INVESTIGATIONS OF NON-LINEAR OPTICS CONTROL KNOBS FOR THE FCC-ee

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Abstract

The non-linear effects that arise from misalignment and field errors have been shown to degrade both the dynamic aperture (DA) and (MA) in simulation of the Future electron-positron Circular Collider machine (FCC-ee). This study focuses on using multipoles, such as sextupoles or octupoles, to control amplitude detuning and higher-order chromaticity as well as investigation of dedicated non-linear correctors. These non-linear parameters are often coupled and the relative strength with which lattice elements act on each non-linearity depends on the local optical parameters. Studies are performed on the placement and strength of non-linear lattice elements to contribute to developing orthogonal correction knobs and first attempts at higher-order corrections to recover the reduced DA and MA are explored.

INTRODUCTION

The Future electron-positron Circular Collider (FCC-ee) is an about 91 km storage ring with thousands of magnetic elements, and will operate at energies ranging from 45.6 GeV to 182.5 GeV per beam [1–3]. The unprecedented size and number of elements leads to extreme importance of tuning systems to address any misalignments and field errors that arise in the construction of such an accelerator. Current magnet estimates state that the arc dipoles will have 10 units of error in normal dipole and quadrupole fields (b_1 and b_2 errors) and 0.5 units of normal sextupole and octupole fields (b_3 and b_4 errors) [4]. One unit is a multipole component of 10^{-4} of the lowest order magnetic field, at a reference radius 0.01 m [5].

It has been found that even when the linear optics are well corrected in the presence of misalignments and field errors, that the momentum acceptance (MA) and dynamic aperture (DA) are still reduced [6–8]. This suggests that non-linear correction schemes will be important to ensure performance reach of the FCC-ee. The effects of b_3 and b_4 errors on the MA and initial corrections of each are discussed in the following. Other tuning studies are ongoing in parallel, see e.g. [9–12].

ARC BEND b_3 ERROR AND CORRECTION

To first order, b_3 errors introduce additional linear chromaticity. Amplitude detuning is also impacted as a second order effect.

The magnet estimates for both random and systematic b_3 errors are 0.5 units. Effects of these errors on the FCC-ee lattice version LCC 106.2.1 [13–15] for the Z energy (45.6 GeV) are explored with simulation using Xsuite and MAD-NG [16, 17]. Tracking of particles to determine MA is performed in Xsuite while MAD-NG is used to calculate high-order optics parameters and Resonance Driving Terms (RDTs). First, the sextupole (b_3) errors are applied to all arc dipoles, and particles are tracked in Xsuite at various momentum offsets ($\Delta p/p = \delta$) and amplitudes for 2500 turns using Synchrotron Radiation (SR) with Quantum Fluctuation (QF). In these scans the particles are initialized with an equal number of horizontal and vertical sigmas amplitude, $N_{\sigma_x} = N_{\sigma_y}$. For each error scenario 10 seeds for the QF are averaged.

As a figure of merit, we consider the area of stable initial conditions for DA and MA, seen in Fig. 1. While applying random b_3 errors does not reduce the MA, systematic b_3 errors reduce this quantity in δ-amplitude space by $\approx$ 60%. The strengths of the four arc sextupole families, two focusing and two defocusing, are adjusted with four 'knobs' to restore the MA. Changing the sextupole strengths, K_2, gives control of the horizontal ($\partial Q_x/\partial J_x$) and cross-term ($\partial Q_{x,y}/\partial J_{y,x}$) amplitude detuning and linear chromaticity ($Q'_{x,y} \equiv dQ_{x,y}/d\delta$). When these amplitude detuning terms and the linear chromaticity are restored to their design values the MA is fully recovered, as seen in Fig. 1.

These corrections result in an absolute change in the sextupole strength of a maximum of $-0.013\ \mathrm{m}^{-3}$ for one of the defocusing families, sd2a, corresponding to roughly 7 % relative to the nominal setting. For the other three families the change is less than 5 %. For increasing systematic b_3 a linear relationship to the required change in sextupole strengths is found, shown in Fig. 2. While this correction is feasible in the Z lattice, for the 182.5 GeV $t\bar{t}$ lattice the arc sextupoles are near their strength limitation by design, so a similar level of correction could require lengthening the sextupoles.

