NON-LINEAR RESONANCE FEED-DOWN - A NEW TECHNIQUE FOR CORRECTING HIGH ORDER ERRORS IN THE LHC

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Abstract

Optics errors from the interaction points (IPs) of the LHC, where β^* is most strongly squeezed, can significantly impact machine performance and protection. In anticipation of the HL-LHC, correction strategies extending up to dodecapole order are being targeted. Direct measurement of high-order resonance driving terms (RDTs) remains challenging, however. Applying crossing angle orbit bumps in the experimental insertions induces feed-down from higher-order errors, increasing the magnitude of lower-order RDTs. Leveraging this effect, a novel correction scheme based on RDT feed-down was implemented for the first time in 2025. Skew-octupole errors were successfully corrected, which enabled optics measurements at the collisions working point, down to an unprecedented level of $\beta^* = 18$ cm. Measurements of feed-down from dodecapole errors, to decapole RDTs were also achieved, opening a practical pathway to efficient corrections of very high-order optics errors.

MOTIVATION

Control of non-linear errors in experimental insertions has long been recognised as critical for collider performance. Such effects were already studied for accelerators such as the Tevatron [1], RHIC [2,3], and SuperKEKB [4–6].

The LHC is especially sensitive to interaction region (IR) optics errors due to the large β-functions in the triplet quadrupoles required to reach low β^* at the IP. This sensitivity will increase in the High-Luminosity LHC (HL-LHC), where further reduction of β^* leads to even larger β-functions in these triplets. As a result, the upgrade targets correction of optics errors up to dodecapolar order [7,8].

Following the successful application of non-linear forced-RDT measurements with the AC-dipole [9] at both injection [10] and top energy [11], attention has shifted towards extending these techniques to higher-order RDTs in order to correct optics errors originating from the IRs. Direct measurement of such higher-order terms, particularly of decapolar order and above, remains experimentally challenging.

In the LHC, commissioning procedures routinely include corrections for lower-order non-linearities, in particular the sextupolar and normal octupolar components [12], by exploiting their feed-down to tune. This feed-down effect originates from the beam traversing a magnet with a transverse offset, arising from either magnet misalignments or a displaced closed orbit, therefore generating effective lower-order field components from higher-order magnetic fields [13].

This approach was first extended to decapolar and dodecapolar error correction via feed-down to amplitude detuning, demonstrating successful results [14], albeit at the cost of significant machine time.

Building on this, it was proposed that higher-order magnetic errors could be corrected indirectly through their feed-down contributions to lower-order non-linear RDTs, therefore reducing operational time while avoiding the need for direct measurement [15].

CORRECTION PRINCIPLE

The correction procedure is based on performing forced-RDT measurements under identical machine conditions, namely at the same tunes, differing only in the applied crossing angle. This enables the determination of the partial derivative of the RDT with respect to the crossing angle.

In parallel, the simulated RDT response to a variation in crossing angle is evaluated with each corrector of interest powered at equal strength for both crossing angle configurations. The excitation of each corrector is chosen to emulate the targeted non-linear error. From these simulations, the corresponding partial derivatives of the RDT with respect to the crossing angle are obtained.

By matching the simulated derivatives to the measured responses, the optimal strengths of the correctors can be determined, therefore providing the required correction, as illustrated in Fig. 1.

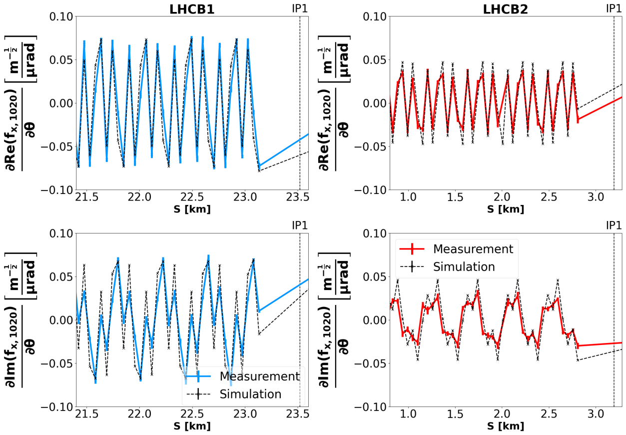

Figure 1: Demonstration of the matching of the partial derivatives in simulation with those in measurement for both beams.

A new GUI was developed to to allow the analysis to be performed online in the CCC during commissioning.

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a_4 ERROR CORRECTION

To assess the feasibility of the method on the real machine, previously acquired measurements used for feed-down to coupling were analysed, in which the f_{0030} RDT was clearly identifiable in the spectrum. A systematic shift of this RDT as a function of the crossing angle was observed, as can be seen in Fig. 2.

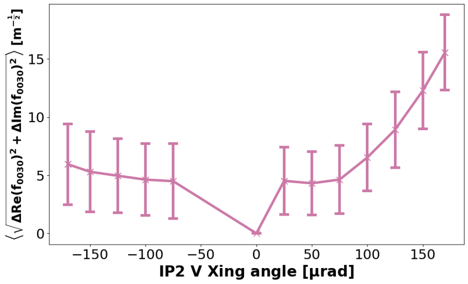

Figure 2: Average f_{0030} RDT amplitude shift vs vertical IP2 crossing angles for LHCB1 (line is visual guide for trend).

