INFLUENCE OF SYNCHRO-BETATRON RESONANCE ON SLOW EXTRACTION OF HIGH-FREQUENCY BUNCHED BEAMS AT SIS18*

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Abstract

Measurements and simulations of the slow extraction of bunched particle beams via tune scan at the GSI heavy ion synchrotron SIS18 were performed to study the temporal structure of the spills. The arrival times of extracted single particles were recorded in relation to the bunch-forming 80 MHz rf signal. It is demonstrated that high-frequency bunching leads to an enlarged synchrotron tune and, hence, to isolated synchro-betatron resonances that are crossed during the tune scan. The resonances affect the temporal structure of the extracted beams on time scales ranging from nanoseconds to seconds.

BUNCHED BEAM SLOW EXTRACTION

Most users of the GSI synchrotron SIS18 demand resonant slow extraction with a typical duration between 1 and 10 s. The tune-scan method is performed by increasing the horizontal tune towards a 3rd integer resonance, i.e., $Q_x \to 4\frac{1}{3}$. For coasting beam extraction, pronounced fluctuations appear on time scales from 10 µs up to several ms. They are related to the current ripple of the quadrupole magnets [1–3]. These fluctuations are referred to as spill micro-structure. Several schemes for mitigating spill micro-structures on coasting beams were examined at GSI [2–6].

In addition, the spill micro-structure mitigation by creating stationary bunches inside the synchrotron has been investigated at GSI [5, 6] and is adopted in ion synchrotrons for cancer treatment; further literature is compiled in [7]. Commonly, acceleration and bunching are performed with ferrite-loaded cavities that provide a maximum rf of $f_{\mathrm{rf,max}} = 5.5$ MHz, corresponding to an rf harmonic number $h = 4$. The modifications of the spill micro-structure for such conditions were previously investigated through measurements and MAD-X simulations [8]. It was also shown experimentally that the ferrite cavity rf imprints bunch-synchronous time structures on the extracted beam of typically 10 ns duration and separated by the bunch period of 200 to 300 ns. These structures are referred to as spill nano-structures and, unfortunately, limit the applicability of the extracted beams for some fixed-target experiments.

In this work, we describe the temporal structure for high-frequency bunching with $h = 90$ in order to reduce the separation between the extracted bunches. The high harmonic numbers are achieved by installing a pillbox-like rf-cavity in SIS18, which provides a frequency range of 80.2 to 81.6 MHz. Tunability is required to match harmonics for the relevant extraction energies between 0.3 and 2 GeV/u. More cavity parameters are described in [9].

During extraction, the horizontal tune Q_x is scanned, and synchro-betatron resonances (SBR) [10–12] are crossed at the resonance condition

$$nQ_x + mQ_s = p \quad \text{with} \quad n, m, p \in \mathbb{Z} \setminus \{0\}\,. \tag{1}$$

where Q_s is low-amplitude synchrotron tune [10, 13]. SBRs are excited due to the coupling of horizontal and longitudinal particle motions by dispersion at the bunching cavity and sextupole fields, as well as chromaticity. For a circulating beam with velocity βc, mass Am_0c^2, ionic charge qe, and gap voltage U_{gap}, the low-amplitude synchrotron tune Q_s is

$$Q_s = \sqrt{\frac{1}{2\pi} \cdot \frac{qe}{Am_0c^2} \cdot \frac{|\eta|}{\beta^2\gamma} \cdot h \cdot U_{\mathrm{gap}}} \propto \sqrt{h \cdot U_{\mathrm{gap}}} \tag{2}$$

with $\eta = 1/\gamma_{\mathrm{tr}}^2 - 1/\gamma^2$ being the phase slip factor and β, γ being the Lorentz factors; $\gamma_{\mathrm{tr}} = 5.58$ is the transition energy. As the synchrotron tune Q_s increases, the spacing between SBRs becomes larger. For the investigated gap voltages, a fraction of particles is at the periphery or even outside the stationary bucket. Hence, the amplitude-dependent decrease of the synchrotron tune is quite relevant for resonant slow extraction.