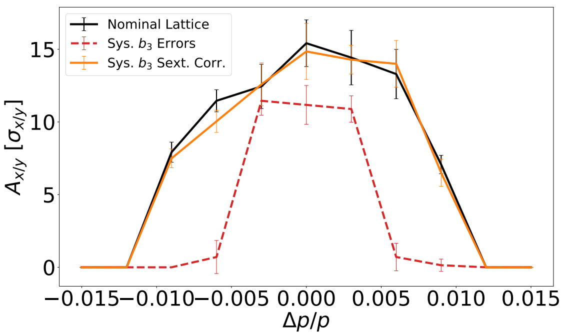

Figure 1: MA of the nominal lattice (black), with systematic b_3 errors (red), and with additional sextupole correction (orange).

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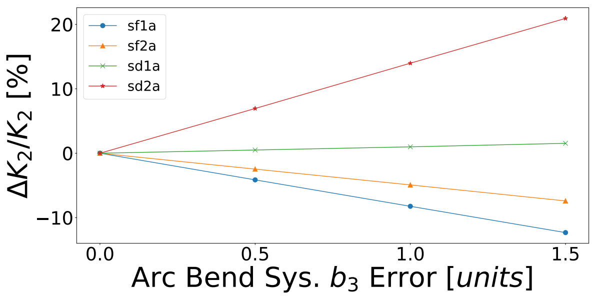

Figure 2: Relative strength change of arc sextupole families to correct systematic b_3 errors.

ARC DIPOLE b_4 ERRORS EFFECT

The impact of b_4 errors on amplitude detuning is in the first order, and unlike sextupole fields they do not directly affect linear chromaticity. Instead they act on second-order chromaticity with a first-order contribution.

For 0.5 units of random b_4 errors, there is negligible effect on the MA. For 0.5 units of systematic b_4, with different sign between inner and outer arcs there is only minor impact on the amplitude detuning and second-order chromaticity ($Q''_{x,y} \equiv d^2Q_{x,y}/d\delta^2$), as summarized in Table 1.

Table 1: High Order Optics Terms

Lattice	$\frac{\partial Q_x}{\partial J_x}$ [10^3m^{-1}]	$\frac{\partial Q_y}{\partial J_y}$ [10^6m^{-1}]	$\frac{\partial Q_{x,y}}{\partial J_{y,x}}$ [10^5m^{-1}]	$\frac{\partial^2 Q_x}{\partial \delta^2}$ [10^3]	$\frac{\partial^2 Q_y}{\partial \delta^2}$ [10^2]
Ideal	-1.5	1.4	1.7	-2.1	2.4
Sys. b4	-1.7	1.4	1.7	-2.2	2.4

Since non-linear optics are not significantly affected, resulting MA with such errors is also not noticeably reduced, shown in Fig. 3. Furthermore, RDTs and their relative change with systematic b_4 errors are evaluated using MAD-NG. The rms change over all elements is below 1%, as shown in Fig. 4. Although with assumed errors the effect on MA is marginal, we note that these simulations are optimistic since only one type of errors is assumed, and no misalignments are assumed. Since the change of RDTs is considered small, and thus requires a very precise and sophisticated optics measurement strategy [18, 19], fully restoring the MA is studied

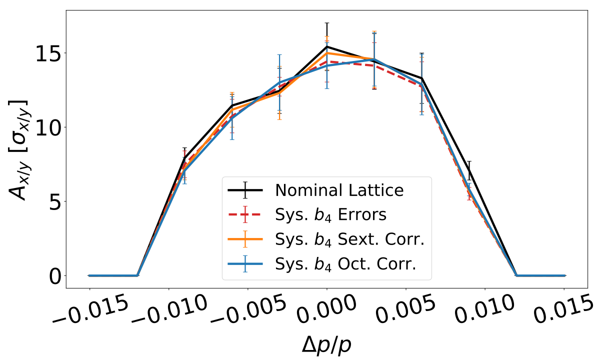

Figure 3: MA of nominal lattice (black), with systematic b_4 errors (red), when corrected with sextupole families (orange), and with inserted octupoles (blue).