This finding motivated a dedicated study aimed at trying to correct some higher-order non-linear errors. The first being of the skew-octupolar order since these are of particular importance for LHC commissioning - these components can adversely affect the dynamic aperture, and limit beam excitation with the AC-dipole. However, a robust and consistently reliable measurement procedure has not yet been established.

Previous studies have investigated the use of second-order feed-down to coupling as an indirect diagnostic [16], but these methods proved unreliable and ambiguous, particularly in distinguishing whether corrections should be applied on the left or right side of the IP. Direct corrections based on RDT measurements have occasionally been successful [17, 18], yet reliable measurements at low β^* remain elusive.

The skew-octupole component is therefore well suited to the present method, as the associated sextupolar-order RDT, arising from feed-down, can be measured with significantly greater reliability. Furthermore, studies in 2024 revealed that the collimator hierarchy was impacted by the presence of the $3Q_y$ resonance [19]. A horizontal orbit offset in the presence of a skew octupole error generates a feed-down skew sextupole component, including the $3Q_y$ resonance. Consequently, the direct correction of this feed-down contribution is of particular interest for HL-LHC operation.

To investigate this effect, measurements of the $3Q_y$ resonance were performed as a function of the horizontal crossing angle at IP1 during the 2025 end-of-squeeze commissioning period. The data was subsequently analysed using the GUI, revealing a linear dependence on the crossing angle, as shown at the BPM-level in Fig. 3.

The observed linear behaviour indicates that the skew octupolar resonance is the dominant feed-down source. A correction was therefore derived using the only available corrector in IP1; the second unit is currently non-operational but is foreseen to be replaced for the HL-LHC. The lack of correctors on both sides of the interaction point precluded the simultaneous correction of LHCB1 and LHCB2, as demonstrated in Fig. 1. Consequently, LHCB1 was selected as the target beam for this test. Figure 4 shows that the measured response is well reproduced by a corrector strength within the maximum powering limits, confirming its suitability for machine implementation.

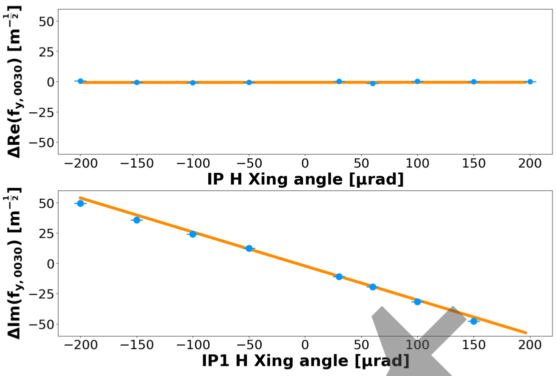

Figure 3: Shift in imaginary and real part of f_{0030} RDT vs horizontal IP1 crossing angles for an example LHCB1 BPM (line is a linear fit for the data).

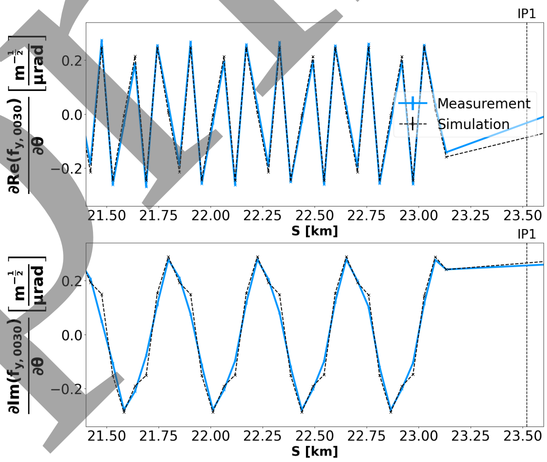

Figure 4: Demonstration of the matching of the partial derivatives vs s in simulation with those in measurement for LHCB1.

With the inclusion of the correction, a marked reduction in the variation of the f_{0030} RDT as a function of crossing angle was observed, as shown in Fig. 5. The correction was also found not to degrade the behaviour of LHCB2.

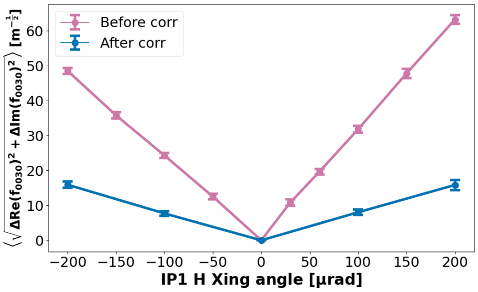

Figure 5: Average f_{0030} RDT amplitude shift vs horizontal IP1 crossing angle for LHCB1 (line is visual guide for trend).

Fortunately, the $Q_x - Q_y$ resonance, corresponding to the f_{1012} RDT, was also clearly visible in the measurements as demonstrated in Fig. 6.