MEASUREMENT SETUP AND ANALYSIS

In measurements with high-frequency bunching, a $^{14}N^{7+}$ beam was injected into SIS18 at 11.4 MeV/u via multi-turn injection, filling the horizontal acceptance, and was subsequently accelerated to the extraction energy of $E = 300$ MeV/u by the ferrite-loaded cavity operating at harmonics $h = 4$. After debunching, the beam was adiabatically rebunched in 0.25 s to build bunches at 81.44 MHz ($h = 90$). The gap voltage was varied between $U_{\mathrm{gap}} = 0.5$ and 13 kV; for comparison, a coasting beam was also measured. At $U_{\mathrm{gap}} = 2$ kV, $Q_s = 0.0038$; other values scale according to Eq. (2). An extraction duration of $t_{\mathrm{ex}} = 8$ s using a tune ramp $\Delta Q_x = 0.0061$ was chosen; the average extraction rate was $r_{\mathrm{av}} \approx 10^6$ s^{-1}. The extracted particles were detected using a BC400 plastic scintillator coupled to a photomultiplier. The particles' arrival times were recorded by a TDC (CAEN V1290N) [14]. The system's overall time resolution is better than 0.3 ns. The data are displayed using the extraction rate as a function of extraction time or relative to the zero-crossing of the bunching frequency.

For the micro-structure evaluation, time bins of $\Delta t = 10$ µs are generated and analysed using the average counts $\langle c\rangle^2(t_{\mathrm{ex}}) = \mu_c^2(t_{\mathrm{ex}})$ and the variance $\langle c^2\rangle(t_{\mathrm{ex}}) = \sigma_c^2(t_{\mathrm{ex}})$

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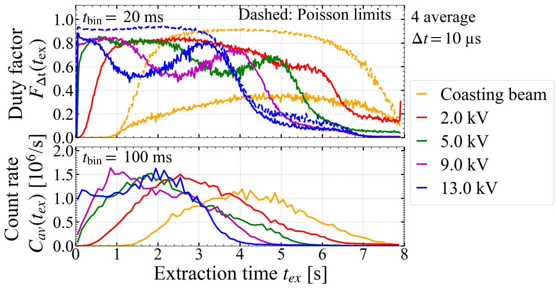

Figure 1: Measured duty factor (top) and count rate (bottom) for Δt=10 µs during extraction for different gap voltages, with evaluation interval of t_{bin} = 20 ms and 100 ms, respectively.

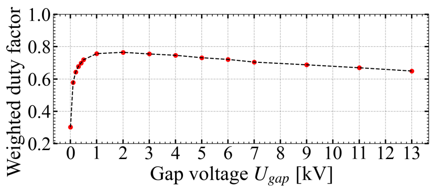

Figure 2: Weighted duty factor for different gap voltages.

within an interval $[t_{\mathrm{ex}}, t_{\mathrm{ex}} + t_{\mathrm{bin}}]$ of length t_{bin} = 20 ms. The common definition of duty factor as a function of extraction time $F_{\Delta t}(t_{\mathrm{ex}})$ and the general Poisson limit $F_{\mathrm{P}}(t_{\mathrm{ex}})$ [1] are

$$F_{\Delta t}(t_{\mathrm{ex}}) = \frac{\mu_c^2(t_{\mathrm{ex}})}{\mu_c^2(t_{\mathrm{ex}}) + \sigma_c^2(t_{\mathrm{ex}})} \text{ and } F_{\mathrm{P}}(t_{\mathrm{ex}}) = \frac{\mu_c(t_{\mathrm{ex}})}{\mu_c(t_{\mathrm{ex}}) + 1}. \tag{3}$$

For the entire spill characterisation, the weighted duty factor F_{w} is defined by a weighted average over one extraction:

$$F_{\mathrm{w}} = \frac{1}{\int \mu_c(t_{\mathrm{ex}})\,\mathrm{d}t_{\mathrm{ex}}} \int \mu_c(t_{\mathrm{ex}})\,F_{\Delta t}(t_{\mathrm{ex}})\,\mathrm{d}t_{\mathrm{ex}} \,. \tag{4}$$

MEASURED SPILL MICRO-STRUCTURE

Figure 1 displays the measured time-dependent duty factors and the corresponding count rates for several gap voltages. The first observation is that increasing the gap voltage leads to the earlier extraction of the beams. This arises from the larger momentum spread and the resulting chromatic horizontal tune spread after bunching. The starting value of the tune ramp was kept constant to prevent systematic uncertainties. In addition, the time-dependent duty factors found for bunched beams created with any gap voltage are significantly higher than those found for the coasting beam.