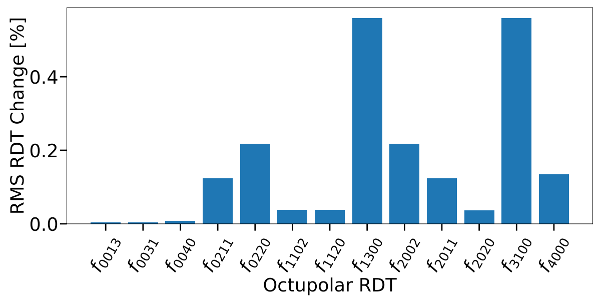

Figure 4: RMS percent change of octupolar RDTs with 0.5 units of systematic b_4 errors.

here by correction of amplitude detuning and second-order chromaticity.

CONTROL OF b_4 ERRORS

Two methods of correction knobs are considered.

Correction with Sextupoles

The same sextupole correction knobs used in the correction of systematic b_3 errors above are used to address the horizontal amplitude detuning and the amplitude detuning cross-term changes from systematic b_4 errors. While the first order chromaticity is not directly affected by the b_4 errors, the sextupole correction for amplitude detuning shifts linear chromaticity, which is therefore corrected here too. The strengths required to address the non-linear optics changes from systematic b_4 are much lower than the systematic b_3 case, as shown in Fig. 5. The percent change in sextupole strengths remains below 1% at three times the estimated error. In Figure 3, it is shown that the MA is preserved with this correction of the non-linear optics.

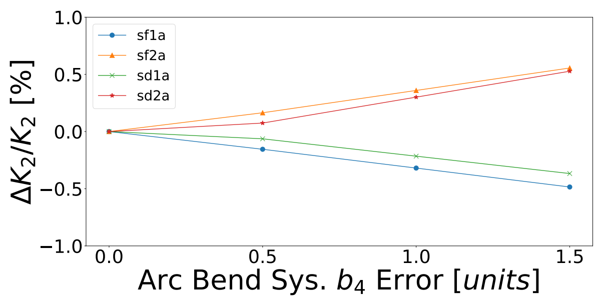

Figure 5: Relative strength change of arc sextupole families to correct systematic b_4 errors.

Identifying Effective Octupole Correctors

A second, novel approach for the LCC lattice, is correcting the amplitude detuning with octupoles. Including octupole correctors in the lattice could be a complementary approach to the sextupole correction discussed above. Preliminary investigations of pessimistic b_4 error assumptions show that octupole correctors are effective and could potentially even expand the MA beyond the current nominal [19], which should be investigated further.

Detailed derivations of the following two formulae can be found in Ref. [20] and the most relevant results are recalled here. The effect of an octupole strength K_3, with a length L, at position s where the beta-function is β_x adds to the amplitude detuning in the horizontal plane according to

$$\frac{\partial Q_x}{\partial J_x} = \frac{1}{16\pi} K_3 L \beta_x^2(s) \ , \tag{1}$$

and reads similar for vertical amplitude detuning. The effect on 2nd-order chromaticity is

$$\frac{\partial^2 Q_x}{\partial \delta^2} = \frac{1}{4\pi} K_3 L \beta_x (D_x(s)^2 - D_y(s)^2), \tag{2}$$

which includes the horizontal and vertical dispersions at the octupole location $D_{x,y}(s)$. For vertical 2nd-order chromaticity β_x is replaced by β_y and the sign is flipped. From these well established formulae we can determine a factor, F, from the ratio of the change in each term due to an octupole with strength K_3,

$$F = \left(\frac{\partial Q_x}{\partial J_x}\right) \Big/ \left(\frac{\partial^2 Q_x}{\partial \delta^2}\right) \ , \tag{3}$$

$$F = \beta_x / (4D_x^2 - 4D_y^2) \ , \tag{4}$$

which simplifies further with $D_y \ll D_x$ to

$$F = \beta_x / (4D_x^2) \ . \tag{5}$$

While this is not a dimensionless ratio, it shows that in order to have an octupole act on amplitude detuning with minimal effect on second-order chromaticity it must be located at a position s with large beta function and low dispersion. On the other hand, for an octupole to have strong impact on 2nd-order chromaticity there must be relatively high dispersion.

