ENTROPY GENERATION ANALYSIS OF HEAT LOADS IN AN SSR2 SUPERCONDUCTING CRYOMODULE*

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Abstract

The SSR2 superconducting cryomodule is analyzed in terms of static and dynamic heat loads and the associated entropy rates at the 2 K stage. The reference cryomodule contains six superconducting single-spoke resonator cavities. Static heat loads arise from conduction through cryomodule supports, couplers, cables, and beam-pipe interfaces, and from radiation through the insulation vacuum. Dynamic heat load is produced by radio-frequency surface dissipation. At 2 K, static and dynamic heat loads of 20 W and 80 W correspond to entropy rates of 10 W/K and 40 W/K, respectively, for a total cold-stage entropy rate of 50 W/K. The dynamic contribution accounts for 80% of the total entropy rate and scales inversely with the intrinsic quality factor Q_0. A coefficient-of-performance analysis shows that 100 W at 2 K requires at least 14.9 kW in the Carnot limit and about 100 kW for a representative real COP of 0.001. The results show that reducing radio-frequency loss, increasing Q_0, minimizing static heat leakage, and improving refrigerator performance directly reduce entropy generation and improve cryomodule efficiency.

INTRODUCTION

Superconducting radio-frequency (SRF) cavities are operated at cryogenic temperatures to achieve high accelerating fields with low RF surface resistance [1]. Although the surface resistance of a superconducting cavity is extremely small, it remains finite. As a result, part of the stored electromagnetic energy is dissipated on the cavity wall and converted into heat. This RF wall loss appears as a dynamic heat load in the helium cooling system. Related studies on zero-temperature phenomena [2], quality-factor conservation under relativistic motion [3], generalized effective temperature [4], unified analysis of quantum and relativistic effects in superconducting cavities [5], and SSR cavity performance [6] have been reported previously.

In addition to dynamic RF losses, the cryomodule receives static heat loads through mechanical supports, couplers, instrumentation cables, beam-pipe interfaces, residual-gas effects, and thermal radiation across the insulation vacuum. A conventional heat-load budget lists these contributions in watts at each temperature stage. From a thermodynamic viewpoint, however, a watt deposited at 2 K is much more severe than a watt deposited at a higher temperature because the entropy associated with heat transfer increases as the heat load is deposited at lower temperature [7]. This penalty is also evident in practical SRF cryogenic systems, where wall-plug conversion factors of approximately 800 W/W at 2 K [8] and 720.3 W/W near 1.8 K [9] have been reported.

This paper converts the SSR2 cryomodule heat-load budget into an entropy-rate budget. The objective is to connect SRF cavity quality factor, RF surface dissipation, static heat leakage, coefficient of performance (COP), and refrigerator irreversibility within a common thermodynamic framework. This approach provides a direct method for evaluating how RF-loss reduction, improved cryomodule thermal design, and higher cryogenic efficiency reduce entropy generation and improve overall accelerator performance [10].

SSR2 CRYOMODULE HEAT-LOAD BASIS

The reference cryomodule considered here contains six SSR2 superconducting single-spoke resonator cavities operated in a 2 K class helium system. The parameters used for the entropy analysis are summarized in Table 1. The reference 2 K heat-load basis consists of 20 W static heat load and 80 W dynamic RF heat load, for a total of 100 W at the cold stage.

Table 1: Representative SSR2 Parameters used in the Entropy Analysis

Parameter	**Value**
T_c	2 K
f	325 MHz
V_{acc}	4.1 MV
E_{acc}	>8.7 MV/m
$Q_{0,ref}$	5×10^9
R/Q	252 Ω
G	123 Ω
N_{cav}	6
Static heat load	20 W
Dynamic RF heat load	80 W

The static load is controlled mainly by cryomodule hardware and thermal shielding. The dynamic load is controlled mainly by RF surface resistance and cavity operating field. For the reference case, the dynamic RF load is the dominant contribution and therefore strongly determines the entropy burden during powered operation.

THERMODYNAMIC FORMULATION

Cold-Stage Entropy Rate

For a heat load deposited at a cold stage, the cold-stage entropy rate is approximated by

$$\dot{S}_{\mathrm{c}} = \frac{\dot{Q}_{\mathrm{c}}}{T_{\mathrm{c}}}, \quad (1)$$

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where the numerator is the heat absorbed at the cold stage. At 2 K, each watt of heat corresponds to 0.5 W/K of entropy rate. When multiple heat loads are deposited at the same temperature, the entropy rates are additive:

$$\dot{S}_{\mathrm{tot}} = \sum_i \frac{\dot{Q}_{\mathrm{i}}}{T_{\mathrm{c}}}. \tag{2}$$

For static heat leakage from a warm reservoir to the cold reservoir, the entropy generated along the heat-transfer path can be written approximately as

$$\dot{S}_{\mathrm{gen,path}} = \dot{Q}\left(\frac{1}{T_{\mathrm{c}}} - \frac{1}{T_{\mathrm{h}}}\right). \tag{3}$$

Since the warm-reservoir temperature is much larger than the cold-stage temperature in a cryogenic system, Eq. (3) is close to the cold-stage entropy rate in Eq. (1). For RF dissipation occurring directly on the superconducting surface, Eq. (1) gives the local entropy rate associated with conversion of ordered RF energy into heat.