However, the duty factors vary during extraction, with a more pronounced dip appearing earlier at higher voltages. The dip position is related to the timing of the SBR crossing during the tune scan; see below.

Using the weighted duty factor for the entire spill characterisation, the improvements compared to the coasting beam are visualised in Fig. 2. The maximum is reached at a relatively low gap voltage of U_{gap} = 2 kV. It reflects a significant increase in the duty factor, low temporal variation, and less pronounced dips.

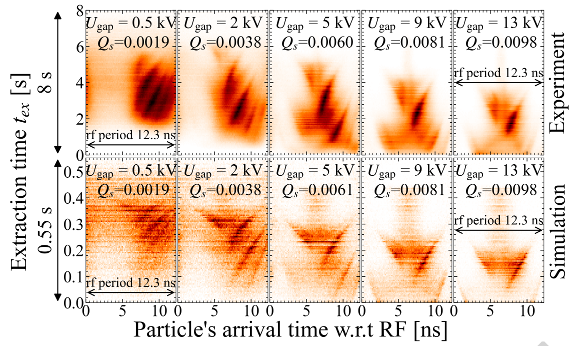

Figure 3: Particle arrival distributions with respect to the rf along the extraction time: measurements (upper row) and simulations (lower row).

MEASURED SPILL NANO-STRUCTURE

The upper row of Fig. 3 depicts the extracted particle arrival distributions at the scintillator, determined with respect to the bunching rf as a function of the extraction time. A noticeable variation in the bunch structure is observed, with separated islands of different centres and widths. The number of islands decreases with pronounced separation at higher gap voltages, i.e., larger low-amplitude synchrotron tunes. To our knowledge, this is the first report of such high-resolution measurements in the literature. According to the simulations depicted in the lower row of Fig. 3 and described below, the structure can be attributed to changes in the circulating beam properties during the SBR crossing and is correlated with variations in the duty factor. Further results are presented in [15, 16]. An additional series of measurements with beam energies ranging from 300 to 900 MeV/u revealed comparable variations in micro- and nano-structures.

SIMULATIONS AND INTERPRETATION

Extensive particle tracking simulations were performed using Xsuite [17], with realistic parameters compiled in Table 1. To improve simulation efficiency, the extraction duration was shortened to t_{ex} = 0.55 s. Realistic quadrupole power supply ripples are included based on previous investigations [2,3,16]. The simulation results depend on ripple amplitude, horizontal emittance, and initial momentum spread. The chosen values are comparable to the measurements.

Figure 4 shows the evolution of the horizontal profile of the circulating beam for different gap voltages. When the SBR condition is reached, the transverse distribution function is significantly modified. In particular, for the lowest order resonance $3Q_x + Q_s = 13$, a twofold reaction occurs: Firstly, a type of halo develops as a subset of particles moving to the periphery. Secondly, the concentration at the core increases. Accordingly, the standard deviation of the resulting non-Gaussian distribution increases, reflecting halo formation and particle extraction, followed by a strong reduction associated with the core concentration.

Figure 5 shows the temporal longitudinal profiles of the circulating bunches. At the resonance condition, the distri-

Table 1: Xsuite Simulation Parameters for SIS18

Parameter	Values
Ion, kin. energy [MeV/u]	$^{14}N^{7+}$, 300
Trans. emi. ε [mm·mrad]	2.5 (hor.), 2.0 (vert.)
$\Delta p/p$ before bunching	10^{-4}
Harmonic number h	90
Cavity voltage U_{gap} [kV]	0 – 13
Tune scan ΔQ_x, ramp	4.328 – 4.3345, linear
Extraction time [s]	0.55 (5×10^5 turns)
Norm. sextupole strength	0.04
Hor. chromaticity ξ_x	−6.45
Power supply ripples	Sine ($n \cdot$ 50Hz) + noise

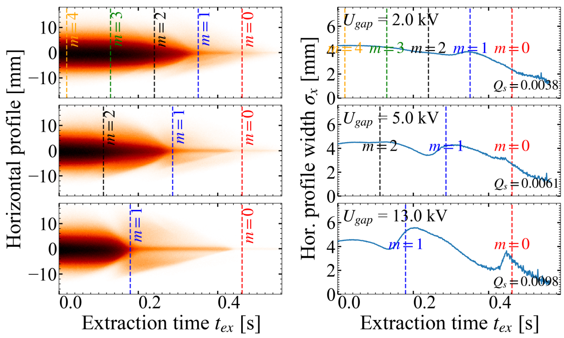

Figure 4: The horizontal profile evolution of the stored beam. The SBRs are marked by lines according to $3Q_x + mQ_s = 13$ using the low-amplitude Q_s.

bution undergoes significant modification. It is dominated by a decrease in bunch length. The formation of the longitudinal halo is less visible than that of the horizontal one. The temporal coincidence confirms the SBR coupling. The increase in bunch width at the end of the extraction is caused by a small fraction of unextracted particles.