Correction with Octupoles

Considering this ratio, it can be seen that the start and end of each straight section, near the dispersion suppressors, are the optimal locations to insert octupoles for correction of amplitude detuning. For control of 2nd-order chromaticity octupole correctors are placed at the first and last focusing and defocusing quadrupoles in the arcs, where dispersion is larger. The positions that the octupoles are added to the model are shown in Fig. 6.

Note that these octupoles are not fully orthogonal, so each corrector will change both amplitude detuning and 2nd-order chromaticity. With these four octupole correction families, the non-linear optical parameters are matched to the nominal values given in Table 1. The non-linear optics terms deviate only slightly from nominal, as shown in Table 1, so the correction is performed with only a single 2 m octupole corrector at each of the locations in Fig. 6. Figure 3 shows that the correction of the non-linear optics is performed with no degradation in MA with this much sparser octupole corrector setup. Using a total of sixteen 2 m octupoles in the ring the correction is achieved with $|K_3| < 3\ \text{m}^{-4}$.

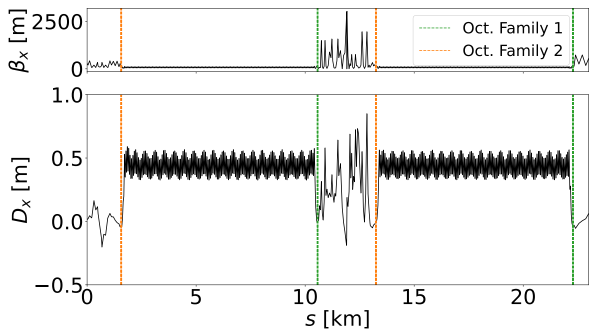

Figure 6: Locations selected for amplitude detuning correction octupoles to maximize Eq. 5 through minimal dispersion (bottom) and large β (top).

CONCLUSIONS

For the FCC-ee LCC 106.2.1 Z lattice it has been shown that the estimated 0.5 units of random and systematic b_3 errors on arc dipoles can be controlled by correction of first-order chromaticity and amplitude detuning requiring less than a 7% change of the arc sextupole strengths. MA is fully recovered with this correction.

Similarly, 0.5 units of random b_4 errors are not seen to degrade the MA. Systematic b_4 errors of the same magnitude, when applied considering the beam crossings, do not strongly degrade the MA, and the MA and non-linear optics parameters of the lattice can be recovered with a similar sextupole correction.

An alternative method of correcting the high-order optics using octupole magnets is also presented. Octupoles are inserted into the model at locations chosen to maximize or minimize their impact on amplitude detuning relative to 2nd-order chromaticity to produce knobs that are as independent as possible. Further studies are planned to explore if some iteration of this octupole corrector scheme can be used to enhance the MA when used in combination with the sextupole corrections in the FCC-ee lattice.

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REFERENCES

[1] M. Benedikt *et al.*, “Future circular collider feasibility study report – Volume 2 accelerators, technical infrastructure and safety”, *Eur. Phys. J. Spec. Top.*, vol. 234, no. 19, pp. 5713–6197, 2025. doi:10.1140/epjs/s11734-025-01967-4

[2] M. Benedikt *et al.*, “Future circular collider feasibility study report – Volume 1 physics, experiments, detectors”, *Eur. Phys. J. C.*, vol. 85, no. 12, p. 1468, 2025. doi:10.1140/epjc/s10052-025-15077-x

