\right]^2 G}{H\left[\left(\frac{R}{\cos\theta}\right)^2 \cdot 4f_{CL} - HG\right]} \right\}^{\frac{1}{4}} \quad (9)$$

Where f_{CL} is the focal length of the cylindrical lens *CL*, and:

$$G = \left(\frac{R}{\cos\theta} - L_3\right) 4L_2\cos^2\theta + L_3^2,$$
$$H = L_3 + \frac{R}{\cos\theta} + 4f_{CL} - 4L_2\cos^2\theta \quad (10)$$

To obtain a circular cavity mode at the beam waist, we only need to select an appropriate focal length f_{CL} so that the $\omega'_{0,s}$ is close to $\omega_{0,t}$, as the results in Fig. 4.

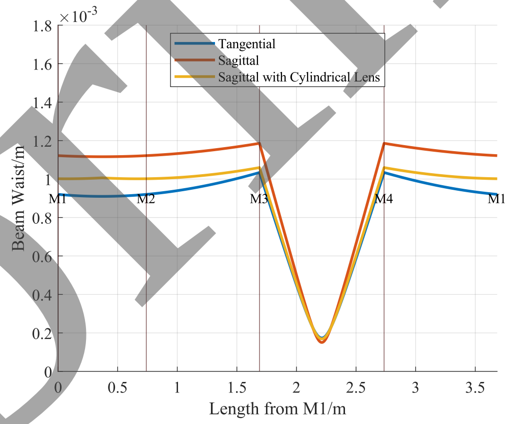

Figure 4: Simulation of gaussian beam radius evolution, with and without the cylindrical lens *CL*

Experimental Setup

To suppress the emergence of elliptical cavity modes, conventional OECs are designed with an incidence angle limited to below 5° [5]. In this paper, to demonstrate the emergence of elliptical cavity modes, a larger incidence angle is selected (Table 1). Figures 5 and 6 show the experimental setup.

Table 1: Optimized Geometries Of The Cavity

L1/mm	L2/mm	L3/mm	L4/mm	θ	*f_{CL}/m*
750.0	950.0	1035	950.0	10.02°	15.00

In the experiment, the spot at the beam waist is difficult to measure. We indirectly evaluate the cavity mode at the beam waist by measuring the spot after *M3*. Under the same cavity geometry, we measured the cavity modes without and with the cylindrical lens *CL*, respectively.

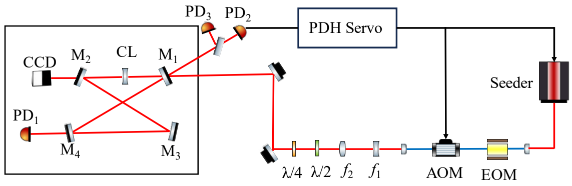

Figure 5: Schematic diagram of the experimental setup. *M1*, *M2*, *M3* and *M4* are the Fabry-Perot cavity mirrors. *CL* is a cylindrical lens. PD1, PD2 and PD3 are photodiodes. EOM is an electro-optic modulator. AOM is an acousto-optic modulator. CCD is a charge coupled device.

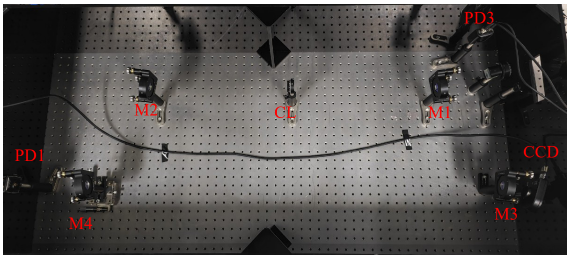

Figure 6: Experimental setup

Figure 7 and Table 2 show the measurement results of the spots at the cavity mirrors *M2* and *M3*, and the simulation results of the cavity mode at the beam waist without the cylindrical lens *CL*. The ellipticities of the three results are 80.28%, 82.67%, and 81.73%, respectively.

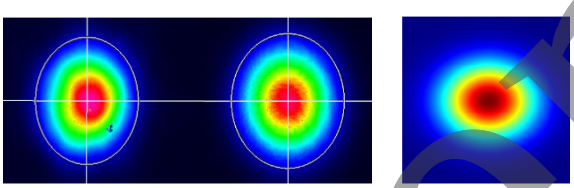

Figure 7: Measured spots at *M2* (left) and *M3* (center), and simulated cavity mode at the beam waist(right), SBT.

Table 2: Spot Size, SBT

Position	Tangential Radius[μm]	Sagittal Radius[μm]	Ellipticity
M2	1097.25	1366.75	80.28%
M3	1207.36	1460.39	82.67%
Beam waist	166.404	136.011	81.73%

After inserting the cylindrical lens *CL*, we re-measured and recalculated the above data. Figure 8 and Table 3 show the measurement results of the spots at the cavity mirrors *M2* and *M3*, and the simulation results of the cavity mode at the beam waist. It can be seen that the ellipticity of the spot at *M3* and the simulation cavity mode at the beam waist are 96.63% and 96.03%, respectively.

Comparing with the results without the cylindrical lens, the insertion of the cylindrical lens can be considered to have a good correcting effect on the elliptical cavity mode. With this method, the incidence angle of the SBT optical cavity can be increased, and its transverse dimension design can also be enlarged, which facilitates broader applications of the optical cavity in various fields.

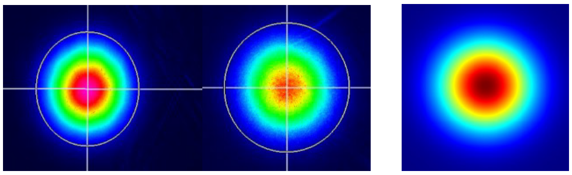

Figure 8: Measured spots at *M2* (left) and *M3* (center), and simulated cavity mode at the beam waist(right), with *CL*.

Table 3: Spot Size, With *CL*

Position	Tangential Radius[μm]	Sagittal Radius[μm]	Ellipticity
M2	1102.76	1226.51	89.91%
M3	1342.00	1388.75	96.63%
Beam waist	166.404	159.793	96.03%

CONCLUSION

This paper proposes a method for correcting the elliptical cavity mode in an SBT planar four-mirror optical cavity using a cylindrical lens. Theoretical calculations, numerical simulations, and experimental validation are carried out for this method. In the experiment, the ellipticity of the spot on the cavity mirror *M3* is corrected from 82.67% to 96.63%. This method is expected to expand the application of high-power, large-size OECs in fields such as ICS and SSMB.

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