Besides the SBR coupling, the evolution of the particle phase space is influenced by the extraction process, as halo particles are extracted rapidly due to larger betatron amplitudes and negative momentum offsets by chromatic coupling [1,3,5]. An application based on analytic results using the Hamiltonian formalism [10–12] may be hindered by proximity to a 3rd integer resonance and due to particle extraction. It might be a comparable situation to tune measurements near a slow extraction setting [18].

The simulated time-dependent duty factors are depicted in Fig. 6. In agreement with the measurements, it increases near the SBR crossing, manifesting as "bumps" at the SBR conditions and "dips" in between. With lower particle statistics in simulations compared to measurements, the increase in the Poisson limits appears near the SBR crossing. This effect is also illustrated by the variation in the count rates; see the lower subfigure. However, it is observable that the difference between the duty factors and the corresponding Poisson limits varies. This confirms that spill quality is affected by the dynamical redistribution of stored particles during the SBR crossing. The extracted temporal structure is related to non-linearities in both horizontal and longitudinal motion. In particular, particles with larger horizontal and longitudinal amplitudes conduct shorter transit times, leading to a reduced temporal quality [1,3–5]. Therefore, high-frequency rf bunching does not uniformly suppress the spill micro-structures during the entire extraction process.

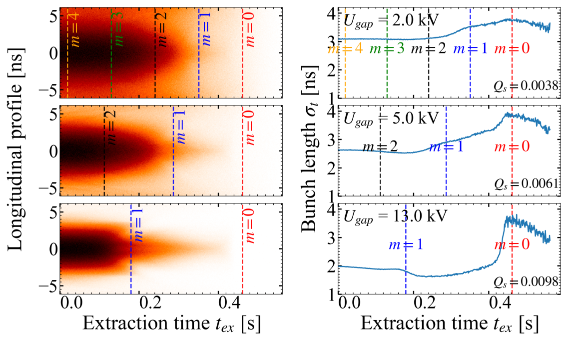

Figure 5: The longitudinal bunch profile evolution of the stored beam. The SBR lines are indicated.

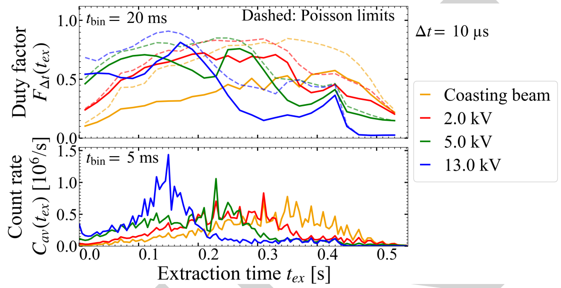

Figure 6: Simulated duty factor (top) and count rate (bottom) for Δt=10 µs during extraction for different gap voltages, with evaluation interval of t_{bin} = 20 ms and 5 ms respectively.

The nano-structure of the extracted bunches for different gap voltages is displayed in Fig. 3. Both centre variation and island structure resemble those found in the measurements. This confirms the correctness of the simulation and interpretation. More simulation outcomes are available [15, 16].

CONCLUSION

The spill smoothness in tune-scan slow extraction can be significantly improved with high-frequency bunching. Systematic measurements with high temporal resolution were performed. High-frequency bunching can lead to an enlarged synchrotron tune and, hence, to isolated synchro-betatron resonances that are crossed during the tune scan. The resonances modify the circulating beam's horizontal and longitudinal phase space, as verified by simulations. As a relevant measurable result, the micro- and nano-structure is systematically modified by SBRs. For low-frequency bunching, similar modifications were detected [15, 16]. However, due to decreased distances between the SBRs lines, the effect is less pronounced. Moreover, the visibility depends on the initial horizontal and longitudinal emittance.

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