[3] M. Benedikt *et al.*, "Future circular collider feasibility study report – Volume 3 civil engineering, implementation and sustainability", *Eur. Phys. J. Spec. Top.*, vol. 234, pp. 5113–5383, 2025. doi:10.1140/epjs/s11734-025-01958-5

[4] L. Todorovic, "FCC-ee collider tolerance analysis", presented at the FCC-ee optics tuning WG meeting, Mar. 2026, https://indico.cern.ch/event/1658268/contributions/6969507,

[5] A. K. Jain, "Basic theory of magnets", in *CAS – CERN Accelerator School: measurement and alignment of accelerator and detector magnets*, Anacapri, Italy, pp. 1–26, Apr. 1997. doi:10.5170/CERN-1998-005.1

[6] S. S. Jagabathuni, "Optics tuning of the FCC-ee in Xsuite", presented at FCC-ee optics tuning WG meeting, Feb. 2026, https://indico.cern.ch/event/1648395/contributions/6928073/,

[7] Kévin. André, P. Raimondi, S. White, and S. Liuzzo, "Tuning strategies for the FCC-ee local chromaticity correction collider lattice", presented at IPAC'26, Deauville, France, May 2026, paper MOP1079, this conference.

[8] T. K. Charles, B. Holzer, R. Tomas, K. Oide, Léon. van Riesen-Haupt, and F. Zimmermann, "Alignment & stability challenges for FCC-ee", *EPJ Techn. Instrum.*, vol. 10, no. 1, Mar. 2023. doi:10.1140/epjti/s40485-023-00096-3

[9] J. Keintzel, "FCC-ee optics tuning - towards the pre-TDR", presented at IPAC'26, Deauville, France, May 2026, paper MOP1039, this conference.

[10] W. V. Goethem, "Impact and correction of a3 random errors in the arc dipoles", presented at FCC-ee optics tuning WG meeting, Feb. 2026, https://indico.cern.ch/event/1654640/contributions/6955556/,

[11] W. V. Goethem, "Impact and correction of systematic b2 errors in the arc dipoles & tapering", presented at FCC-ee optics tuning WG meeting, Apr. 2026, https://indico.cern.ch/event/1667088/contributions/7021275,

[12] T. Prebibaj, "Initial studies on resonance driving term correction knobs for FCC-ee beam-beam and imperfections", presented at IPAC'26, Deauville, France, May 2026, paper MOP1040, this conference.

[13] P. Raimondi, S. M. Liuzzo, L. Farvacque, S. White, and M. Hofer, "Local chromatic correction optics for future circular collider e^+e^-", *Phys. Rev. Accel. Beams*, vol. 28, no. 2, p. 021002, Feb. 2025. doi:10.1103/PhysRevAccelBeams.28.021002

[14] J. Keintzel, "Defining the new FCC-ee baseline optics", presented at IPAC'26, Deauville, France, May 2026, paper MOV1002, this conference.

[15] G. Iadarola, "FCC-ee lattice repository releases", 2026, https://gitlab.cern.ch/acc-models/fcc/acc-models-fcc-ee/-/releases,

[16] G. Iadarola *et al.*, "Xsuite: an integrated beam physics simulation framework", in *Proc. HB'23*, Geneva, Switzerland, Oct. 2023, pp. 73–80. doi:10.18429/JACoW-HB2023-TUA2I1

[17] L. Deniau, "MAD-NG, a standalone multiplatform tool for linear and non-linear optics design and optimisation", *arXiv*, Jan. 2025. doi:10.48550/arXiv.2412.16006

[18] J. Keintzel, "Progress on optics measurements for the FCC-ee", presented at IPAC'26, Deauville, France, May 2026, paper MOP1038, this conference.

[19] P. Hunchak, J. Keintzel, and M. Boland, "FCC-ee non-linear optics updates", presented at FCC-ee optics tuning WG meeting, Apr. 2026, https://indico.cern.ch/event/1667088/contributions/7021276,

[20] J. Dilly and M. le Garrec, "On the derivation of Amplitude Detuning and Chromaticity Formulas for Particle Accelerators", *arXiv*, Jan. 2023. doi:10.48550/arXiv.2301.09132