RF Loss and Quality Factor

For an SRF cavity operated at a given accelerating voltage, the RF power dissipated on the cavity surface is

$$P_{\mathrm{diss}} = \frac{V_{\mathrm{acc}}{}^2}{(R/Q)Q_0}. \tag{4}$$

The corresponding dynamic entropy rate at the cold stage is

$$\dot{S}_{\mathrm{dyn}} = \frac{V_{\mathrm{acc}}{}^2}{(R/Q)Q_0 T_{\mathrm{c}}}. \tag{5}$$

At fixed accelerating voltage, geometry, and temperature, the dynamic heat load and dynamic entropy rate scale inversely with the intrinsic quality factor:

$$\dot{S}_{\mathrm{dyn}} \propto P_{\mathrm{diss}} \propto \frac{1}{Q_0}. \tag{6}$$

Thus, improving Q_0 directly reduces RF dissipation, dynamic heat load, entropy generation, and required refrigeration power.

COP and Refrigerator Irreversibility

The ideal coefficient of performance of a refrigerator operating between a cold stage and a warm reservoir is the Carnot COP:

$$\mathrm{COP}_{\mathrm{Carnot}} = \frac{T_{\mathrm{c}}}{T_{\mathrm{h}} - T_{\mathrm{c}}}. \tag{7}$$

The minimum input power needed to remove heat from the cold stage is

$$\dot{W}_{\mathrm{min}} = \frac{\dot{Q}_{\mathrm{c}}}{\mathrm{COP}_{\mathrm{Carnot}}}. \tag{8}$$

For a real refrigerator, the real COP is lower than the Carnot COP. The refrigerator entropy generation due to irreversibility can be expressed as

$$\dot{S}_{\mathrm{gen,ref}} = \left(\frac{\dot{Q}_{\mathrm{c}}}{T_{\mathrm{h}}}\right)\left[\frac{1}{\mathrm{COP}_{\mathrm{real}}} - \frac{1}{\mathrm{COP}_{\mathrm{Carnot}}}\right]. \tag{9}$$

Equation (9) becomes zero only in the reversible limit, where the real refrigerator reaches the Carnot COP. Therefore, the cold-stage entropy load and the refrigerator entropy generation should be treated separately. The cold-stage entropy load is associated with heat deposited in the cryomodule, whereas the refrigerator entropy generation arises from irreversibilities in the real refrigeration process while removing that heat load.

RESULTS

Entropy Rates at 2 K

At 2 K, the 20 W static load gives 10 W/K, while the 80 W dynamic load gives 40 W/K. The total 2 K heat load of 100 W corresponds to 50 W/K. As summarized in Table 2, the dynamic RF heat load accounts for 80% of both the total heat load and the total entropy rate.

Table 2: Static, Dynamic, and Total Entropy Rates at 2 K

Item	**Heat**	**Entropy**	**Share**
Static	20 W	10 W/K	20%
Dynamic	80 W	40 W/K	80%
Total	100 W	50 W/K	100%

The average dynamic heat load per cavity is 80 W/6 = 13.33 W. This corresponds to 6.67 W/K per cavity at 2 K. The per-cavity estimate is listed in Table 3 and is consistent with Eq. (4) for an accelerating voltage of 4.1 MV, R/Q = 252 Ω, and $Q_0 = 5 \times 10^9$.

Table 3: Per-Cavity Dynamic Heat-Load Estimate

Quantity	**Value**
Dynamic heat load, module	80 W
Number of cavities	6
Dynamic heat load, cavity	13.33 W
Entropy rate, cavity	6.67 W/K
Dominant control	Q_0 and RF surface resistance

Effect of Q_0 Improvement

Because the dynamic load scales as $1/Q_0$, changes in Q_0 produce nearly linear changes in dynamic entropy generation. Table 4 shows the scaling while the static heat load is kept fixed at 20 W. Doubling Q_0 decreases the total entropy rate from 50 W/K to 30 W/K. A fourfold improvement in Q_0 decreases the total entropy rate to 20 W/K in this simplified model.