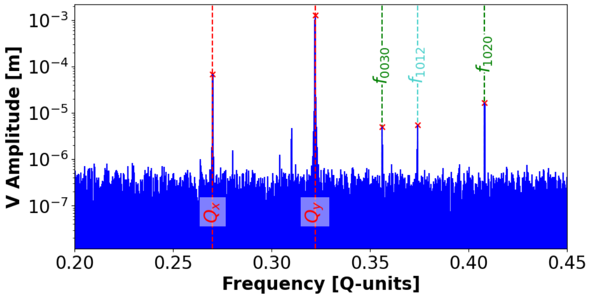

Figure 6: Turn-by-Turn frequency spectrum for LHCB1 showing the visibility of the f_{1012} RDT.

Figure 7 compares the conditions before and after the correction and demonstrates a clear reduction in the amplitude of the skew octupolar f_{1012} RDT.

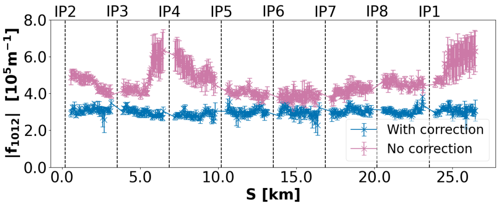

Figure 7: f_{1012} RDT amplitude vs s for LHCB1.

b_6 ERROR BENCHMARKING

Following its successful demonstration during the 2025 commissioning, the method was investigated for application to more challenging higher-order errors. The correction of dodecapolar components, while crucial, remains difficult in practice.

Dedicated machine development time was utilised to measure the dodecapolar error feed-down contribution to the decapolar RDT. For this purpose, the working point was positioned in close proximity to the decapole resonance. The procedure was analogous to that previously employed for the a_4 feed-down to a_3, with larger excitation amplitudes applied to enhance the visibility of the relevant spectral lines.

Analysis of the measurements showed that the normal decapolar resonance $-Q_x + 4Q_y$, associated with the RDT f_{0140}, is clearly visible in the Turn-by-Turn frequency spectra for different crossing angle configurations, where an example spectrum is shown in Fig. 8.

The f_{0140} RDT was observed to vary during a horizontal crossing angle scan at IP1. An example of its distribution around the ring for two crossing angle settings is shown in Fig. 9.

This behaviour exhibits an approximately linear dependence, as shown in Fig. 10. It is consistent with feed-down from dodecapolar field errors in the interaction region, providing a proof-of-principle for a correction scheme relevant to the HL-LHC within the present LHC.

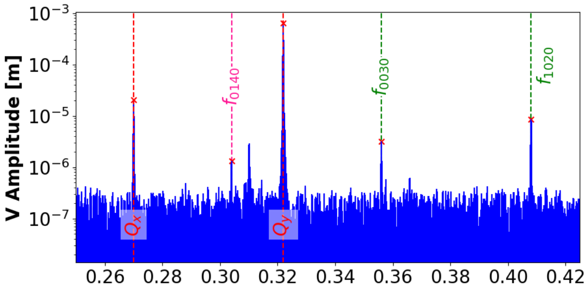

Figure 8: Turn-by-Turn frequency spectrum for LHCB1 showing the visibility of the f_{0140} RDT.

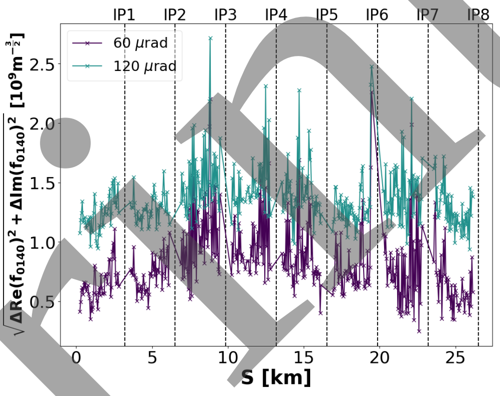

Figure 9: f_{0140} RDT amplitude vs s for LHCB1.

Tracking simulations were performed using the same configuration, with b_6 and b_7 WISE field errors [20] included on top of the baseline model to validate the predictions. The results indicate a dominant contribution from the b_6 components (Fig. 10), in good agreement with the measurements.

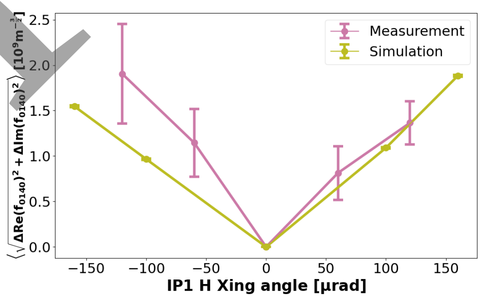

Figure 10: Average f_{0140} RDT amplitude shift vs horizontal IP1 crossing angle for LHCB2 (line is visual guide for trend).

CONCLUSION

A novel correction methodology for higher-order field errors in the LHC interaction regions was developed and validated through beam-based measurements. The approach enables identification and compensation of non-linear errors via feed-down mechanisms, particularly the skew octupolar resonance, without direct measurement of higher-order RDTs, while remaining compatible with routine operational time constraints. The measured dodecapole feed-down shows good agreement with simulations.

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