Table 4: Q_0 Scaling of Dynamic Heat Load and Entropy Rate

$Q_0/Q_{0,\mathrm{ref}}$	**Dynamic heat**	**Dynamic $\dot{S}$**	**Total $\dot{S}$**
0.5	160 W	80 W/K	90 W/K
1	80 W	40 W/K	50 W/K
2	40 W	20 W/K	30 W/K
4	20 W	10 W/K	20 W/K

COP-Based Interpretation

The COP calculation shows the system-level penalty of a 2 K heat load. For a refrigerator operating between 2 K and 300 K, the Carnot COP is:

$$\mathrm{COP}_{\mathrm{Carnot}} = \frac{2}{300-2} = 0.0067. \tag{10}$$

For a total 2 K heat load of 100 W, the ideal minimum work is approximately:

$$\dot{W}_{\min} = \frac{100\ \mathrm{W}}{0.0067} \approx 14.9\ \mathrm{kW}. \tag{11}$$

If a representative real 2 K COP of 0.001 is used, the real input work becomes:

$$\dot{W}_{\mathrm{real}} = \frac{100\ \mathrm{W}}{0.001} = 100\ \mathrm{kW}. \tag{12}$$

The corresponding refrigerator entropy generation is approximately 283.6 W/K. This value is much larger than the 50 W/K cold-stage entropy load because the real refrigerator is much less efficient than the Carnot limit.

Table 5: COP-Based Input Power and Refrigerator Entropy Generation

Item	Carnot power	Real power	Entropy gen.
Static	2.98 kW	20 kW	56.7 W/K
Dynamic	11.92 kW	80 kW	226.9 W/K
Total	14.90 kW	100 kW	283.6 W/K

Table 5 summarizes the static, dynamic, and total contributions to the Carnot minimum input power, real input power, and refrigerator entropy generation for a representative real 2 K COP of 0.001.

The real-COP estimate is consistent with the published SRF cryogenic practice cited above. Saini et al. used approximately 800 W of wall-plug power per watt dissipated at 2 K when evaluating radiation heat loads in a cryomodule. Chojnacki et al. estimated cryomodule wall-plug power using helium-refrigerator vendor COP values and reported a 1.8 K COP factor of 720.3 W/W. Pierini described the Carnot and entropy-pumping basis of cryogenic refrigeration for superconducting RF systems.

DISCUSSION

The entropy-generation view clarifies why the 2 K stage is the most thermodynamically expensive location for heat deposition. Heat that reaches 2 K imposes a large entropy burden and requires substantial wall-plug power to remove. Therefore, thermal intercepts at 50-80 K and other intermediate-temperature stages are important, even when their heat loads are larger in watts.

Dynamic entropy generation is reduced most effectively by improving cavity performance. Surface preparation, residual-resistance reduction, trapped-flux mitigation, clean assembly, optimized cooldown, magnetic shielding, and stable RF operation all support higher Q_0. Since the dynamic entropy rate is proportional to $1/Q_0$, an improvement in Q_0 provides a direct reduction in dynamic heat load and cold-stage entropy rate.

Static entropy generation is reduced through cryomodule thermal design. Low-thermal-conductivity supports, longer conduction paths, optimized cable routing, low-emissivity surfaces, multilayer insulation, line-of-sight radiation blocking, and well-designed thermal intercepts decrease heat leakage to the 2 K helium system. The main entropy-reduction design levers are summarized in Table 6.

Table 6: Design Levers for Entropy Reduction

Source	Design lever	Effect
Dynamic RF	Increase Q_0; reduce surface resistance	Lower RF loss and dynamic entropy rate
Conduction	Long paths; low-k supports	Lower static heat leak to 2 K
Radiation	MLI; low-emissivity surfaces	Reduce radiative heat load
Interfaces	Thermal intercepts	Move heat to warmer stages
Refrigerator	Improve COP	Reduce input power and refrigerator entropy generation

CONCLUSION

An entropy-generation analysis was applied to the static and dynamic heat loads of an SSR2 superconducting cryomodule. For the reference heat-load budget of 20 W static load and 80 W dynamic RF load at 2 K, the corresponding cold-stage entropy rates are 10 W/K and 40 W/K, respectively. The total cold-stage entropy rate is 50 W/K.

The dynamic RF heat load from six cavities contributes about 80% of the total entropy rate. Because the dynamic heat load scales inversely with Q_0, improving the intrinsic quality factor directly reduces RF dissipation, heat load, entropy generation, and required refrigeration power. COP analysis further shows that 100 W at 2 K requires at least 14.9 kW in the Carnot limit but approximately 100 kW for a real COP of 0.001, producing approximately 283.6 W/K of refrigerator entropy generation. This estimate is consistent with the cryogenic conversion factors and thermodynamic SRF cryogenics framework discussed above. These results show that entropy analysis is a useful framework for connecting cavity RF performance, cryomodule heat-load reduction, and accelerator cryogenic efficiency.

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